

Puzzle:

Consider a vector field $\vec{F}(x, y, z) = (x^2 - yz, y^2 - xz, z^2 - xy)$.

1. **Divergence Check:** Compute the divergence of \vec{F} and determine if the field is solenoidal (divergence-free).
2. **Curl Challenge:** Find the curl of \vec{F} and check if the field is irrotational (curl-free).
3. **Conservative or Not:** Is \vec{F} a conservative field? Justify your answer.
4. **Line Integral:** If \vec{F} is conservative, find the scalar potential function $\phi(x, y, z)$ such that $\nabla\phi = \vec{F}$.