

🌟 Puzzle 1: Classic Contour Integral

Evaluate the following integral using contour integration techniques:

$$I = \int_{-\infty}^{\infty} \frac{dx}{x^2 + 1}$$

Hint: Consider a semicircular contour in the upper half-plane and use the residue theorem.

🎯 Puzzle 2: Tricky Residues

Use the residue theorem to compute:

$$\int_C \frac{e^z}{z^2 + 4} dz$$

where C is the positively oriented circle $|z| = 3$.

1. Identify the poles of the integrand.
2. Calculate the residues at each pole.
3. Apply the residue theorem to find the value of the integral.

🔥 Puzzle 3: Real Integral with a Complex Twist

Evaluate the real integral:

$$J = \int_0^{\infty} \frac{x^{a-1}}{x+1} dx, \quad 0 < a < 1$$

using a keyhole contour around the branch cut along the negative real axis.

🌟 Puzzle 4: Improper Integral Challenge

Show that:

$$\int_{-\infty}^{\infty} \frac{\sin x}{x} dx = \pi$$

Hint: Use a semicircular contour in the upper half-plane, and consider a function like:

$$f(z) = \frac{e^{iz}}{z}$$