19EC502 -Transmission Lines and Antennas

UNIT -I -Derivations of Stub Matching

STUB MATCHING:

In general, the Source (or) i/p impedance is a fixed one. By Choosing the Value of load impedance to be equal to the i/p impedance, impedance matching is achieved.

In certain cases (especially if the load is an antenna), the load impedance is also fixed. If the load impedance is not equal to the Complex Conjugate of the i/p impedance, the maximum Power transfer will not take place. This is known as mismatching. So, it is necessary to introduce some form of an impedance transforming section b/w the Source and load to achieve impedance matching. Such a section is called an impedance matching device. Another means of accomplishing impedance matching is the use of an open (or) Short Circuited line of suitable length, called stub at a designated distance from the load. this called stub matching.

There are two types of stub matching:

(i) Single Stub matching (ii) Double stub matching

SINGLE

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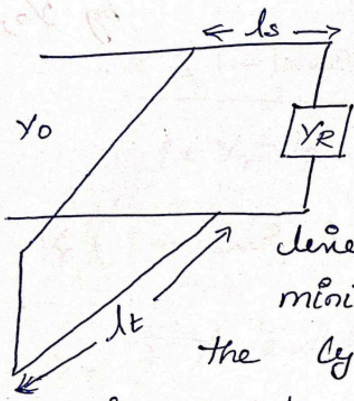
SINGLE STUB MATCHING :

A transmission line having a characteristic admittance Y_0 terminated with load Conductance Y_R (load resistance Z_R). Since Y_R is different from Y_0, standing waves are setup in b/w Source and load.



The i/p admittance of a transmission line is changing from maximum to minimum Conductance and so on and the cycle repeats for every $\lambda/2$.

When the line is transversed from the point of maximum or minimum Conductance to that of minimum (or) maximum Conductance, there will be a point at which the real part of the admittance is equal to the characteristic admittance (i.e $R = Z_0$)

The i/p impedance at any point of a transmission line is given by,

$$Z_s = Z_0 \frac{Z_R + Z_0 \tanh \gamma l}{Z_0 + Z_R \tanh \gamma l}$$

The i/p admittance is $Y_S = Y_0 \dfrac{Y_R + Y_0 \tanh \gamma l}{Y_0 + Y_R \tanh \gamma l}$

For Propagation $\gamma = j\beta$ $(\alpha = 0)$

$$Y_s = Y_0 \frac{Y_R + jY_0 \tan\beta l}{Y_0 + jY_R \tan\beta l}$$

For normalization, the above expression is divided by Y_0,

$$\frac{Y_s}{Y_0} = \frac{Y_R + jY_0 \tan\beta l}{Y_0 + jY_R \tan\beta l}$$

$$Y_{in} = \frac{\frac{Y_R}{Y_0} + j\tan\beta l}{1 + j\frac{Y_R}{Y_0}\tan\beta l}$$

where $\frac{Y_s}{Y_0} = Y_{in}$, normalized i/p admittance

$\frac{Y_R}{Y_0} = Y_r$, normalized load admittance.

$$Y_{in} = \frac{Y_r + j\tan\beta l}{1 + jY_r\tan\beta l}$$

This can be written as,

$$= \frac{Y_r + j\tan\beta l}{1 + jY_r\tan\beta l} \cdot \frac{(1 - jY_r\tan\beta l)}{(1 - jY_r\tan\beta l)}$$

$$= \frac{Y_r(1 + \tan^2\beta l) + j(1 - Y_r)^2\tan\beta l}{1 + Y_r^2\tan^2\beta l}$$

$Y_s = Y_0$

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$$\frac{Y_s}{Y_0} = 1$$

$$\therefore Y_{in} = 1$$

The stub has to be located at a point where the real part of Y_{in} is equal to unity.

$$\therefore \frac{Y_r(1+\tan^2\beta l_s)}{1+Y_r^2\tan^2\beta l_s} = 1$$

$$Y_r(1+\tan^2\beta l_s) = 1 + Y_r^2\tan^2\beta l_s.$$

$$Y_r + Y_r\tan^2\beta l_s = 1 + Y_r^2\tan^2\beta l_s.$$

$$Y_r\tan^2\beta l_s - Y_r^2\tan^2\beta l_s = 1 - Y_r$$

$$\tan^2\beta l_s(Y_r - Y_r^2) = 1 - Y_r$$

$$Y_r(1-Y_r)\tan^2\beta l_s = 1 - Y_r$$

$$Y_r\tan^2\beta l_s = 1$$

$$\Rightarrow \tan^2\beta l_s = 1/Y_r \Rightarrow \tan\beta l_s = \frac{1}{\sqrt{Y_r}}$$

$$\Rightarrow \sqrt{\frac{Y_0}{Y_r}} \qquad \left(\because Y_r = \frac{Y_R}{Y_0}\right)$$

$$\beta l_s = \tan^{-1}\sqrt{\frac{Y_0}{Y_R}}$$

$$\frac{2\pi}{\lambda}.l_s = \tan^{-1}\sqrt{\frac{Y_0}{Y_R}}$$

The location of the stub l_s is given by,

$$l_s = \frac{\lambda}{2\pi}\tan^{-1}\sqrt{\frac{Z_R}{Z_0}} \qquad \left[\because Z_R = \frac{1}{Y_R};\ Z_0 = \frac{1}{Y_0}\right]$$

The Susceptance at the location of the stub

$$\frac{S_s}{Y_0} = \frac{(1-Y_r^2)\tan\beta l_s}{1+Y_r^2\tan^2\beta l_s}$$

$$= \frac{(1-Y_r^2)\sqrt{\frac{Y_0}{Y_R}}}{1+Y_r^2\frac{Y_0}{Y_R}} = \frac{\left(1-\frac{Y_R^2}{Y_0^2}\right)\left(\sqrt{\frac{Y_0}{Y_R}}\right)}{1+\frac{Y_R^2}{Y_0^2}\cdot\frac{Y_0}{Y_R}}$$

$$= \frac{\left(1-\frac{Y_R^2}{Y_0^2}\right)\sqrt{\frac{Y_0}{Y_R}}}{1+\frac{Y_R}{Y_0}}$$

$$= \left(1-\frac{Y_R}{Y_0}\right)\sqrt{\frac{Y_0}{Y_R}}$$

$$= \frac{Y_0-Y_R}{Y_0}\sqrt{\frac{Y_0}{Y_R}}$$

The Susceptance of the stub is,

$$S_s = (Y_0 - Y_R)\sqrt{\frac{Y_0}{Y_R}}$$

This, can be obtained either by an open circuited (or) short circuited stub.

Advantages: (i) It radiates less power.

(ii) It's effective length may be varied by means of a shorting bar.

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The Susceptance of a short Circuited stub is equated to $Y_0 \cot \beta l_t$.

$$(Y_0 - Y_R)\sqrt{\frac{Y_0}{Y_R}} = Y_0 \cot \beta l_t$$

$$\frac{Y_0 - Y_R}{Y_0}\sqrt{\frac{Y_0}{Y_R}} = \cot \beta l_t$$

$$\cot \beta l_t = (Y_0 - Y_R) \cdot \frac{1}{\sqrt{Y_0 Y_R}}$$

$$= \frac{Z_R - Z_0}{Z_R \cdot Z_0} \sqrt{Z_0 \cdot Z_R} = \frac{Z_R - Z_0}{\sqrt{Z_0 \cdot Z_R}} \qquad \frac{1}{Z_0} - \frac{1}{Z_R} = \frac{Z_R - Z_0}{Z_0 \cdot Z_R}$$

$$\tan \beta l_t = \frac{\sqrt{Z_R \cdot Z_0}}{Z_R - Z_0}$$

$$\beta l_t = \tan^{-1} \frac{\sqrt{Z_0 \cdot Z_R}}{Z_R - Z_0}$$

The length of the stub is given by,

$$l_t = \frac{\lambda}{2\pi} \tan^{-1}\left[\frac{\sqrt{Z_0 \cdot Z_R}}{Z_R - Z_0}\right]$$

LOCATION & LENGTH OF THE STUB USING REFLECTION CO-EFFICIENT

The i/p impedance of the line is given by,

$$Z_i = Z_0 \cdot \frac{1+k\,e^{-2\gamma l}}{1-k\,e^{-2\gamma l}}$$

For loseless line $\alpha=0$, $\gamma=j\beta$ and $k=|k|e^{j\phi}$

where ϕ is the angle of reflection co-efficient.

$$Z_l = Z_0 \frac{1+|k|\,e^{j\phi}\,e^{-j2\beta l}}{1-|k|\,e^{j(\phi-2\beta l)}}$$

$$= Z_0 \frac{1+|k|\,e^{j(\phi-2\beta l)}}{1-|k|\,e^{j\phi}\cdot e^{-j2\beta l}}$$

The i/p admittance is given by,

$$Y_i = G_0 \frac{1-|k|\,e^{j(\phi-2\beta l)}}{1+|k|\,e^{j(\phi-2\beta l)}}$$

where the characteristic conductance is

$$G_0 = \frac{1}{Z_0} = \frac{1}{R_0}$$

$$Y_i = G_0 \frac{1-|k|\left[\cos(\phi-2\beta l)+j\sin(\phi-2\beta l)\right]}{1+|k|\left[\cos(\phi-2\beta l)+j\sin(\phi-2\beta l)\right]}$$

$$= G_0 \frac{1-|k|\left[\cos(\phi-2\beta l)-j\,|k|\sin(\phi-2\beta l)\right]}{1+|k|\left[\cos(\phi-2\beta l)+j|k|\sin(\phi-2\beta l)\right]}$$

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Multiplying the numerator and denominator by,

$1+|k|\left[\cos(\phi-2\beta l)-j|k|\sin(\phi-2\beta l)\right]$

$$Y_i = G_0 \frac{1-|k|^2 - 2j|k|\sin(\phi-2\beta l)}{1+|k|^2+2|k|\cos(\phi-2\beta l)}$$

Since $Y_i = G_i + jS_i$, then

$$\frac{Y_i}{G_0} = \frac{G_i}{G_0} + j\frac{S_i}{G_0} = \frac{1-|k|^2-2j|k|\sin(\phi-2\beta l)}{1+|k|^2+2|k|\cos(\phi-2\beta l)}$$

Equating the real parts,

$$\frac{G_i}{G_0} = \frac{1-|k|^2}{1+|k|^2+2|k|\cos(\phi-2\beta l)}$$

Equating the imaginary parts,

$$\therefore \frac{S_i}{G_0} = -\frac{2|k|\sin(\phi-2\beta l)}{1+|k|^2+2|k|\cos(\phi-2\beta l)}$$

At the location of stub $Z_i = Z_0$, for matching

Since there is no reflection, $l = l_s$

$$\therefore G_i = G_0$$

$$\frac{G_i}{G_0} = 1$$

$$\frac{1-|K|^2}{1+|K|^2+2|K|\cos(\phi-2\beta l_s)} = 1$$

$$1-|K|^2 = 1+|K|^2+2|K|\cos(\phi-2\beta l_s)$$

$$2|K|\cos(\phi-2\beta l_s) = -2|K|^2$$

$$\cos(\phi-2\beta l_s) = -|K|$$

$$\phi-2\beta l_s = \cos^{-1}(-|K|)$$

But $\cos^{-1}(-|K|) = -\pi+\cos^{-1}|K|$

$$\therefore \phi-2\beta l_s = -\pi+\cos^{-1}|K|$$

$$2\beta l_s = \phi+\pi-\cos^{-1}|K|$$

$$l_s = \frac{\phi+\pi-\cos^{-1}|K|}{2\beta}$$

(or) $$\boxed{l_s = \frac{\lambda}{4\pi}\left[\phi+\pi-\cos^{-1}|K|\right]}$$

The normalized susceptance (imaginary part) eqn is,

$$\frac{S_i}{G_0} = -\frac{2|K|\sin(\phi-2\beta l)}{1+|K|^2+2|K|\cos(\phi-2\beta l)}$$

But $(\phi-2\beta l_s) = -\pi+\cos^{-1}|K|$ and

$$\cos(\phi - 2\beta l_s) = -|k|$$

$$\therefore \frac{S_i}{G_0} = \frac{-2|k| \sin(-\pi + \cos^{-1}|k|)}{1+|k|^2 + 2|k|(-|k|)}$$

$$= \frac{2|k| \sin(\cos^{-1}|k|)}{1+|k|^2 - 2|k|^2}$$

Let $\cos^{-1}|k| = \theta$, then $|k| = \cos\theta$ and

$$\sin(\cos^{-1}|k|) = \sin\theta$$

$$= \sqrt{1-\cos^2\theta} = \sqrt{1-|k|^2}$$

$$\therefore \frac{S_i}{G_0} = \frac{2|k|\sqrt{1-|k|^2}}{1-|k|^2}$$

$$S_i = G_0 \frac{2|k|}{\sqrt{1-|k|^2}}$$

The susceptance of the stub is $G_0 \cot \beta l_s$

$$G_0 \cot \beta l_s = G_0 \frac{2|k|}{\sqrt{1-|k|^2}}$$

$$\frac{1}{\tan \beta l_s} = \frac{2|k|}{\sqrt{1-|k|^2}}$$

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$$\tan \beta l_s = \frac{\sqrt{1-|K|^2}}{2|K|}$$

$$\beta l_s = \tan^{-1} \frac{\sqrt{1-|K|^2}}{2|K|}$$

$$l_s = \frac{1}{\beta} \tan^{-1} \frac{\sqrt{1-|K|^2}}{2|K|}$$

$$l_s = \frac{\lambda}{2\pi} \tan^{-1} \frac{\sqrt{1-|K|^2}}{2|K|}$$

The location of the stub 'l_s' and length of the stub 'l_t' can be determined, if the reflection co-efficient and frequency are known.

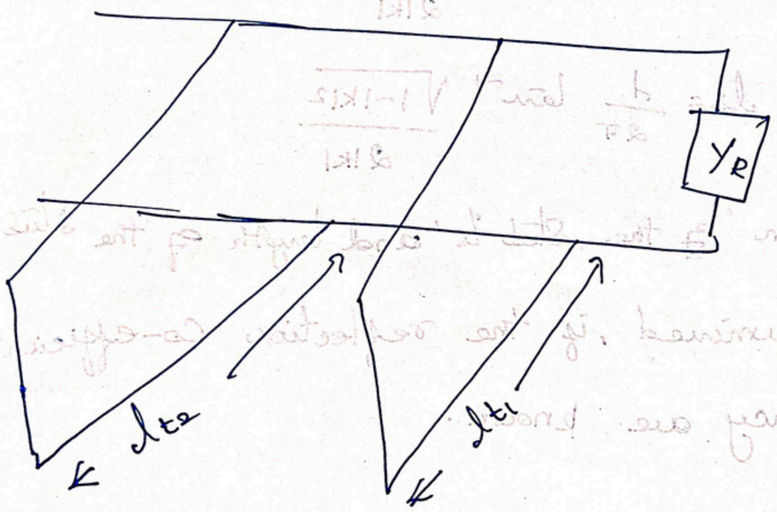
DRAWBACKS:

(i) Single stub matching is applicable for single frequency. For variable frequency the location of stub is not fixed. (i.e., changing)

(ii) For final adjustment the stub has to be moved along the line slightly. So it is possible only in open wire lines.

DOUBLE STUB MATCHING

To avoid the disadvantages of single matching, double stub matching is introduced. Double stub matching is one in which two short circuited stubs, whose length are adjustable independently are fixed.



CIRCLE DIAGRAM:

The i/p impedance for the TL is given by,

$$Z_s = \frac{V}{I} = \frac{\frac{V_R(Z_R+Z_0)}{2Z_R}\left[e^{\gamma x} + k e^{-\gamma x}\right]}{\frac{I_R(Z_R+Z_0)}{2Z_0}\left[e^{\gamma x} - k e^{-\gamma x}\right]}$$

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$$= \frac{V_R Z_0 [e^{\gamma x} + k e^{-\gamma x}]}{I_R Z_R [e^{\gamma x} - k e^{-\gamma x}]}$$

$[\because I_R Z_R = V_R]$

$$= \frac{V_R Z_0 \, e^{\gamma x} [1 + k \, e^{-\gamma x} \cdot e^{-\gamma x}]}{V_R \, e^{\gamma x} [1 - k e^{-\gamma x} \cdot e^{-\gamma x}]}$$

$$= Z_0 \frac{[1 + k e^{-2\gamma x}]}{1 - k e^{-2\gamma x}}$$

The i/p impedance of the TL is given by,

$$Z_s = Z_0 \frac{1 + k e^{-2\gamma x}}{1 - k e^{-2\gamma x}}$$

For a lossless line $\gamma = j\beta$ $(\because \alpha = 0)$

The normalised i/p impedance is obtained by $\div Z_s$ by its characteristic impedance Z_0.

$$Z_{in} = \frac{Z_s}{Z_0} = \frac{1 + k e^{-2\beta x}}{1 - k e^{-2\beta x}}$$

$$Z_{in}(1 - k e^{-2\beta x}) = 1 + k e^{-2\beta x}$$

$$Z_{in} - 1 = k e^{-2\beta x}(1 + Z_{in})$$

$$k e^{-j2\beta x} = \frac{Z_{in} - 1}{Z_{in} + 1}$$

But Z_{in} is a complex quantity. It can be represented by,

$$Z_{in} = R + jX$$

$$k e^{-j2\beta x} = \frac{R + jX - 1}{R + jX + 1} = \frac{(R-1) + jX}{(R+1) + jX}$$

The above eqn leads to two sets of circles. They are S circle and βx circle.

$$k e^{-j2\beta x} = \left[\frac{(R-1) + jX}{(R+1) + jX}\right]\left[\frac{(R+1) - jX}{(R+1) - jX}\right]$$

$$= \frac{R^2 - 1 + jX(R+1) - jX(R-1) + X^2}{(R+1)^2 + X^2}$$

$$= \frac{R^2 - 1 + X^2}{(R+1)^2 + X^2} + j\frac{2X}{(R+1)^2 + X^2}$$

By converting rectangular co-ordinates into polar co-ordinates.

$$k e^{-j2\beta x} = \sqrt{\frac{(R-1)^2+x^2}{(R+1)^2+x^2}} \tan^{-1}\left[\frac{\frac{2x}{(R+1)^2+x^2}}{\frac{(R^2-1)+x^2}{(R+1)^2+x^2}}\right]$$

Constant S circles are obtained by equating the magnitude,

$$k^2 = \frac{(R-1)^2+x^2}{(R+1)^2+x^2}$$

$$k^2(R+1)^2 + k^2x^2 = (R-1)^2+x^2$$

$$k^2(R^2+2R+1)+k^2x^2 = R^2+1-2R+x^2$$

$$k^2(R^2+2R+1+x^2) = R^2+1-2R+x^2$$

$$k^2(R^2+x^2+2R+1) = R^2+x^2-2R+1$$

$$R^2(k^2-1)+x^2(k^2-1)+2R(k^2+1)+k^2-1=0$$

$\div$ by k^2-1

$$R^2+x^2+2R\left(\frac{k^2+1}{k^2-1}\right)+1=0$$

The reflection Co-efficient can be written in terms of the S.W. SWR,

$$|K| = \frac{S-1}{S+1}$$

$$\frac{K^2+1}{K^2-1} = \frac{\left(\frac{S-1}{S+1}\right)^2+1}{\left(\frac{S-1}{S+1}\right)^2-1} = \frac{(S-1)^2+(S+1)^2}{(S-1)^2-(S+1)^2}$$

$$= \frac{S^2-2S+1+S^2+2S+1}{S^2-2S+1-S^2-2S-1}$$

$$= 2(S^2+1) / -4S$$

$$\frac{K^2+1}{K^2-1} = -\frac{(S^2+1)}{2S}$$

Sub this value in the main eqn is,

$$R^2+X^2-2R\frac{(S^2+1)}{2S}+1=0$$

$$R^2+X^2-2R\frac{(S^2+1)}{2S} = -1$$

Adding $\left(\frac{(S^2+1)}{2S}\right)^2$ on b.s

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$$R^2 - 2R\left(\frac{S^2+1}{2S}\right) + \left(\frac{(S^2+1)}{2S}\right)^2 + X^2 = -1 + \left(\frac{(S^2+1)^2}{2S}\right)$$

$$\Rightarrow \left[R - \frac{(S^2+1)}{2S}\right]^2 + X^2 = \frac{-4S^2 + S^4 + 2S^2 + 1}{4S^2}$$

$$= \frac{S^4 - 2S^2 + 1}{4S^2}$$

$$= \left(\frac{S^2-1}{2S}\right)^2$$

$$\left[R - \frac{(S^2+1)}{2S}\right]^2 + X^2 = \left(\frac{S^2-1}{2S}\right)^2$$

This the eqn of the S Circle whose radius is

$$\frac{S^2-1}{2S} = \frac{S - \frac{1}{S}}{2}$$ & centre is $$\frac{S^2+1}{2S} = S + \frac{1}{S}$$