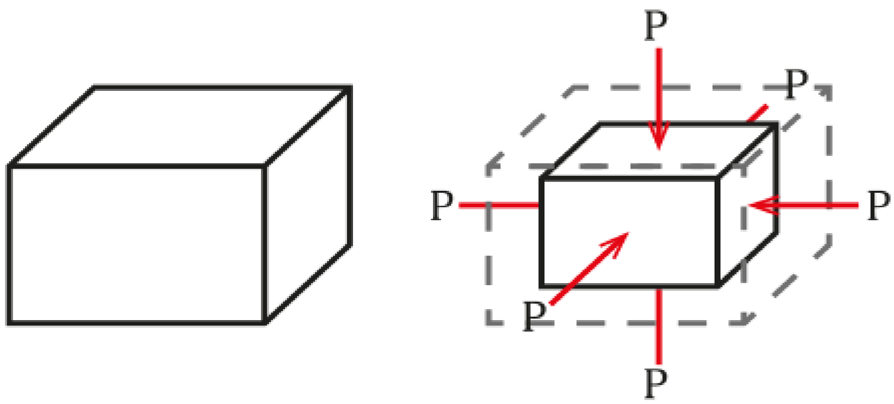
Volumetric stress and volumetric strain:



If the forces acting on an object deform it in such a way that there is a change to the volume of that object, then we are talking about volumetric stress.

Volumetric Stress is equal to the following pressure:

$$\text{Volumetric Stress} = \frac{\text{Load}}{\text{Area}} = \text{Pressure} = \text{dP}$$

- ## Volumetric Strain:

If the load applied cause volume change then the strain is called **volumetric strain** ,When there is volumetric stress, volumetric deformation or volume strain changes the volume of the body. Mathematically, we define that change as:

$$\text{Volumetric Strain} = -\frac{\text{Change in Volume}}{\text{Original Volume}} = \frac{dV}{V}$$

Similar to Tensile Strain, Volumetric Strain also has **no units**.

- ## Bulk's Modulus of Elasticity:

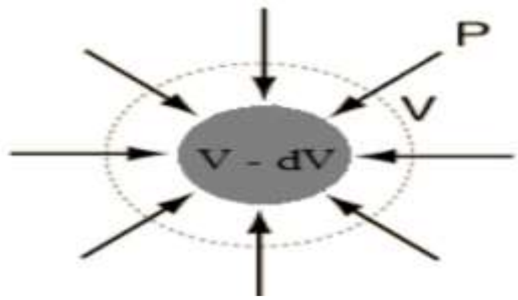
Bulk's Modulus is a numerical constant that describes the elastic properties of a solid or fluid when it is under pressure on all surfaces. The applied pressure reduces the volume of the material.

Mathematically, Bulk's Modulus is defined as:

$$\text{Bulk's Modulus} = \frac{\text{Pressure}}{\text{Strain}} = \frac{\Delta P}{\Delta V/V} = B$$

- B = Bulk modulus in N/m^2 or Pa
- ΔP = Change of the pressure that applied on the material
- ΔV = Change of the volume of the material
- V = Initial volume of the material

When the value is independent of pressure, this equation is basically a specific form of Hooke's law of elasticity.



Bulk modulus of elasticity:

Denoted by "K", so that its constant through the elastic limit and its equal to:

$$K = \frac{\text{Volumetric stress}}{\text{Volumetric strain}}$$

$$K = -\frac{dP}{\frac{dV}{V}} = -V\frac{dP}{dV}$$

Negative sign shows decrease in volume.

Characteristics of Bulk Modulus of Elasticity:

- Within the elastic limit, it is the ratio of volumetric stress to volumetric strain.
- It is associated with the change in the volume of a body.
- It exists in solids, liquids, and gases.
- It determines how much the body will compress under a given amount of external pressure.
- The bulk modulus of a material of a body is given by

Compressibility:

The reciprocal of bulk modulus of elasticity is called as compressibility. Mathematically

$$\text{Compressibility} = 1 / K$$

Its S.I. unit is $m^2 N^{-1}$ or Pa-1 and its dimensions are $[L^{-1}M^{-1}T^2]$.

Example – 1:

A solid rubber ball has its volume reduced by 14.5% when subjected to uniform stress of 1.45×10^4 N/m². Find the bulk modulus for rubber.

Given: Volumetric strain = 14.5 % = 14.5×10^{-2}, Volumetric stress = 1.45×10^4 N/m²,

To Find: Bulk modulus of elasticity =?

Solution:

Bulk modulus of elasticity = K = Volumetric stress / Volumetric strain

$$\therefore\ K = (1.45 \times 10^4) / (14.5 \times 10^{-2}) = 10^5 \text{ N/m}^2$$

Ans: Bulk modulus of elasticity of rubber is 10^5 N/m²

Example 2:

What pressure should be applied to a lead block to reduce its volume by 10% Bulk modulus for lead = 6×10^9 N/m²?

Given: Volumetric strain = 10 % = 10×10^{-2} , Bulk modulus of elasticity = 6×10^9 N/m².

To Find: Pressure intensity =?

Solution:

Bulk modulus of elasticity = K = Volumetric stress / Volumetric strain

$$\therefore \text{Volumetric stress} = K \times \text{Volumetric strain}$$

$$\therefore \text{Pressure intensity} = K \times \text{Volumetric strain}$$

$$\therefore \text{Pressure intensity} = 6 \times 10^9 \times 10 \times 10^{-2}$$

$$\therefore \text{Pressure intensity} = 6 \times 10^8 \text{ N/m}^2$$

Ans: Pressure intensity is 6×10^8 N/m²

Example 3:

A volume of 5 litres of water is compressed by a pressure of 20 atmospheres. If the bulk modulus of water is 20×10^8 N/m². , find the change produced in the volume of water. Density of Mercury = 13,600 kg/m³; g = 9.8 m/s². Normal atmospheric pressure = 75 cm of mercury.

Given: Original Volume = 5 L = 5×10^{-3} m³, Pressure = dP = 20 atm = $20 \times 75 \times 10^{-2} \times 13600 \times 9.8$ N/m², Bulk modulus of elasticity of water = 20×10^8 N/m².

To Find: Change in volume = dV =?

Solution:

$$\text{Volumetric Stress} = \text{Pressure intensity} = dP$$

$$\text{Bulk modulus of elasticity} = K = (dP \times V)/\, dV$$

$$\therefore \text{Change in volume} = dV = (dP \times V)/\, K$$

$$\therefore \quad dV = 5 \times 10^{-6}\ \text{m}^3 = 5\ \text{cc}$$

EX.4:

A volume of 10^{-3} m³ of water is subjected to a pressure of 10 atmospheres. The change in volume is 10^{-6} m³. Find the bulk modulus of water. Atm. pressure = 10^5 N/m².

Given: Original Volume = 10^{-3} m³, Pressure = dP = 10 atm = $10 \times 76 \times 10^{-2} \times 13600 \times 9.8$ N/m², Change in volume = dV = 10^{-6} m³,

To Find: Bulk modulus of elasticity of water =?

Solution:

$$\text{Volumetric Stress} = \text{Pressure intensity} = dP$$

$$\text{Bulk modulus of elasticity} = K = (dP \times V)/\ dV$$

$$\therefore K = (10 \times 76 \times 10^{-2} \times 13600 \times 9.8 \times 10^{-3})/\ 10^{-6}$$

$$\therefore K = 1.01 \times 10^{9}\ \text{N/m}^2$$

Ans: Bulk modulus of elasticity of water is 1.01×10^9 N/m²

Example 5:

Find the increase in the pressure required to decrease volume of mercury by 0.001%. Bulk modulus of mercury = 2.8×10^{10} N/m².

Given: Volumetric strain = 0.001% = $0.001 \times 10^{-2} = 10^{-5}$, Bulk modulus of elasticity = 2.8×10^{10} N/m².

To Find: Pressure intensity =?

Solution:

Bulk modulus of elasticity = K = Volumetric stress / Volumetric strain

$$\therefore \text{Volumetric stress} = K \times \text{Volumetric strain}$$

$$\therefore \text{Pressure intensity} = K \times \text{Volumetric strain}$$

$$\therefore \text{Pressure intensity} = 2.8 \times 10^{10} \times 10^{-5}$$

$$\therefore \text{Pressure intensity} = 2.8 \times 10^{5}\ \text{N/m}^2$$

Ans: Pressure intensity is 2.8×10^{5} N/m²