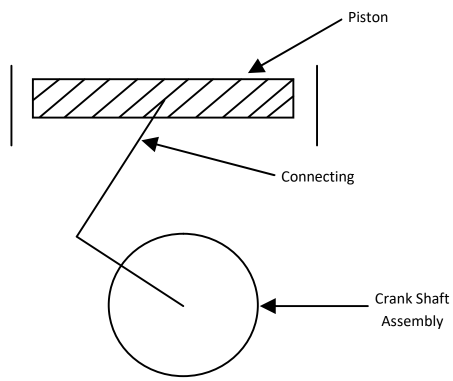
THEORY OF MACHINES

Simple Mechanism

A. **Kinematic Links:** A part of machines, which moves relative to other parts is known as a Kinematic Link. A link may consist of several parts, which are rigidly fastened together, so that they do not move relative each other.



In the picture, piston along with connecting rod attached with crank shaft assembly is an example of line should not be a ridig body, but it must be a restrained body.

Thus line should have the following characteristics:-

a) Should have relative motion.
b) Must be a resistant body

B.

Links

- Rigid lines [does not undergo any deformation while transmitting motion]
- Flexible lines [It is partly deformed in a way not to affect transmission of motion
- Fluid line [operationable for fluid in motion as exmp: In case of hydraulic press, jack

C. **Kinematic Pair:** The two links and elements of a machine, when in contact with each other, are said to form a pair. If the relative motion between them is complete or successfully constrained, then the pair is defined as Kinematic Pair.

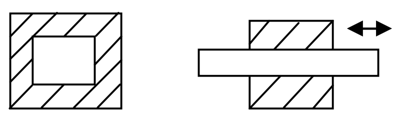
Now what we mean by constrained motion?

Let discuss in a Tabular format:-

Constrained Motion

Completely Constrained Motion

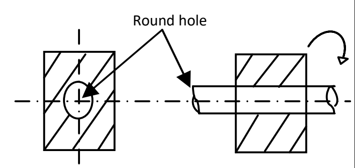
When the motion between a pair is limited to a definite direction, irrespective of the direction of force applied, then the motion is said to be a completely constrained motion.



Square bar in square hole only provide reciprocating motion.

In-completely Constrained Motion

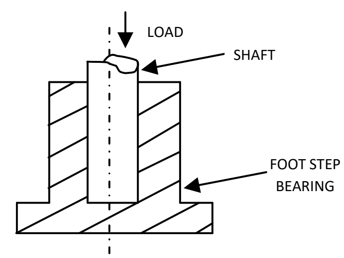
When the motion between a pair can take place in more than one direction, then the motion is called incompletely constrained motion. The change in the direction of impressed force may alter the direction of respective motion between the pair.



The shaft placed in hole may rotate or may have a reciprocation motion.

Successfully Completely Constrained Motion

When the motion between the elements, forming a pair, is such that the constrained motion is not completed by itself, then the motion is successfully constrained motion



Consider a Footstep Bearing with a shaft in it as in Fig. The shaft may rotate or move upward. So this is an example of incompletely constrained motion. But if a load is put on the shaft as in Pic. Then the upward movement will be restricted and ultimately it becomes a successfully constrained motion.

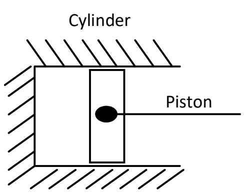
D. <u>Classification of Pairs</u>:

<u>Classification of Kinematic pair:</u>

Based on type of relative motion between the elements

- **Sliding Pair**

  When two elements of a pair are connected in such a way that one can only slide w.r.t the other than they form a sliding pair.

  Piston cylinder relative movement is an example of sliding pair.

- **Turning Pair**

  One element can only twin or revolve around the other line.

  Exmp: Cycle wheels turning over their axle.

- **Rolling Pair**

  On rolls over the other fixed line.

  Exmp.: Ball & roller bearing.

- **Screw Pair**

  When elements are connected in such a way that one element can twin over the other by screw thread.

  Exmp.: Nut & Bolt

- **Spherical Pair**

  When element turns or swivels over the fixed element

  Exmp.: Ball & Socket Joint

According to the type of contact between elements

Lower Pair:

When the two elements of a pair have a surface contact when their relative motion takes place and when one element slides over the surface of the other element, the pair formed, known as Lower Pair.

Exmp.:- Cyl. & Piston pair.

Higher Pair:

When the two elements of a pair have a line or point contact at the time of relative motion and the motion between them is partial turning & partial sliding, then it is a higher pair.

Exmp.:- Ball & Roller bearing, cam profile

According to the type of clousure

Self closed pair

(When connected mechanically)

Force closed pair

(When elements are not connected mechanically but by external force

Exmp.: Cam & Follower assembly)

We have already discussed about links, pairs and joints. Next we will learn about kenematic chains and degree of freedom.

E. **Degree of Freedom:** An object in space has six degrees of freedom, i.e, translation motion along X,Y,Z axis (3DOFs) and rotary motion about x,y,z

The rigid body is space, but due to formations of linkages, one or more DOF is lost due to the presence of constraint on the body. The total number of constraints cannot be zero as the body has to be fixed at some place to make the linkage possible. Thus the degree of freedom is given by:

$$\boxed{DOF = 6 - (\text{Number of constraints})} \qquad (1)$$

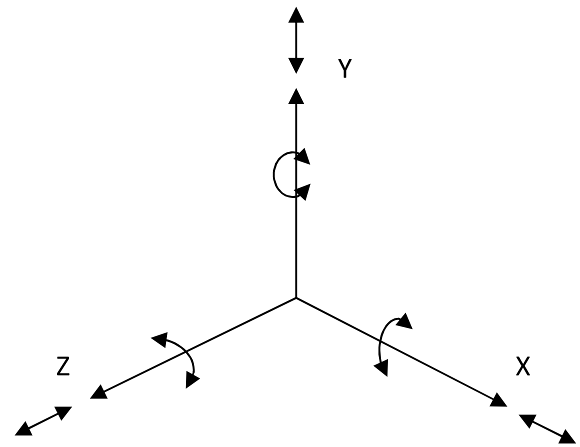
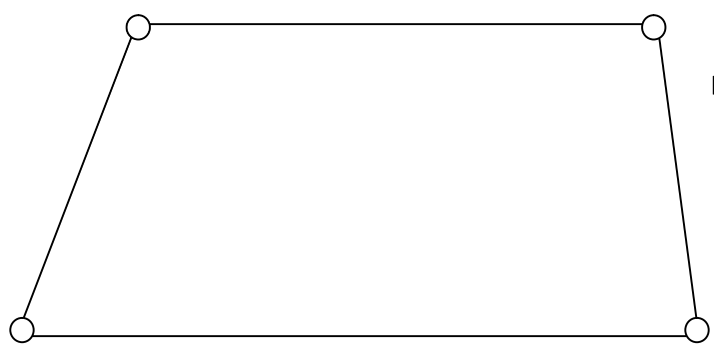


Fig.: Degree of freedom

F. **Kinematic Chain:** A kinematic chain is an assembly of links which are inter connected through joints or pairs, in which the relative motions between the links is possible and motion of each link relative to the other is definite.



Kinematic chain

G. **Degree of freedom in a mechanism:** Degree of freedom of a mechanism in space can be explained as follows –

Let, N = Total number of links in a mechanism

F = Degree of freedom (DOF)

J1 = No. of pairs having one DOF

J2 = No. of pairs having two DOF and so on

Now let consider a situation, when one of the links is fixed in a mechanism, then the number of movable links are = (N-1)
(Because each movable links has six DOF)

Now while discussing types of joints in a chain, lets first discuss:

Binary Joint: When two links are joined at the same connection, the joint is known as binary joint.

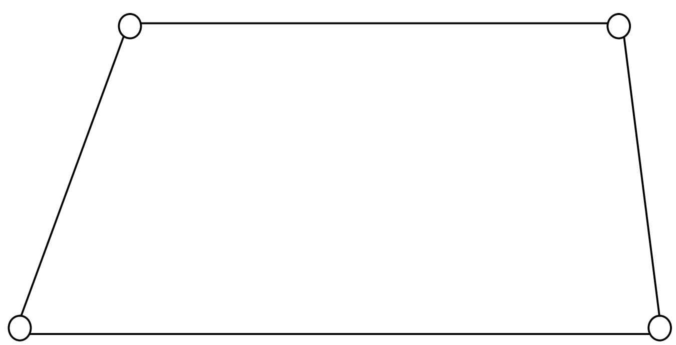


Fig.: Binary jt

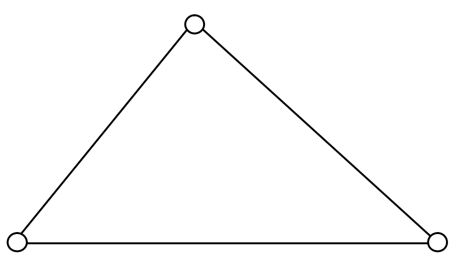


Fig.: Ternary jt

Fig. at the top, which has 4 binary joints and four links. This is basically constitute a 4-Bar link mechanism.

To determine the nature of chain (locked, kinematic or unconstrained) let take help of a relation derived by A.W.klein.

$$\boxed{J + h/2 = 3/2L - 2} \quad \text{——— (2)}$$

Where, j = No. of binary jt
H = No. of higher pair
L = No. of links

When h = 0, i.e, no. of higher pair, then equ. (2) becomes

$$\boxed{J = 3/2L - 2} \quad \text{——— (3)}$$

Now from the above fig. we can find

L = 4, J = 4 & h = 0

∴ From the equ. (3), putting the values

4 = 3/2 X L – 2

=> 4 = 6-2 = 4 => L.H.S = R.H.S

As L.H.S = R.H.S, thus the chain is a kinematic chain or constrained chain. If L.H.S ≠ R.H.L, then it would have been an unconstrained chain.

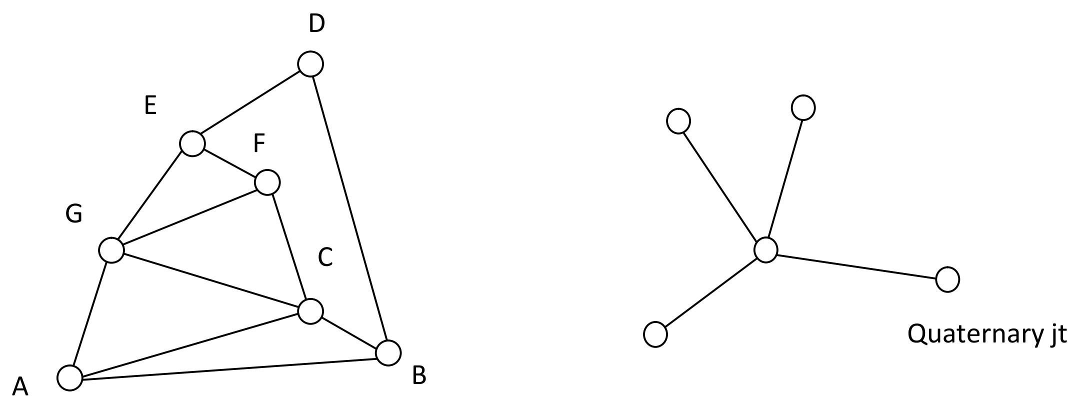
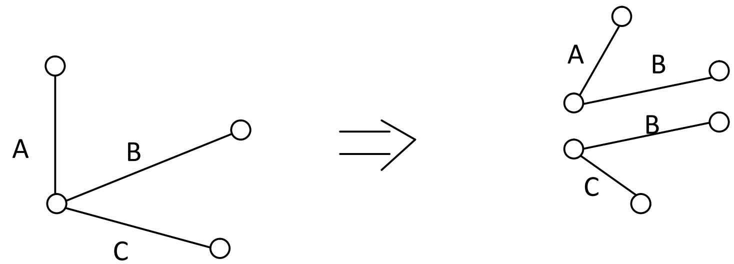


Fig.- Locked chain having binary, ternary, quaternary jt.

Ternary Joints: When three links are joined together at the same connection, then the joint is known as ternary jt.
Quarternary Joints: When four links are joined at the same connection, the joint is is called quaternary joint.

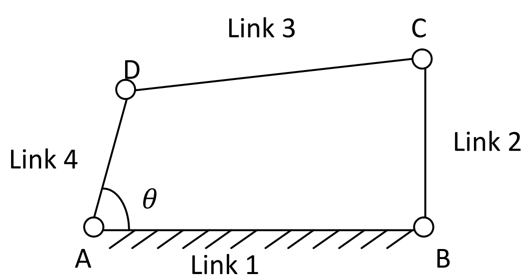
Generally a ternary joints is equivalent to two binary joints.



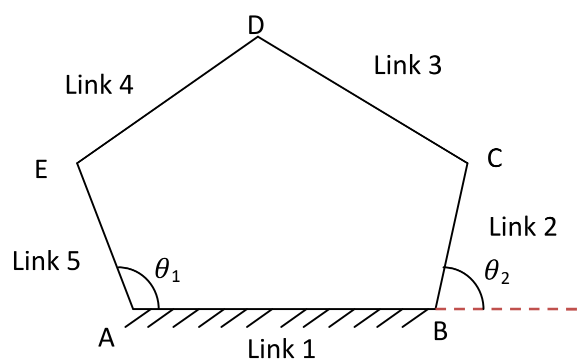
Similarly a quaternary joint is equivalent to three binary joints. In general, when L numbers of links are joined at the same connection, the joint is equivalent to (l-1) binary joints. By considering these relations, we can easily calculate if a link is closed or constrained from eqn. (2) & eqn. (3).

H. **Mechanism:** When one of the links of a kinematic chain is fixed, the chain is known as mechanism, It may be used for transmitting or transforming motion, eg. Engine indicators or type writers etc.
   - A mechanism with four links is known as a simple mechanism.
   - A mechanism with more than four links is known as compound mechanism
   - When a mechanism is required to transmit power or to do some particular type of work, then it because a machine.

I. Number of degrees of freedom for plane mechanism: In the design or analysis of a mechanism, one of the most important concern is the number of degrees of freedom of the mechanism. It is defined as the number of input parameter which must be independently controlled in order to bring the mechanism into a useful engineering purpose. It is possible to determine the number of DOF of a mechanism directly from the number of links & the number of joints which it includes:



(a) Four bar chain



(b) Five bar chain

Consider a four bar chain as shown in fig (a). A little consideration will show that only one variable such as θ is needed to define the relative problem of all the links. In other words, we can say the number of DOF for 4-bar chain is one. Now let us consider a 5-bar chain, two variations such as θ_1 & θ_2 are needed to define complete the relative position of all the number of degrees of freedom is two.

In order to let you understand the issue of number of DOF, consider two independent links AB, CD in a plane motion, as shown in Fig.

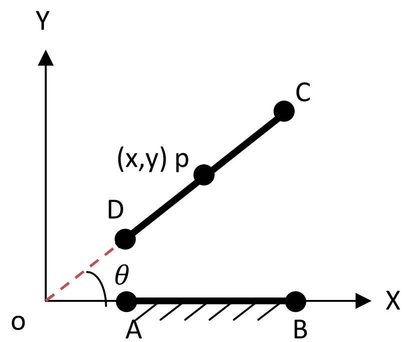


Fig (a)

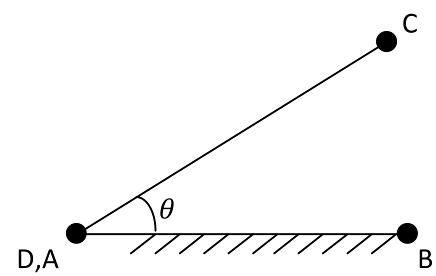
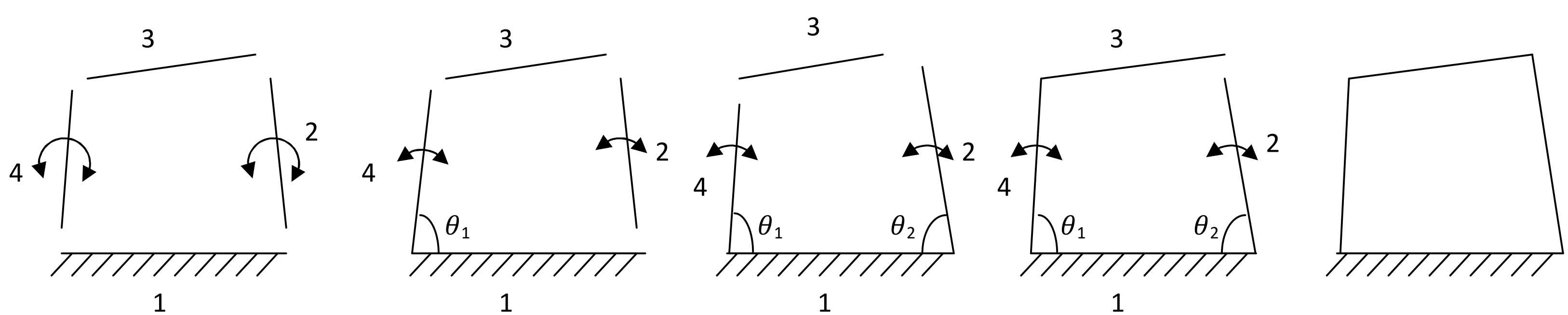


Fig (b)

The link AB with co-ordinate sys. Oxy is taken as fixed link or reference link. The position of point P on the moving link CD can be completely specified by three variable, i.e, the co-ordinates x,y and the inclination angle θ with the x-x axis. In other words, we can say that each link of a mechanism has three DOF in general before it gets connected to any other link. But when the CD line gets joined with line AB by a turning pair at A, as shown in fig.(b), the position of CD is now gets defined by a single variable θ and thus have one DOF.

From above, we can understand that when a link is connected to a fixed link by a turning pair (lower pair), two DOF's are getting destroyed. This can be clearly understood from the below fig. where the resulting DOF is n=1 for the constructed 4-bar mechanism.



(a) N = 3+3+3=9

(e) N = 1+3+3=7

(d) N = 1+3+1=5

(c) N = 1+1+1=3

(b) N = 1

Fig: 4-bar mechanism

Now consider a plane mechanism with "l" number of links. Since in a mechanism, one of the links is to be fixed, therefore the number of movable links will be (l-1) and the total number of DOF will be = 3 X (l-1), before they are connected to each other. In general a mechanism with "l" number of links connected by "j" numbers of binary joints (lower pairs) & "h" number of higher pairs, then the number of DOF of a mechanism is given by:

$$\boxed{N=3(l-1)-2j-h} \quad \text{———} \quad (4)$$

This eqn. in last page is known as the Kutzbach Giterion for the movability of a mechanism having plane motion. If there are no two DOF pairs (higher pair) then h=0, thus pathing h=0 in eqn.(4) we get:

$$\boxed{N=3(l-1)-2j} \quad \text{———} \quad (5)$$

J. Application of Kutzbach Giterion to plane mechanism to plane mechanism: Now let us determine, number of degrees of freedom or movability fro same simple mechanism having no higher pair (n=0). Thus means we will follow eqn.(5).

a) **Three bar mechanism:** As shown in Fig. there are three links and three binary joints.

$\therefore L = 3 \ \& \ j = 3$

$n = 3(3-1)-2X3$

$=> n = (3X2) – (2X3) = 0$

Thus were n = 0, then the mechanism forms a structure & no relative motion between the links is possible.

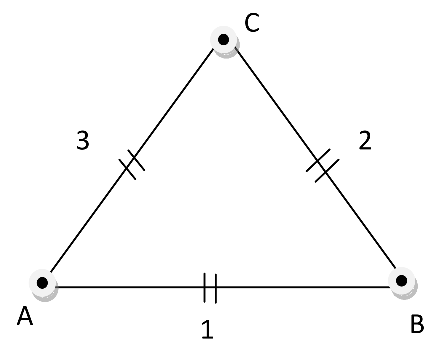


Fig. eqn. (5)

b) **Four bar mechanism:** As per the fig. there are four links and binary joints avl.

$\therefore L = 4, j = 4$

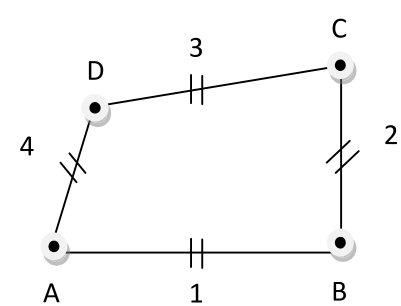
$\therefore$ from eqn. (5) we get

$N = 3(L-1) – 2 Xj$

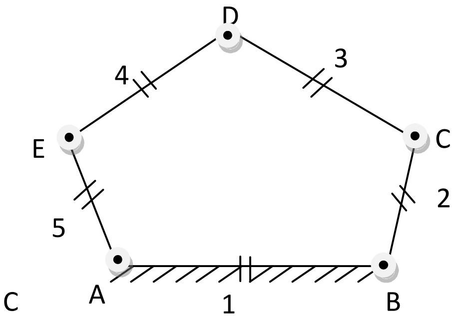
$=> n = 3(4-1) – 2X4$

$=> n = 3X3 – 8 => n = 9 - 8=1$

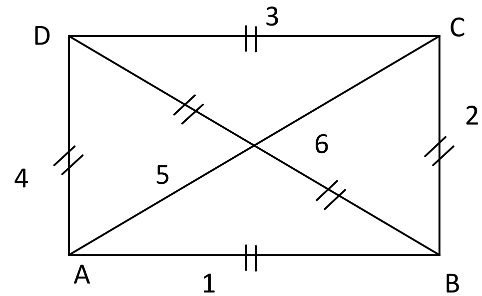
When n=1, then the mechanism can be driven by a single input motion.



c) Five bar mechanism: As per fig. l=5 & j = 5 from eqn. (5)
$N = 3X(5-1) - 2X5$
$\Rightarrow n = 12-10=2$
When n=2, the two separate input motions are necessary to produce constrained motions for the mechanism.



d) Six bar mechanism: As per the fig. we have
$l = 6$ & $j = 4X2 = 8$
(As we have 4 ternary joints and one tenerary joints is equivalent to 2 binary joint)



$n = 3X(l-1) - 2Xj$
$\Rightarrow n = 3X5-2X8 \Rightarrow n = -1$
When n=(-1) or less, then there are redundant constraints in the chain and it forms a statically interminate structure.

e) Inversion of mechanism: The method of obtaining different mechanism by fixing different links in a kinematic chain is known as inversion of mechanism. It can be noted that relative motions between the various links is not changed in any manner through the process of inversion, but their absolute motions may be changed drastically.

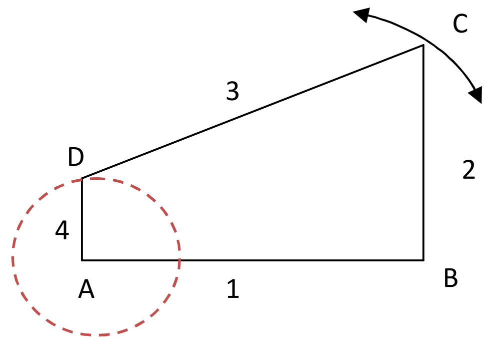
The part of mechanism, which initially moves with respect to the frame or fixed link, is called driver and part of the mechanism to which motion is transmitted is called follower.

L)

<u>Types of Kinematic chain</u>

- 4-Bar chain
- Single slider crank chain
- Double slider crank chain

- 4-Bar chain or Quadric cycle chain:



The simplest form of kinematic chain is a four bar chain or quadric cycle chain as per fig. It consists of four links, each of them forms a turning pair at A,B,C,D. The links may of different lengths. For this consideration, let us consider link DA – shortest link AB,CD are intermediate links.

According to Grashof's law for a four bar link mechanism, the sum of the shortest and the longest links lengths should not be greater than the sum of remaining two links lengths if there is to be continuous relative motion between the two links.
If length of links AB,BC,CD,DAB are a,b,c,d then as per Grashof's Law.

$$(b+d) \ngtr (a+c)$$

$$\text{Or } (b+d) \leq (a+c)$$

Now let us use this four bar link mechanism to understand the

Fig.: 4-Bar link mechanism & piston – connecting rod mechanism

Let us consider AB link as rigid one. The shortest link DA is the turning pair main link which is generally known as driver. This link can be considered as equivalent to motion of crank’s rotating motion.
The longest link BC, which is connected with DA, via CD will have a reciprocating piston’s reciprocating the connecting rod, which is helping to convert the rotating motion of crank to reciprocating motion of the piston.