Stresses on inclined planes

The following figure shows the various stresses that acts on a inclined plane

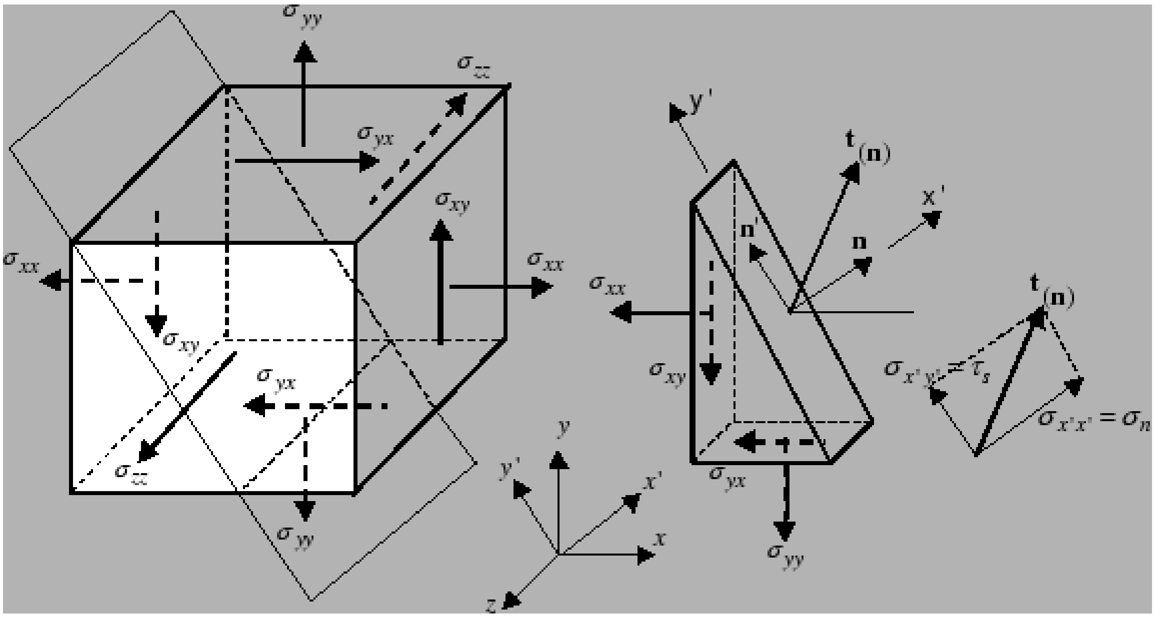



Figure:Stress on inclined planes

**Sign conventions:**

Normal stresses $\sigma_\theta$ positive for tension.

Shear stresses $\tau_\theta$ positive when they tend to produce counterclockwise rotation of the material.

**Let us consider at a point in a strained material, there is a horizontal tensile stress of 100 N/mm$^2$ and an unknown vertical stress. There is also a shear stress of 30 N/mm$^2$ on these planes. On a plane inclined at 30° to the vertical, the normal stress is found to be 90 N/mm$^2$ tensile. Find the unknown vertical stress and also principal stresses and maximum shear stress.**

**Given :**

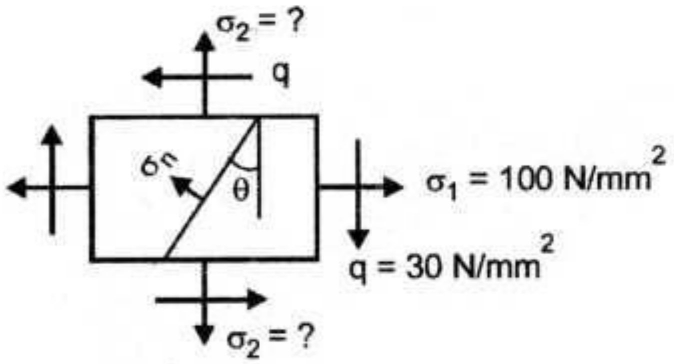
$\sigma_1$ = 100 N/mm$^2$$\sigma_2$ = ?q = 30 N/mm$^2$

$\theta = 30° \sigma_n = 90\ N/mm^2 \sigma_{P1} = ?$

**$\sigma_{P2} = ? (\sigma_t)_{max} = ?$**

To find: unknown vertical stress, ($\sigma_2$)

## Solution:




It is given,

$\sigma_n = 90\ N/mm^2$ at $\theta = 30°$

Normal stress $\sigma_n$ on any plane $\theta$ is given by,

$$\sigma_n = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2}\cos 2\theta + q \sin 2\theta$$

$$\therefore\ 90 = \frac{100 + \sigma_2}{2} + \frac{100 - \sigma_2}{2}\cos 60 + 30 \sin 60 \qquad (\because \theta = 30)$$

$$\text{or } 64 = \frac{100 + \sigma_2}{2} + \left(\frac{100 - \sigma_2}{2} \times \frac{1}{2}\right) = \frac{100 + \sigma_2}{2} + \frac{100 - \sigma_2}{4}$$

$$\text{or } 64 = \frac{(200 + 2\sigma_2) + (100 - \sigma_2)}{4}$$

or $256 = 300 + \sigma_2$

Solving, $\sigma_2 = -44$ N/mm$^2$ (compressive)

Tangential Stress

Tangential stress, $$\sigma_t = \frac{\sigma_1 - \sigma_2}{2}\sin 2\theta - q\cos 2\theta$$

$$= \frac{100-(-44)}{2}\sin 60 - 30\cos 60$$

$$= \frac{144}{2}\sin 60 - 30\cos 60$$

= 62.35 – 15 = 47.35 N/mm$^2$

**Principal Stresses**

Major principal stress,

$$\sigma_{p1} = \frac{\sigma_1 + \sigma_2}{2} + \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + q^2}$$

$$= \frac{100 + (-44)}{2} + \sqrt{\left(\frac{100-(-44)}{2}\right)^2 + 30^2}$$

$$= 28 + \sqrt{5184 + 900} = 106 \text{ N/mm}^2$$

Minor principal stress,

$$\sigma_{p2} = \frac{\sigma_1 + \sigma_2}{2} - \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + q^2}$$

$$= 28 - \sqrt{5184 + 900} = -50 \text{ N/mm}^2$$

**Maximum shear stresses**

Maximum shear stress,

$$(\sigma_t)_{max} = \frac{1}{2}\sqrt{(\sigma_1 - \sigma_2)^2 + 4q^2}$$

$$= \frac{1}{2}\sqrt{\{100 - (-44)\}^2 + 4 \times 30^2} = 73.54 \text{ N/mm}^2$$