19EC502 -Transmission Lines and Antennas

Unit II- Guided Waves

Characteristic of TE and TM WAVES

The properties of the TE and TM waves between parallel conducting planes are altogether different than those of the uniform plane waves in the free space.

From the expressions of the field components of transverse electric waves or transverse magnetic waves, it is clear that there is either sinusoidal or co-sinusoidal variations or standing-wave distribution of each components of E or H in the X-direction

In y direction, none of the field components vary in magnitude or phase which is according to assumption made earlier

Thus x-y plane is an equiphase plane for each of the field components. The meaning of equiphase plane is that for all points on the plane , the maximum value of sinusoidal variation of any field component will reach its maximum value at same instant

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All the previous equation are derived under the assumption that

$$h^2 = \bar{\gamma}^2 + \omega^2 \mu \varepsilon \quad \text{...(1)}$$

But from the previous sections we know that the value of h is restriced to $\frac{m\pi}{a}$.
Hence we can write,

$$h^2 = \bar{\gamma}^2 + \omega^2 \mu \varepsilon = \left(\frac{m\pi}{a}\right)^2 \quad \text{...(2)}$$

$$\therefore \quad \bar{\gamma}^2 = \left(\frac{m\pi}{a}\right)^2 - \omega^2 \mu \varepsilon$$

$$\therefore \quad \boxed{\bar{\gamma} = \sqrt{\left(\frac{m\pi}{a}\right)^2 - \omega^2 \mu \varepsilon}} \quad \text{... (3)}$$

At the lower frequencies, the value of factor με is found to be less than (mπ/a)
Thus becomes real with value equal to the attenuation constant a. Under such condition, B = 0. Thus there is only attenuation suffered by the wave, without any propagation. In contrast to this, at higher frequencies, the value of the factor w^2 με becomes greater that that of the factor ma making purely imaginary. Thus when y is purely imaginary, its value equals to jß be. phase constant. Under such condition, attenuation constant is given by a = 0.

Thus, the lower frequencies are attenuated completely, with no propagation; while the higher frequencies are allowed to propagate with appropriate phase shift only, we can conclude that, the system acts as High Pass Filter (HPF). Let the cut-off frequency of the high pass filter be fc.

The cut-off frequency can be defined as the frequency at which the propagation constant changes from being real to imaginary. In other words, it is the frequency below which signal suffers only attenuation while above it waves just start propagating. It is interesting to note that at f = fc, value of the propagation constant is zero.

Thus,

For, $f < f_c$, $\left(\frac{m\pi}{a}\right)^2 > \omega^2\mu\varepsilon$, $\bar{\gamma} = \alpha$ and $\beta = 0$

For, $f > f_c$, $\left(\frac{m\pi}{a}\right)^2 < \omega^2\mu\varepsilon$, $\bar{\gamma} = j\beta$ and $\alpha = 0$

for, $f = f_c$, $\frac{m\pi}{a} = \omega^2 \mu \varepsilon$, $\bar{\gamma} = 0$

Hence, at $f = f_c$, rewriting equation (3), we get

$$0 = \sqrt{\left(\frac{m\pi}{a}\right)^2 - \omega_c^2 \mu \varepsilon}$$

Sq. on both side

$$\left(\frac{m\pi}{a}\right)^2 - (\omega_c^2 \mu \varepsilon)^2 = 0.$$

$$\left(\frac{m\pi}{a}\right)^2 = \omega_c^2 \mu \varepsilon \quad \text{—(4)}$$

$$\frac{m\pi}{a} = \omega_c \sqrt{\mu \varepsilon}$$

$\omega_c = 2\pi f_c$

$$\frac{m\pi}{a} = 2\pi f_c \sqrt{\mu \varepsilon}$$

$$f_c = \frac{m\pi}{a \cdot 2\pi \sqrt{\mu \varepsilon}} = \frac{m}{2a\sqrt{\mu \varepsilon}}$$

$$\boxed{f_c = \frac{m}{2a\sqrt{\mu \varepsilon}} \text{ Hz}} \quad \text{—(5)}$$

In general, putting the value of $\left(\frac{m\pi}{a}\right)^2$ as $\omega_c^2 \mu \varepsilon$ from eqn (4), in eqn (3), we get

$$\bar{\gamma} = \sqrt{\omega_c^2 \mu \varepsilon - \omega^2 \mu \varepsilon}$$

$$\bar{\gamma} = \sqrt{\mu \varepsilon (\omega_c^2 - \omega^2)}$$

$$\bar{\gamma} = j\sqrt{\mu \varepsilon} \sqrt{(\omega^2 - \omega_c^2)}$$

$\boxed{\omega = 2\pi f}$

$$\bar{\gamma} = j\sqrt{\mu \varepsilon} \sqrt{(2\pi f)^2 - (2\pi f_c)^2}$$

$\boxed{\beta = 2\pi\sqrt{\mu\varepsilon}\sqrt{f^2 - f_c^2}}$

$$\bar{\gamma} = j\sqrt{\mu \varepsilon}\, 2\pi \sqrt{(f^2 - f_c^2)} = j\beta \quad \text{—(6)}$$

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Hence the phase shift constant can be written by comparison as given below

$$\beta = 2\pi \sqrt{\mu\varepsilon} \sqrt{f^2 - f_c^2} \quad \text{—} (7)$$

The wavelength is defined as the distance travelled for the phase shift through 2π radians.
Thus wavelength is given by,

$$\lambda = \frac{2\pi}{\beta} \quad \text{—} (8)$$

The cut-off wavelength (λ_c) is given by,

$$\lambda_c = \frac{v}{f_c}$$ where $v = \frac{1}{\sqrt{\mu\varepsilon}}$ = velocity of propagation

But the cut-off frequency is given by,

$$f_c = \frac{m}{2a\sqrt{\mu\varepsilon}} = \frac{mv}{2a} \qquad \therefore v = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\lambda_c = \frac{v}{f_c} = \frac{2a}{m} \quad \text{—} (9)$$

Thus from eqn (9), the distance of separation is given by,

$$a = m \frac{\lambda_c}{2}$$

From eqn (10), we can conclude that the integer m indicates the number of the half wavelength variation of the either electric or magnetic field along x-direction.