SNS COLLEGE OF ENGINEERING

Kurumbapalayam (Po), Coimbatore – 641 107

AN AUTONOMOUS INSTITUTION

Approved by AICTE, New Delhi and Affiliated to Anna University, Chennai

23MAT102-Complex Analysis and Laplace Transforms

Academic Year 2024 – 2025 (even semester)

QUESTION BANK (IAE-2)

UNIT II

COMPLEX VARIABLES

Part-A

1. Find the image of the circle $|z| = 3$ transformation w=2z.
2. Find the image of the circle $|z| = 3$ transformation w=5z.
3. Find the critical points of the transformation $\omega = z^2 - \frac{1}{z^2}$.
4. Define conformal mapping.
5. Find the critical point of the transformation $w^2 = (z-\alpha)(z-\beta)$
6. Find the image of $x = 0$ under the mapping $w = \frac{1}{z}$.
7. Find the critical point of the transformation $w = z^2$.
8. Find the fixed point of the transformation $w = \frac{z-1}{z+1}$.
9. Find the image of the circle $|z-\alpha| = r$ by the mapping $w = z + c$, where c is a constant.
10. Find the invariant point of the mapping $w = \frac{1-z}{1+z}$.
11. Define the critical point of the transformation $w = 1 + \frac{1}{z}$.

Part-B

1. Find the image of $|z-2i| = 2$ under the mapping w = $\frac{1}{z}$.
2. Under the transformation w = $\frac{1}{z}$ find the image of the circle $|z-1| = 1$.
3. What is the image of the line x=2 under under the transformation w = $\frac{1}{z}$.
4. Find the image of the infinite strip i) $\frac{1}{4} < y < \frac{1}{2}$ ii) $0 < y < \frac{1}{2}$ under the transformation w = $\frac{1}{z}$
5. Show that the transformation w = $\frac{1}{z}$ transforms of circles and straight lines in the Z plane into circles or straight lines in the W plane.

6. Determine the image of $1 < x < 2$ under the transformation $w = \frac{1}{z}$.

7. Find the bilinear transformation which maps the points, $z = \infty$ $z = i$, $z = 0$ on to the points $w = 0$, $w = i$, $w = \infty$

8. Find the bilinear transformation that maps 1, i and -1 of the z-plane on to 0, 1, ∞ of the w-plane.

9. Find the bilinear transformation that transforms the points $z = 1, i, -1$ of the z-plane into the points $w = 2, i, -2$ of the w-plane.

10. Find the bilinear transformation which maps the point z = 0,1.-1 onto the points w= -1,0,∞ Find also the invariant points of the transformation.

11. Find the bilinear transformation which maps the point 1, i, -1 onto the points i, 0, -i Find also the invariant points of the transformation.

UNIT III
COMPLEX INTEGRATION
Part-A

1. State Cauchy's integral formula.
2. Evaluate $\oint \frac{e^z}{z-1} dz$, where C is $|z + 3| = 1$
3. Expand $f(z) = sin\ z$ in a Taylor series about origin.
4. What is meant by essential singularity? Give an example.
5. Find the singular points of f(z) = $f = \frac{sin\ z}{z}$
6. Find the nature of singular points of $\oint \frac{e^z}{z-1} dz$
7. State Cauchy's residue theorem.
8. Determine the residue of $f(z) = \frac{z+1}{(z-1)(z+2)}$ at z=1.
9. Calculate the residue of $f(z) = \frac{e^{2z}}{(z+1)^2}$ at its poles.
10. Find the residue of $f(z) = tan\ z\ at\ z = \frac{\pi}{2}$
11. Find the residue of $f(z) = \frac{1-e^{-z}}{z^2}$ at z = 0

Part B

1. By using Cauchy's integral formula, evaluate $\int_c \frac{zdz}{(z-2)(z-3)^2}$ where C is $|z - 3| = \frac{1}{2}$.
2. Use Cauchy's integral formula to evaluate $\int_c \frac{sin\pi z^2 + cos\pi z^2}{(z-2)(z-3)} dz$, where C is the circle $|z| = 4$.
3. Using Cauchy's integral formula evaluate $\int_c \frac{e^{2z}}{(z+1)^4} dz$ where C is $|z| = 2$

4. Evaluate $\int_C \frac{z+1}{z^2+2z+4} dz$where C is the circle $|z+1+i| = 2$ Using Cauchy's integral formula.
5. Obtain the Taylor's series for$f(z) = log(1+z)$ about $z = 0$.
6. Expand $cosz$ as a Taylor's series about the points i) $z = 0$ ii) $z = \frac{\pi}{4}$
7. Expand $f(z) = \frac{1}{z^2}$ as Taylor' series about the point $z = 2$
8. Find the Laurent's expansion of $f(z) = \frac{7z-2}{(z-1)z(z-2)}$ in the region $1 < |z+1| < 3$.
9. Find the Laurent series expansion of $f(z) = \frac{z^2-1}{(z+2)(z+3)}$valid in the regions $2 < |z| < 3$ and

   $|z| > 3$

10. Find the Laurent series expansion of $f(z) = \frac{1}{z^2+4z+3}$ valid in the region$|z| < 1$and$0 < |z+1| < 2$.
11. Expand $\frac{1}{(z-1)(z-2)}$ in a Laurent series valid for

    (i) $|z| < 1$ (ii) $1 < |z| < 2$
12. Expand as a Laurent's series the function $f(z) = \frac{z}{z^2-3z+2}$ in the regions

    (i) $|z| < 1$ ii) $1 < |z| < 2$ iii) $|z| > 2$

13. Using Cauchy's residue Theorem, evaluate $\int_C \frac{z-1}{(z-1)^2(z-2)} dz$ where C is $|z-i| = 2$.
14. Using Cauchy's residue theorem evaluate $\int_C \frac{12z-7}{(2z+3)(z-1)^2} dz$ where C is $|z| = 2$.
15. Evaluate $\int_C \frac{zdz}{(z^2+1)^2}$ where C is the circle $|z-i| = 1$ using Cauchy's residue theorem.