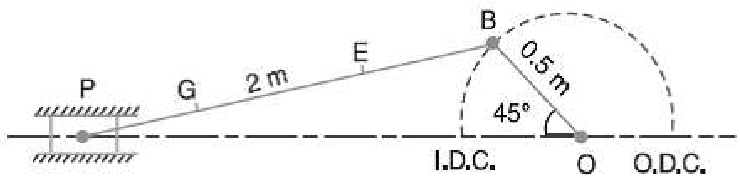
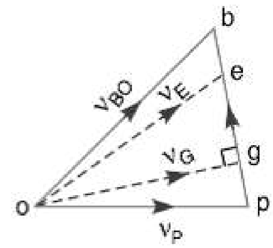


(a) Space diagram.



(b) Velocity diagram.

$$\text{vector } oa = v_{AO} = v_A = 1.76 \text{ m/s}$$

$$v_D = \text{vector } od = 1.6 \text{ m/s}$$

$$v_{DB} = \text{vector } bd = 1.7 \text{ m/s}$$

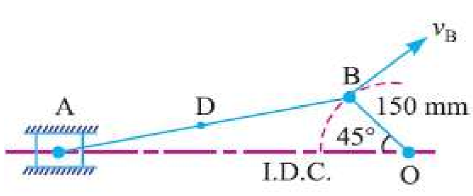
$$\omega_{BD} = \frac{v_{DB}}{BD} = \frac{1.7}{0.046} = 36.96 \text{ rad/s (Clockwise about } B)$$

Example *The crank of a slider crank mechanism rotates clockwise at a constant speed of 300 r.p.m. The crank is 150 mm and the connecting rod is 600 mm long. Determine : 1. linear velocity and acceleration of the midpoint of the connecting rod, and 2. angular velocity and angular acceleration of the connecting rod, at a crank angle of 45° from inner dead centre position.*

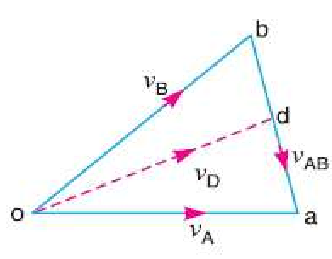
Solution. Given : N_{BO} = 300 r.p.m. or $\omega_{BO} = 2\pi \times 300/60 = 31.42$ rad/s; OB = 150 mm 0.15 m ; BA = 600 mm = 0.6 m

We know that linear velocity of B with respect to O or velocity of B,

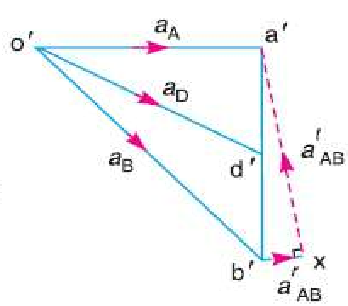
$$v_{BO} = v_B = \omega_{BO} \times OB = 31.42 \times 0.15 = 4.713 \text{ m/s}$$



(a) Space diagram.



(b) Velocity diagram.



(c) Acceleration diagram.

$$\text{vector } ob = v_{BO} = v_B = 4.713 \text{ m/s}$$

$$v_{AB} = \text{vector } ba = 3.4 \text{ m/s}$$

$$\text{Velocity of } A,\ v_A = \text{vector } oa = 4 \text{ m/s}$$

$$v_D = \text{vector } od = 4.1 \text{ m/s}$$

$$a^r_{BO} = a_B = \frac{v^2_{BO}}{OB} = \frac{(4.713)^2}{0.15} = 148.1 \text{ m/s}^2$$

$$a^r_{AB} = \frac{v^2_{AB}}{BA} = \frac{(3.4)^2}{0.6} = 19.3 \text{ m/s}^2$$

$$\text{vector } o'b' = a^r_{BO} = a_B = 148.1 \text{ m/s}^2$$

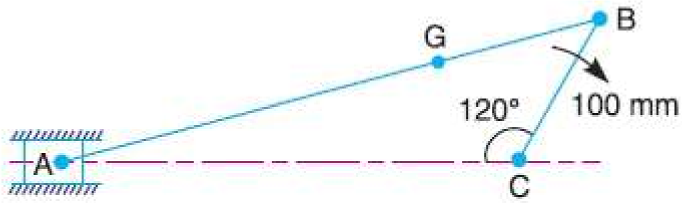
$$a_D = \text{vector } o'd' = 117 \text{ m/s}^2$$

$$\omega_{AB} = \frac{v_{AB}}{BA} = \frac{3.4}{0.6} = 5.67 \text{ rad/s}^2 \text{ (Anticlockwise about } B)$$

$$a^t_{AB} = 103 \text{ m/s}^2$$

$$\alpha_{AB} = \frac{a^t_{AB}}{BA} = \frac{103}{0.6} = 171.67 \text{ rad/s}^2 \text{ (Clockwise about } B)$$

An engine mechanism is shown in Fig. 8.5. The crank CB = 100 mm and the connecting rod BA = 300 mm with centre of gravity G, 100 mm from B. In the position shown, the crankshaft has a speed of 75 rad/s and an angular acceleration of 1200 rad/s^2. Find: **1.** *velocity of G and angular velocity of AB, and* **2.** *acceleration of G and angular acceleration of AB.*



Solution. Given : $\omega_{BC} = 75$ rad/s ; $\alpha_{BC} = 1200$ rad/s^2, $CB = 100$ mm $= 0.1$ m; $BA = 300$ mm $= 0.3$ m

We know that velocity of B with respect to C or velocity of B,

$$v_{BC} = v_B = \omega_{BC} \times CB = 75 \times 0.1 = 7.5 \text{ m/s} \quad \text{...(Perpendicular to } BC\text{)}$$

Since the angular acceleration of the crankshaft, $\alpha_{BC} = 1200$ rad/s^2, therefore tangential component of the acceleration of B with respect to C,

$$a^t_{BC} = \alpha_{BC} \times CB = 1200 \times 0.1 = 120 \text{ m/s}^2$$

Note: When the angular acceleration is not given, then there will be no tangential component of the acceleration.

1. *Velocity of G and angular velocity of AB*

First of all, draw the space diagram, to some suitable scale, as shown in Fig. 8.6 (*a*). Now the velocity diagram, as shown in Fig. 8.6 (*b*), is drawn as discussed below:

1. Draw vector *cb* perpendicular to *CB*, to some suitable scale, to represent the velocity of B with respect to C or velocity of B (*i.e.* v_{BC} *or* v_B), such that

$$\text{vector } cb = v_{BC} = v_B = 7.5 \text{ m/s}$$

2. From point *b*, draw vector *ba* perpendicular to *BA* to represent the velocity of *A* with respect to *B i.e.* v_{AB}, and from point *c*, draw vector *ca* parallel to the path of motion of *A* (which is along *AC*) to represent the velocity of *A i.e.* v_A. The vectors *ba* and *ca* intersect at *a*.

3. Since the point *G* lies on *AB*, therefore divide vector *ab* at *g* in the same ratio as *G* divides *AB* in the space diagram. In other words,

$$ag / ab = AG / AB$$

The vector *cg* represents the velocity of *G*.

By measurement, we find that velocity of *G*,

$$v_G = \text{vector } cg = 6.8 \text{ m/s} \quad \textbf{Ans.}$$

2. From point b, draw vector ba perpendicular to BA to represent the velocity of A with respect to B *i.e.* v_{AB}, and from point c, draw vector ca parallel to the path of motion of A (which is along AC) to represent the velocity of A *i.e.* v_A. The vectors ba and ca intersect at a.

3. Since the point G lies on AB, therefore divide vector ab at g in the same ratio as G divides AB in the space diagram. In other words,

$$ag / ab = AG / AB$$

The vector cg represents the velocity of G.

By measurement, we find that velocity of G,

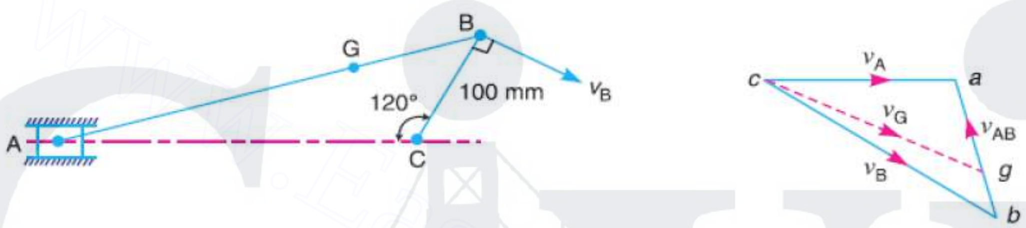
$$v_G = \text{vector } cg = 6.8 \text{ m/s} \quad \textbf{Ans.}$$

From velocity diagram, we find that velocity of A with respect to B,

$$v_{AB} = \text{vector } ba = 4 \text{ m/s}$$

We know that angular velocity of AB,

$$\omega_{AB} = \frac{v_{AB}}{BA} = \frac{4}{0.3} = 13.3 \text{ rad/s (Clockwise)} \quad \textbf{Ans.}$$



2. *Acceleration of G and angular acceleration of AB*

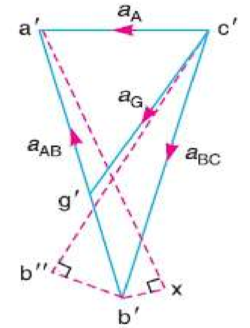
We know that radial component of the acceleration of B with respect to C,

$$^*a^r_{BC} = \frac{v^2_{BC}}{CB} = \frac{(7.5)^2}{0.1} = 562.5 \text{ m/s}^2$$

and radial component of the acceleration of A with respect to B,

$$a^r_{AB} = \frac{v^2_{AB}}{BA} = \frac{4^2}{0.3} = 53.3 \text{ m/s}^2$$

Now the acceleration diagram, as shown in Fig. 8.6 (*c*), is drawn as discussed below:



(*c*) Acceleration diagram.

Fig. 8.6

1. Draw vector $c'\,b''$ parallel to CB, to some suitable scale, to represent the radial component of the acceleration of B with respect to C, *i.e.* a^r_{BC}, such that

$$\text{vector } c'b'' = a^r_{BC} = 562.5 \text{ m/s}^2$$

2. From point b'', draw vector $b''\,b'$ perpendicular to vector $c'\,b''$ or CB to represent the tangential component of the acceleration of B with respect to C *i.e.* a^t_{BC}, such that

$$\text{vector } b''b' = a^t_{BC} = 120 \text{ m/s}^2 \quad \text{... (Given)}$$

3. Join $c'\,b'$. The vector $c'\,b'$ represents the total acceleration of B with respect to C *i.e.* a_{BC}.

4. From point b', draw vector $b'\,x$ parallel to BA to represent radial component of the acceleration of A with respect to B *i.e.* a^r_{AB} such that

$$\text{vector } b'x = a^r_{AB} = 53.3 \text{ m/s}^2$$

5. From point x, draw vector xa' perpendicular to vector $b'x$ or BA to represent tangential component of the acceleration of A with respect to B *i.e.* a^t_{AB}, whose magnitude is not yet known.

6. Now draw vector $c'a'$ parallel to the path of motion of A (which is along AC) to represent the acceleration of A *i.e.* a_A.The vectors xa' and $c'a'$ intersect at a'. Join $b'\,a'$. The vector $b'\,a'$ represents the acceleration of A with respect to B *i.e.* a_{AB}.

7. In order to find the acceleratio of G, divide vector $a'\,b'$ in g' in the same ratio as G divides BA in Fig. 8.6 (*a*). Join $c'\,g'$. The vector $c'\,g'$ represents the acceleration of G.

By measurement, we find that acceleration of G,

$$a_G = \text{vector } c'\,g' = 414 \text{ m/s}^2 \textbf{ Ans.}$$

From acceleration diagram, we find that tangential component of the acceleration of A with respect to B,

$$a^t_{AB} = \text{vector } xa' = 546 \text{ m/s}^2 \quad \text{...(By measurement)}$$

∴ Angular acceleration of AB,

$$\alpha_{AB} = \frac{a^t_{AB}}{BA} = \frac{546}{0.3} = 1820 \text{ rad/s}^2 \text{ (Clockwise) } \textbf{Ans.}$$