SNS COLLEGE OF ENGINEERING

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AN AUTONOMOUS INSTITUTION

Approved by AICTE, New Delhi and Affiliated to Anna University, Chennai

23MAT102 - COMPLEX ANALYSIS AND LAPLACE TRANSFORMS

QUESTION BANK

INTERNAL ASSESSMENT EXAMINATION I

UNIT I

PART A

1. Find the directional derivative of $\varphi = x^2yz + 4xz^2 + zx$ at the point (1,-2,-1) in the direction vector $\overrightarrow{2i} - \vec{j} - 2\vec{k}$.
2. Find the directional derivative of $\varphi = 3x^2 + 2y - 3z$ at (1 , 1 ,1) in the direction of $2\vec{i} + 2\vec{j} - \vec{k}$.
3. Find the unit normal vector to xy=z^2 at (1,1,-1).
4. If $\vec{F} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$, then find the divergence.
5. Find the unit normal vector of the surface $x^2 + y^2 - z = 1$ at (1, 1,1).
6. Find the unit normal vector of xy=z^2 at (1,1,-1).
7. Show that $\vec{F} = yz\vec{i} + zx\vec{j} + xy\vec{k}$ is irrotational.
8. Prove that Curl(gradΦ) = 0.
9. State Green's theorem in a plane.
10. State Stokes' theorem.
11. State Gauss Divergence theorem.
12. Prove that $\vec{F} = (x^2 - y^2 + x)\vec{i} - (2xy + y)\vec{j}$ is irrotational.
13. If $\vec{F} = (y^2 \cos x + z^3)\vec{i} + (2y\, sinx - 4)\vec{j} + (3xz^2)\vec{k}$, then find its scalar potential.

PART B

1. Verify Green's theorem $\int_c (xy + y^2)dx + x^2\, dy$ where C is the closed curve of the region bounded by y=x and y=x^2.
2. Verify Greens theorem, for $\int_C [(3x - 8y^2)dx + (4y - 6xy)dy]$ where C is the boundary of the region by the lines x=0, y=0, x+y=1 in the xy plane.
3. Using Greens theorem, evaluate $\int_c [(y - sin\, x)dx + cos\, x\, dy]$ for where C is the triangle bounded by y=0, x=π/2 y=2x/π.
4. Using Green's theorem, evaluate ∫c (x^2 – y^2)dx +2xy dy where C is the closed curve of the region bounded by y^2 =x and y=x^2.
5. Verify Greens theorem in the XY plane, for $\int_C [(3x^2 - 8y^2)dx + (4y - 6xy)dy]$ where C is the boundary of the region defined by x= y^2, y=x^2 in the xy plane.
6. Verify Gauss divergence theorem for $\vec{F} = (x^2 - yz)\vec{i} + (y^2 - xz)\vec{j} + (z^2 - xy)\vec{k}$. and S is the surface of the rectangular parallelepiped bounded by $x = 0, x = a, y = 0, y = b, z = 0\, and\, z = c$.

7. Verify Gauss divergence theorem for $\vec{F} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$, where S is the surface of the cuboid formed by the planes $x = 0, x = a, y = 0, y = b, z = 0, z = c$
8. Verify Gauss Divergence theorem for $\vec{F} = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$ taken over the cube bounded by the planes x=0,x=1,y=0,y=1,z = 0,z=1.
9. Verify Gauss Divergence theorem for $\vec{F} = xy^2\,\vec{i} + yz^2\,\vec{j} + zx^2\,\vec{k}$ over the region bounded by x = 0, x= 1, y= 0, y= 2, z= 0,z= 3
10. Verify Gauss Divergence theorem for $\vec{F} = x^2\,\vec{i} + z\,\vec{j} + yz\,\vec{k}$ over the cube bounded by x = 0, x= 1, y= 0, y= 1, z= 0,z= 1
11. Verify Stoke's theorem for $\vec{F} = (x^2 - y^2)\vec{i} + 2xy\vec{j}$,where S is the rectangle in the xy-plane formed by the lines x=0 ,x=a,y=0 and y=b.
12. Verify Stoke's theorem for $\vec{F} = (y - z + 2)\vec{i} + (yz + 4)\vec{j} - xz\,\vec{k}$ where S is the open surfaces of the cube x = 0, x= 1, y= 0, y= 1, z= 0,z= 1 not included in the XOY plane.
13. Verify Stoke's theorem for $\vec{F} = x^2\vec{i} + xy\vec{j}$ integrated round the square in the z= 0 plane whose sides are along the lines x = 0, y= 0, x = a, y = a
14. Verify Stoke's theorem for $\vec{F} = (x^2 - y^2)\vec{i} + 2xy\vec{j} + xyz\,\vec{k}$, over the surface of the box bounded by the planes x=0 ,x=a, y=0 , y=b ,z=c above the xy plane.
15. Verify Stoke's theorem for $\vec{F} = (x^2 + y^2)\vec{i} - 2xy\vec{j}$,where S is the rectangle in the xy-plane formed by the lines x=+a,x=-a,y=0 and y=b

Unit-II

Part-A

1. Is the $f(z) = |z|^2$ analytic function . Justify.
2. Show that the function f(z)= $\overline{z}$ is nowhere differentiable
3. Determine whether the function 2xy + i$(x^2 - y^2)$ is analytic or not.
4. Test the analyticity of the function w=sinz
5. Test the analyticity of log z
6. If $u(x, y) = 3x^2y + 2x^2 - y^3 - 2y^2$, verify whether u is harmonic.
7. Examine whether y+ e^x cosy is harmonic.

Part-B

1. Prove that every analytic function w=u(x,y)+ i v(x,y) can be expressed as a function of z alone.
2. An analytic function whose real part is constant must itself be a constant.
3. Show that the function u=$\frac{1}{2}$log $(x^2 + y^2)$ is harmonic and determine its conjugate. Also find f(z).
4. Determine the analytic function whose real part is $\frac{\sin 2x}{\cosh 2y - \cos 2x}$
5. Find the Regular function whose imaginary part is e^{-x} (x cos y + ysiny)
6. Find the analytic function for which $\frac{\sin 2x}{\cosh 2y - \cos 2x}$ is the real part .Hence determine the analytic function u+iv for which u+v is the above function.
7. Show that u = e^{-x} (x cos y + ysiny) is harmonic function. Hence find the analytic function f(z) = u + iv.

8. Find the analytic function whose real part is e^x (x cos y – y siny).
9. Find the analytic function f=u+iv given that $u(x,y) = e^{2x}(x \sin 2y + y \cos 2y)$.
10. If f(z) is a regular function of z, then prove that $\nabla^2 |f(z)|^2 = 4|f'(z)|^2$