Problems

Let us look to solve the problem based on the given statement that astrain induced in an M.S bar of rectangular section having width equal to twice the depth is 2.5 × 10–5. The bar is subjected to a tensile load of 4 KN. And let us find the section dimensions of the bar. Take E = 0.2×106 N/mm2.

Given:b = 2d

Solution:

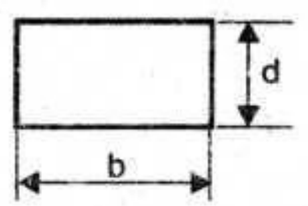
Strain, $e = 2.5 \times 10^{-5}$ $P = 4 \times 10^{3}$ N,

$E = 0.2 \times 10^{6}$ N/mm^2.

Stress, $\sigma = e\,E$

$= 2.5 \times 10^{-5} \times 0.2 \times 10^{-6}$

$= 5$ N/mm^2



$$\text{Stress} = \frac{P}{\text{Area of cross section}} \quad \frac{P}{b \times d}$$

$$\text{or} \qquad 5 = \frac{4 \times 10^3}{(2d \times d)} \qquad (\because b = 2d)$$

$$\text{or} \qquad 12d^2 = \frac{4 \times 10^3}{5} = 800$$

Solving, d = 20 mm

breadth, b= 2d = 2 × 20 = 40 mm.

Here we discuss strain problem with the given problem statement that asquare steel rod 20 mm×20 mm in section is to carry an axial load (compression) of 100 KN. Calculate the shortening in a length of 50 mm.Take E = 2.14×10^8 KN/m^2.

GivenA = 20 × 20 = 400 mm^2,P = 100 × 10^3 N

l = 50 mm,

E = 2.14 × 10^8 KN/m^2 = 2.14 × 10^5 N/mm^2.

δl = ?

Solution:

Change in length, $\delta \ell = \frac{P\ell}{AE} = \frac{100 \times 10^3 \times 50}{400 \times 2.14 \times 10^5}$

= 0.0584 mm

Let us solve the stress and strain problem with the given problem statement that a bar of 30 mm diameter is subjected to a pull of 60 KN. The measured extension on gauge length of 200 mm is 0. 1 mm and change in diameter is 0.004 mm. Calculate Young's modulus.

Given,d = 30 mm,P = 60 × 10^3 N, 1 = 200 mm

δl = 0.1 mm,δd = 0.004 mm.E = ?.

Solution:

Strain, $e = \frac{\delta \ell}{\ell} = \frac{0.1}{200}$

Stress, $\sigma = \frac{P}{A} = \frac{60 \times 10^3}{\left(\frac{\pi}{4} \times 30^2\right)}$

Young's Modulus, $E = \sigma/_e = \left(\frac{60 \times 10^3 \times 4}{\pi \times 30^2}\right) \div \left(\frac{0.1}{200}\right)$

$$= \frac{60 \times 10^3 \times 4}{\pi \times 30^2} \times \frac{200}{0.1}$$

= 169765 N/mm^2 ***(Ans)***

Let us see the strain problem with the given problem statement to calculate the instantaneous stress produced in a bar of cross sectional area 1000 mm2 and 3m long by the sudden application of a tensile load of unknown magnitude. if the instantaneous extension is 1.5 mm. Also find the corresponding load. Take E = 200 G pa.

Given,A = 1000 mm^2

L = 3m = 3000 mm

δl = 1.5 mm

E = 200 × 10^3 N/mm^2

σ = ?P = ?

Solution:

Using the equation, $\delta \ell = \frac{P\ell}{AE}$

or $$\delta \ell = \frac{\sigma \ell}{E} \qquad (\because P/A = \sigma)$$

$$1.5 = \frac{\sigma \times 3000}{200 \times 10^3}$$

Solvingσ = 100 N/mm^2 (Ans)

$$\begin{bmatrix}\text{Stress due to suddenly} \\ \text{applied load}\end{bmatrix} = \begin{bmatrix}2 \times \text{Stress due to gradually} \\ \text{applied load}\end{bmatrix}$$

$$\text{i.e., } \sigma = \frac{2P}{A}$$

$\therefore \quad 100 = \frac{2P}{1000}$; solving, P = 50 × 10^3 N

= 50 KN

We will discuss the problem of poisson's ratio and young's modulus with the given problem statement that the following data relate to a bar subjected to a tensile test.Diameter of the bar, d = 30 mm.Tensile load, P = 54 KN,Gauge length, l = 300 mm,Extension of the bar, dl = 0.112 mm,change in diameter, dd = 0.00366 mm. Calculate (i) the Poisson's ratio (ii) the values of Bulk, Young's and Shear Modulus.

Solution:

Direct stress, $\sigma = P/A = \frac{54 \times 10^3}{\frac{\pi}{4} \times 30^2}$

= 76.39 N/mm^2

Longitudinal strain, $e = \frac{\delta l}{l} = \frac{0.112}{300}$

= 3.73×10^{-4}

∴ Young's Modulus, $E = \frac{\sigma}{e} = \frac{76.39}{3.73 \times 10^{-4}}$ = 2.047×10^5 N/mm^2

Lateral strain $= \frac{\delta d}{d} = \frac{0.00366}{30}$ = 1.22×10^{-4}

∴Poisson's ratio, $\frac{1}{m} = \frac{\text{Lateral Strain}}{\text{Longitudinal Strain}}$

$$= \frac{1.22 \times 10^{-4}}{3.73 \times 10^{-4}} = 0.327 \text{ (Ams).}$$

To find Bulk Modulus

$$E = 3K\left(1 - \frac{2}{m}\right)$$

or$2.047 \times 10^5 = 3K \{1 - (2 \times 0.327)\}$

Solving, $K = 1.972 \times 10^5$ N/mm^2

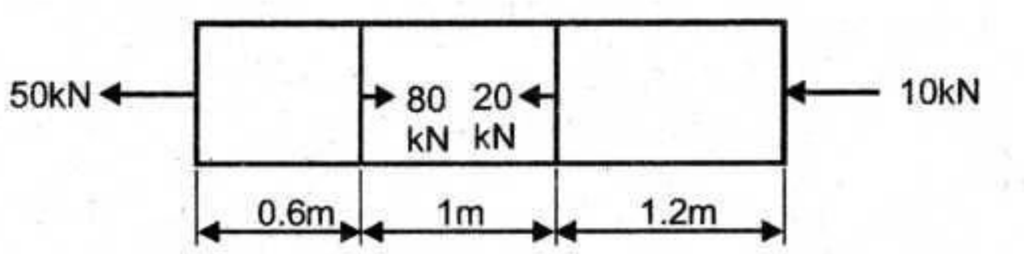
To find Shear Modulus

$$E = 2C\left(1 + \frac{1}{m}\right)$$

or $2.047 \times 10^5 = 2C(l + 0.327)$

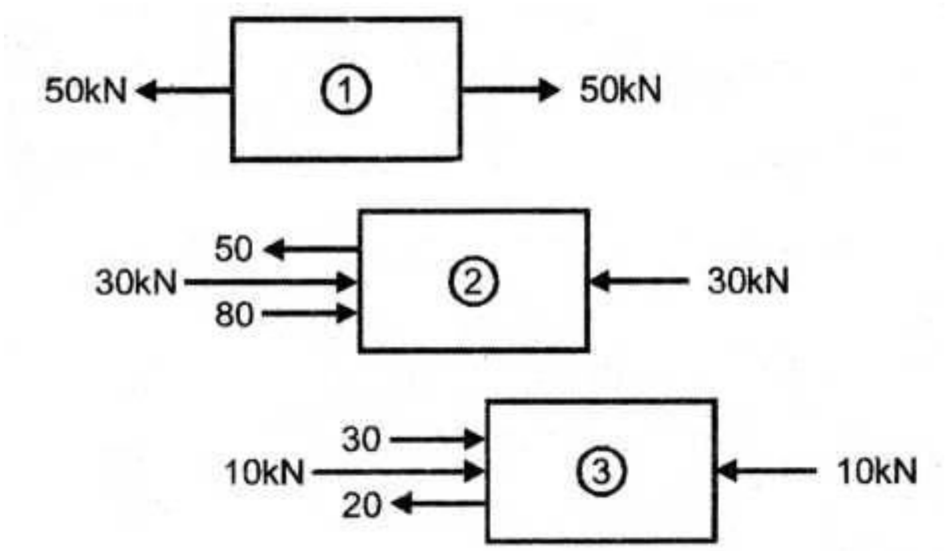
Solving, $C = 0.771 \times 10^5$ N/mm^2

Let us discuss Elongation problem **with the given problem statement that a brass bar having cross sectional area of 1000 mm2 is subjected to axial forces as shown in figure below. Find the elongation of the bar. Take $E = 1.05 \times 10^5$**



Solution:

The forces acting on the given bar is taken as below.



Total elongation of the bar, $\delta 1 = \delta 1_1 \pm \delta 1_2 \pm \delta 1_3$

$$\therefore\ \delta l = \frac{P_1 l_1}{AE} - \frac{P_2 l_2}{AE} - \frac{P_3 l_3}{AE}$$ (– for contraction and + for elongation)

$$= \frac{1}{AE}(P_1 l_1 - P_2 l_2 - P_3 l_3)$$ (A, E constant)

$$= \frac{1}{1000 \times 1.05 \times 10^5}\left(\begin{matrix}(50 \times 10^3 \times 600) - (30 \times 10^3 \times 1000) - \\ (10 \times 10^3 \times 1200)\end{matrix}\right)$$

= – 0.1l4 mm

= 0.114 (Contraction).

Let's consider the problem that a compound tube consists of a steel tube of 140 mm internal diameter and 5 mm thickness and an outer brass tube of 150 mm internal diameter and 5 mm thick. The two tubes are of same length. Compound tube carries an axial load of 600 KN. Find the stresses carried by each tube and amount of shortening. Length of the tube is 120 mm.Es = 2 × 10^5 N/mm^2 and Eb = 1 × 10^5 N/mm^2.

Given :

Steel tubeBrass tube

d_i = 140 mmd_i = 150 mm

d_o = 150 mmd_o = 160 mm

Given,**P** = 600 × 10^3 N

E_s = 2 × 10^5 N/mm^2

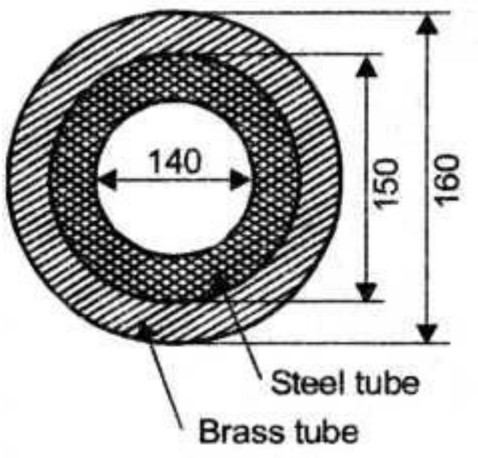
E_b = 1 × 10^5 N/mm^2

$1_s = 1_b = 120$ mm

σ_S = ?

σ_b = ?

Solution:



Area of cross section of steel tube,

$$A_s = \frac{\pi}{4}(150^2 - 140^2) = 2276 \text{ mm}^2$$

Area of cross section of Brass tube,

$$A_b = \frac{\pi}{4}\left(160^2 - 150^2\right) = 2433 \text{ mm}^2$$

Using $\delta l_s = \delta l_b$

i.e., $$\frac{P_s\, l_s}{A_s\, E_s} = \frac{P_b\, l_b}{A_b\, E_b}$$

or $$\frac{P_s \times 120}{2276 \times 2 \times 10^5} = \frac{P_b \times 120}{2433 \times 1 \times 10^5}$$

or $$P_s = \left(\frac{2276 \times 2 \times 10^5}{2433 \times 1 \times 10^5}\right) P_b$$

$$\boxed{P_s = 1.87\, P_b}$$

UsingP = $P_s + P_b$

or600 × $10^3 = (1.87\, P_b) + P_b$

Solving, $P_b = 209.06 \times 10^3$ N

∴$P_s = (1.87 \times 209.06 \times 10^3)$

$= 390.94 \times 10^3$ N

∴ Stress in steel tube, $\sigma_s = \frac{P_s}{A_s}$

$$= \frac{390.94 \times 10^3}{2276}$$ *= 171.7 7 N/mm²*

Stress in brass tube, $\sigma_b = \frac{P_b}{A_b}$

$$= \frac{209.06 \times 10^3}{2433}$$ = 85.93 N/mm^2

Shortening length of bar,

$$\delta l = \frac{P_s l_s}{A_s E_s} \quad \left(\text{or } \frac{P_b l_b}{A_b E_b} \right)$$

$$= \frac{390.94 \times 10^3 \times 120}{2276 \times 2 \times 10^5}$$

= 0.103 mm.

Let us discuss about compressive stress problem with the given problem statement that a reinforced short concrete column 400 mm × 400mm in section is reinforced by 4 longitudinal 50 mm diameter round steel bars placed at each corner. If the column carries a compressive load of 300 KN, and Young's modulus of elasticity of steel is 15 times that of concrete, determine i) Load carried and ii)The compressive stress produced in the concrete and steel bars.

Given, $A_s = 4 \times \frac{\pi}{4} \times 50^2$ = 7854 mm^2

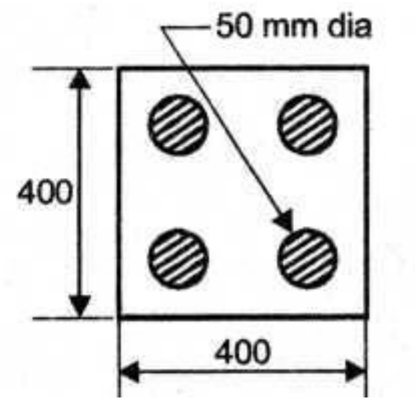
$A_c = A - A_s$

= 400^2 – 7854 = 152146 mm^2

P = 300 × 10^{3}N

$E_s = 15\ E_e$, P_s = ? P_c = ? σ_s = ? σ_c = ?.

Solution:



Using the condition

Strain in steel = Strain in concrete

i.e., $e_s = e_c$ $\left(\because e_s = \sigma / E\right)$

or $$\frac{\sigma_S}{E_S} = \frac{\sigma_C}{E_C}$$

or $$\frac{\sigma_S}{15E_C} = \frac{\sigma_C}{E_C}$$

($\because E_S = 15\ E_C$)

or $\sigma_S = 15\sigma_C$

Using the condition,

Total load on column = Load on steel + Load on concrete

i.e., $P = \sigma_S\ A_S + \sigma_C A_C$

or $300 \times 10^3 = (15\sigma_C \times 7854) + (\sigma_C \times 152146)$

Solving, $$\sigma_C = \frac{300 \times 10^3}{269956}$$

$= 1.11\ N/mm^2$ (Ans)

∴ Stress in steel, $\sigma_S = 15\sigma_C$

$= 15 \times 1.11 = 16.67\ N/mm^2$ (Ans)

Load carried by steel, $P_S = \sigma_s A_S$

$= 16.67 \times 7854$

$= 130.93$ KN (Ans)

Load carried by concrete,

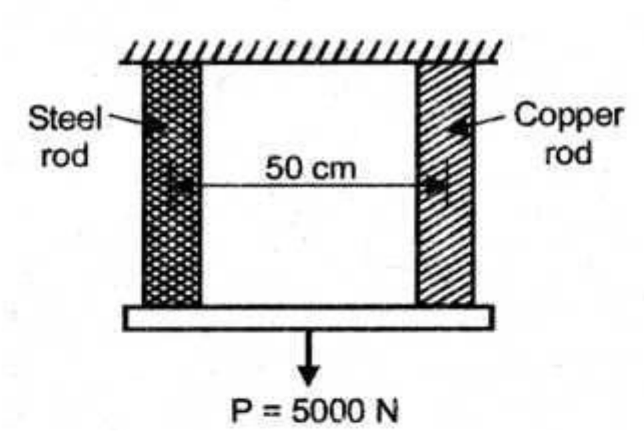
$P_C = \sigma_C A_C$

$= 1.11 \times 152146$

$= 168.89$ KN

let us look at another stress problem with the given problem statement that two vertical rods one of steel and the other of copper are each rigidly fixed at the top and 50 cm apart. Diameters and length of each rod are 2 cm and 4 cm respectively. A cross bar fixed to the rods at the lower ends carries a load of 5000 N such that the cross bar remains horizontal even after loading. Find the stress in each rod and the position of the load on the bar, Take E for steel = 2 × 105 N/mm2 and E for copper = 1 × 105 N/mm2.

Given :



Steel rodCopper rod

d_s = 2cm = 20 mmd_c = 20 mm

l_S = 4m = 4000 mml_C = 4000 mm

σ_S = ?σ_C=?

$E_S = 2 \times 10^5 N/mm^2 E_c = 1 \times 10^5 N/mm^2$

Solution:

Using the condition

Strain in steel rod = Strain in copper rod

i.e.,$e_S = e_C$

or $$\frac{\sigma_S}{E_S} = \frac{\sigma_C}{E_C}$$

or $$\frac{\sigma_S}{2 \times 10^5} = \frac{\sigma_C}{1 \times 10^5} \text{ or } \boxed{\sigma_S = 2\sigma_C}$$

Using the condition,

Total load = Load on steel + Load on copper

i.e.,**$P = P_S + P_C$**

$5000 = (\sigma_S A_S) + (\sigma_C A_C)$

$$\begin{aligned} A_C &= A_S \\ &= \pi/4 \times 20^2 \\ &= 314 \text{ mm}^2 \end{aligned}$$

$= (\sigma_S \times 314) + (\sigma_C \times 314)$

$5000 = (2\sigma_C \times 314) + (\sigma_C \times 314)(\sigma_S = 2\sigma_c)$

Solving, $\sigma_C = 5.31$ N/mm^2 (Ans)

∴ Stress in copper, $\sigma_S = 2\sigma_C$

$= 2 \times 5.31 = 10.62$ N/mm^2 (Ans)

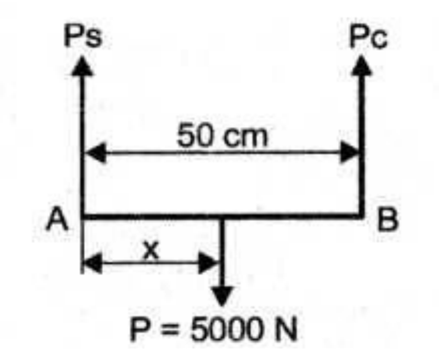
∴ Load on steel, $P_S = \sigma_S A_S$

$= 10.62 \times 314 = 3334$ N

Load on copper, $P_C = \sigma_C A_C$

= 5.31 × 314 = 1666 N

Position of the load to keep the cross bar remains horizontal



AB is the crossbar. P_S and P_C are the loads taken by steel rod and copper rod respectively, at a distance of 50 cm.

Let P is the load applied at a distance of x cm from A (i.e., from the steel rod) as shown in figure.

Applying the equilibrium equation at A,

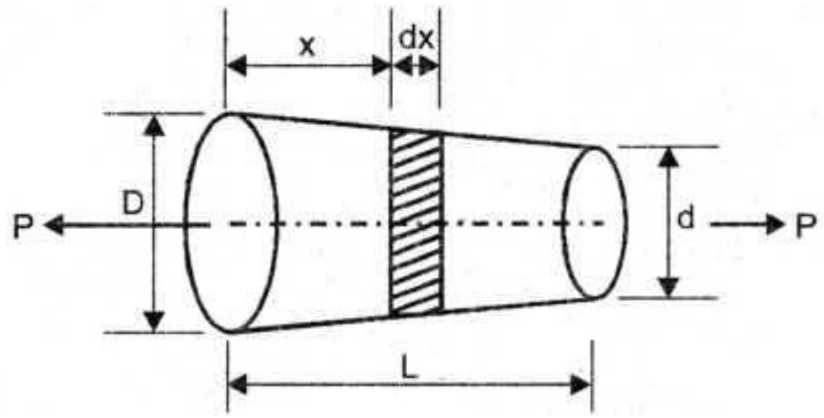
$\Sigma M_A = 0 \; (\circlearrowleft +)$

$5000 \times x = P_C \times 50$

$5000 \times x = 1666 \times 50$ ($\because P_C = 1666$ N) Solving, $x = 16.67$ cm

Here we learn Elongation problem with the given statement that aTapered circular bar tapers uniformly from a diameter 'd' at its small end to D at its big end. The length of the bar is 'L'. Derive an expression for the elongation of the bar due to an axial tensile force 'P'.

Solution:



The tapered rod is shown in Figure.

Consider a small element of length 'dx' at a distance x from the left end.

Diameter of the bar at the element, Dx $= D - \left(\frac{D-d}{L}\right)$

orD_x = D – kxwhere $k = \frac{D-d}{L}$

Area of cross section at the element,

$$A_x = \frac{\pi}{4}(D_x)^2 = \frac{\pi}{4}(D-kx)^2$$

Stress at the section, $\sigma_x = \frac{\text{Load}}{A_x}$

$$\therefore \sigma_x = \frac{P}{\frac{\pi}{4}(D - kx)^2} = \frac{4P}{\pi(D - kx)^2}$$

Strain in the element, $e_x = \frac{\text{Stress}}{E}$

$$\therefore e_x = \frac{4P}{\pi E(D - kx)^2} \times \frac{1}{E}$$

$$= \frac{4P}{\pi E(D - kx)^2}$$

Extension of the element, = Strain × Original length

= e_x × dx

$$\text{or} \quad (\delta l)_x = \frac{4P}{\pi E(D - kx)^2}.dx \qquad ....(1)$$

To find the total elongation of the bar integrate the equation (1) between the limits 0 and L,

∴ Total elongation,

$$\delta \ell = \int_0^L \frac{4P}{\pi E(D - kx)^2} dx$$

$$= \frac{4P}{\pi E} \int_0^L (D - kx)^{-2} dx$$

Integrating and substituting $k = \frac{D - d}{L}$

We get $\delta\ell = \frac{4PL}{\pi EDd}$