5. Four Bar Mechanism

A co-planar four-link mechanism with four revolute joints (R-R-R-R) is known as a four bar mechanism. Like the slider-crank mechanism and its kinematic inversions (R-R-R-P) it is one of the basic building blocks of many machines.

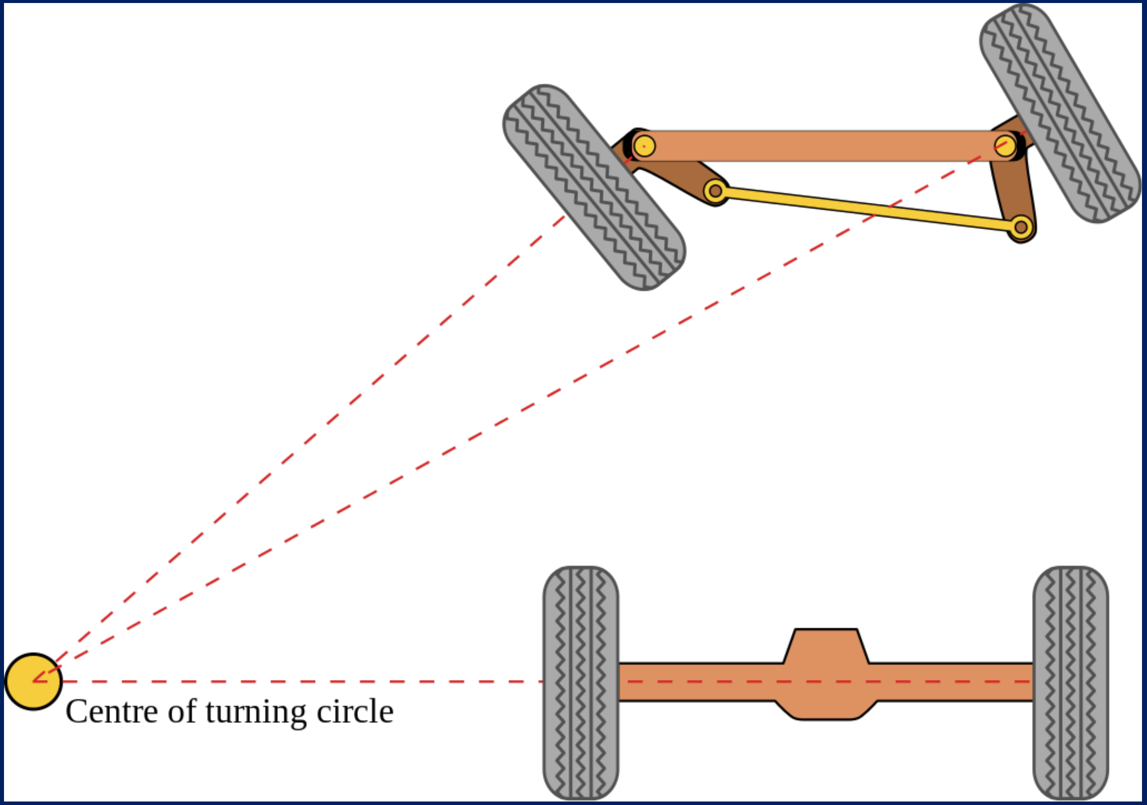
After covering kinematic and force analysis in general we will concentrate on four-bar mechanisms.

5. Four Bar Mechanism

All uses of a four-bar can be categorized in one of these three groups:

1. Correlation of Crank Angles / Function Generation

$$\theta_{14} = f(\theta_{12})$$



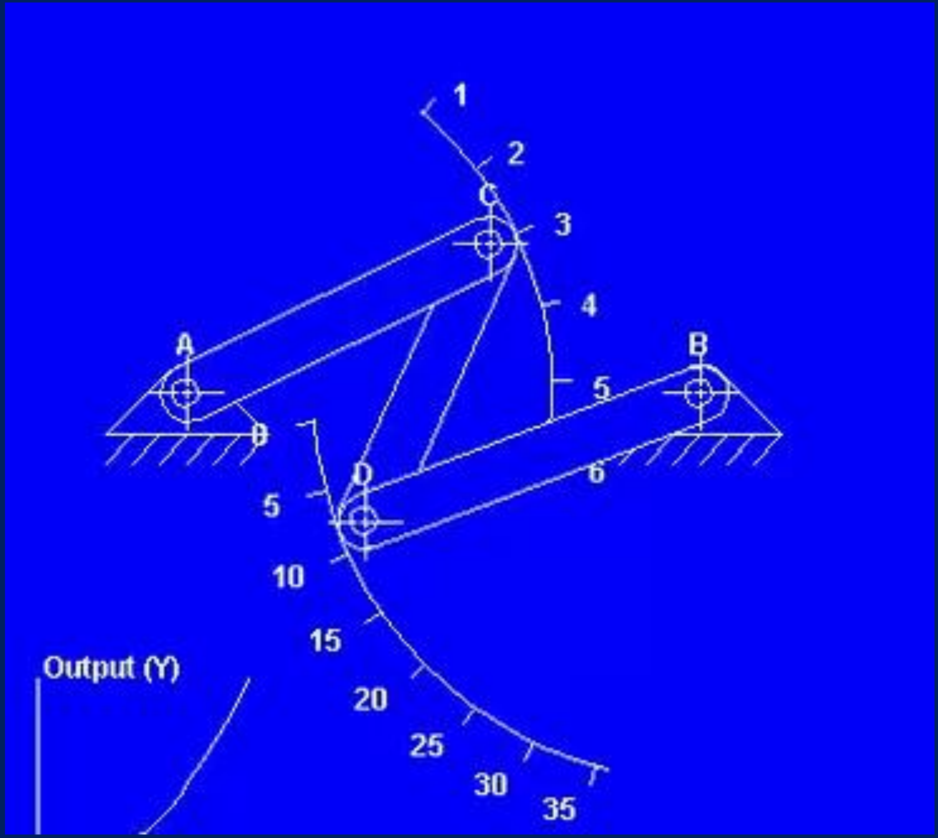
5. Four Bar Mechanism

All uses of a four-bar can be categorized in one of these three groups:

1. Correlation of Crank Angles / Function Generation

$\theta_{14} = f(\theta_{12})$

$y = x^2$



https://www.youtube.com/watch?v=HAdjYblt3hM

5. Four Bar Mechanism

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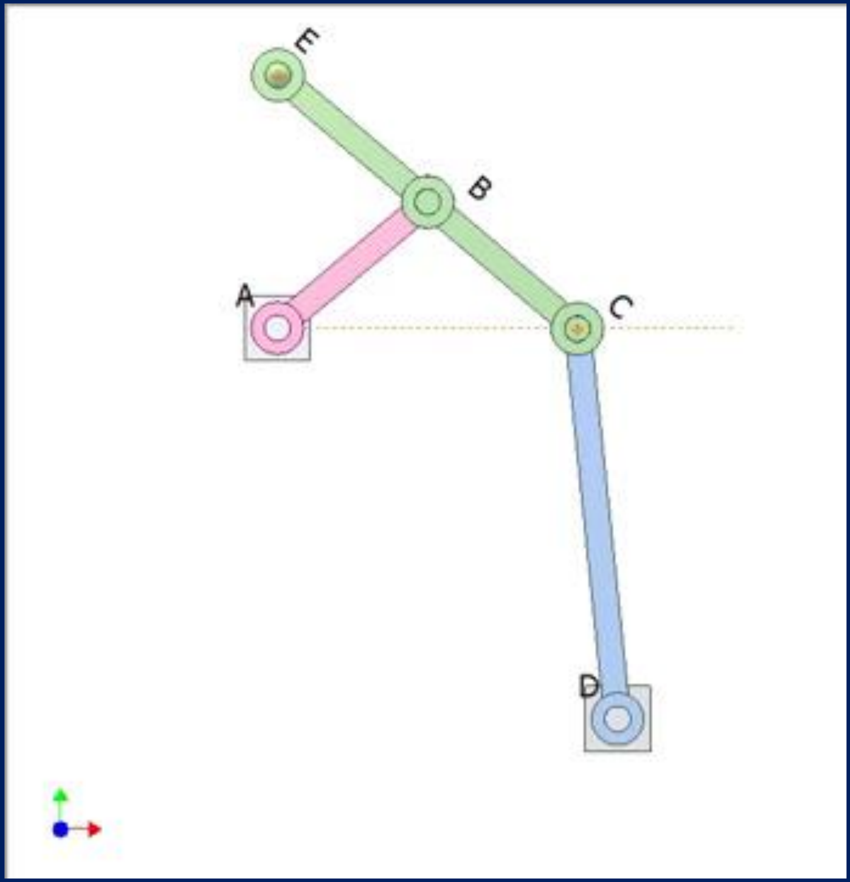
https://www.youtube.com/watch?v=noL5D4jLW6A

5. Four Bar Mechanism

All uses of a four-bar can be categorized in one of these three groups:

2. Coupler point curve



https://www.youtube.com/watch?v=4U8aREBFIDE

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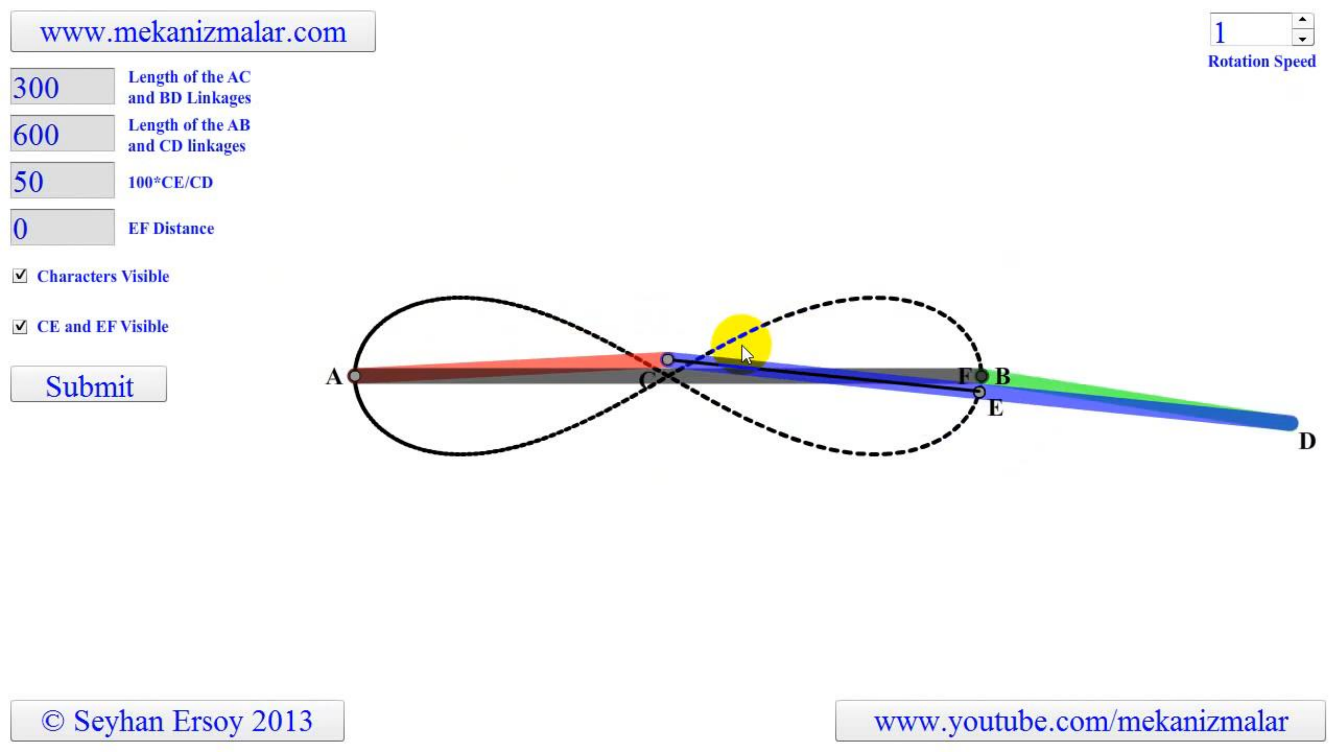

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5. Four Bar Mechanism

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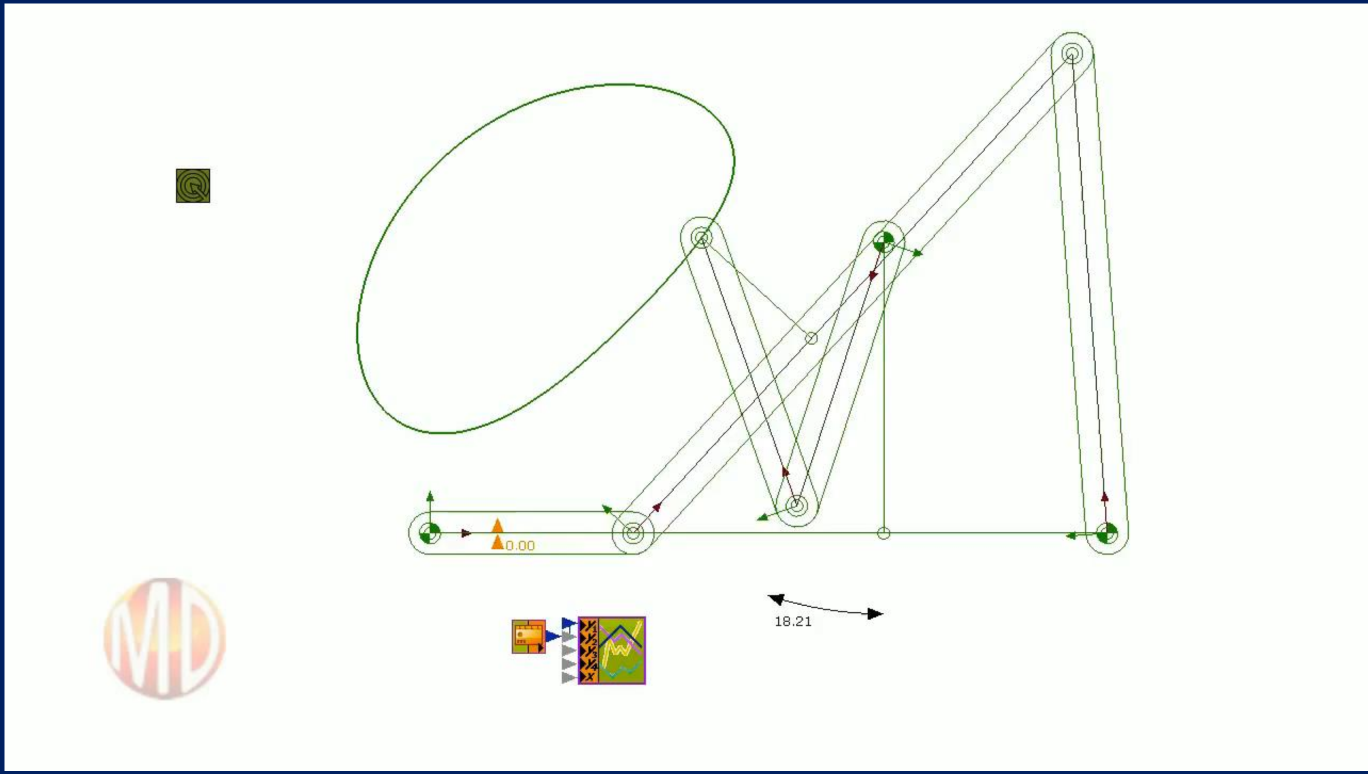
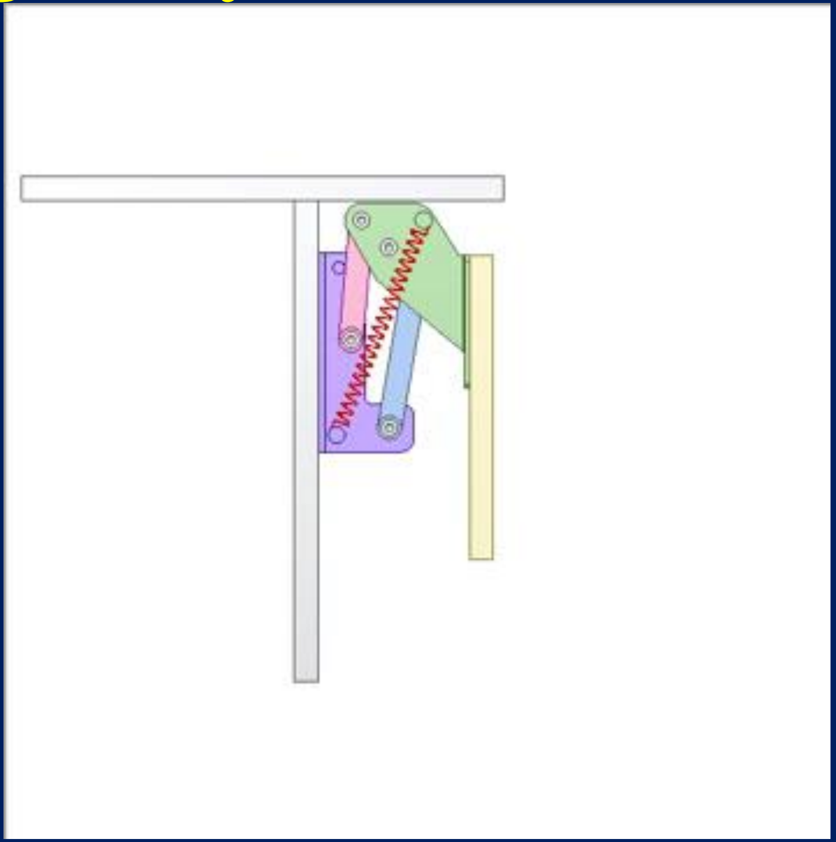


https://www.youtube.com/watch?v=ESpEFJZp-co

5. Four Bar Mechanism

All uses of a four-bar can be categorized in one of these three groups:

3. Positions of Coupler Link / Rigid Body Guidance



https://www.youtube.com/watch?v=TLROAYXxkvA

5. Four Bar Mechanism

Grashof's Rule:

The motion characteristics of a four-bar mechanism will depend on the ratio of link lengths.

The links that are connected to the fixed link may have one of these two types of motion:

- The link may have a full rotation around the fixed axis. This is called a *crank* (crank is also used for the input link)
- The link may oscillate between two limiting angles. This is called a *rocker*.

5. Four Bar Mechanism

Grashof's Rule:

A four-bar mechanism may have one of the following three types of motion:

- Both links connected to the fixed link can have full rotation. This is called *double crank* or *drag link* mechanism.
- Both links connected to the fixed link can oscillate between two limiting positions. This is called *double rocker* mechanism.
- One of the links connected to the fixed link oscillates between two limiting positions while other can make full rotation. This is called *crank rocker* mechanism.

5. Four Bar Mechanism

Grashof's Rule:

ℓ: Length of the longest link

s: Length of the shortest link

p, q: lengths of the two intermediate links

1. If $\ell + s < p + q$
   a. Two different crank-rocker mechanisms are possible. In either case shortest link is the crank and the fixed link is either of the adjacent links.
   b. One drag link (double crank) is possible when the shortest link is fixed.
   c. One double rocker mechanism is possible when the link opposite to the shortest is fixed.

5. Four Bar Mechanism

Grashof's Rule:

2. If $\ell + s > p + q$

   Only four different double rocker mechanisms are possible.

3. If $\ell + s = p + q$

   Same as case 1 however these mechanisms suffer from a condition known as *change point position*. At this position all the link center lines are collinear and this is a kinematically singular (undetermined) position. The follower at this position may rotate in either direction. The sign (σ) that determines the closure is subject to change at this position.

5. Four Bar Mechanism

Dead-Center Positions of a *Crank Rocker* Mechanism:

Grashof's Rule

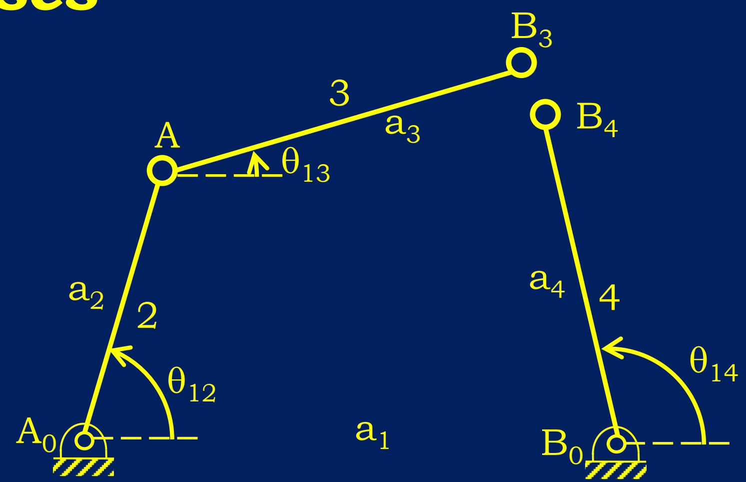
$\ell + s < p + q$, s is the crank

a. Two different crank-rocker mechanisms are possible. In either case shortest link is the crank and the fixed link is either of the adjacent links.

2. Kinematic Analysis

Velocity and Acceleration Analyses

Four-Bar Mechanism



$a_2e^{i\theta_{12}} + a_3e^{i\theta_{13}} = a_1 + a_4e^{i\theta_{14}}$

Re: $a_2cos\theta_{12} + a_3cos\theta_{13} = a_1 + a_4cos\theta_{14}$ (1)

Im: $a_2sin\theta_{12} + a_3sin\theta_{13} = a_4sin\theta_{14}$ (2)

$(\dot{1})$: $-\dot{\theta}_{12}a_2sin\theta_{12} - \dot{\theta}_{13}a_3sin\theta_{13} = -\dot{\theta}_{14}a_4sin\theta_{14}$

$(\dot{2})$: $\dot{\theta}_{12}a_2cos\theta_{12} + \dot{\theta}_{13}a_3cos\theta_{13} = \dot{\theta}_{14}a_4cos\theta_{14}$

Let θ_{12}, $\dot{\theta}_{12}$ and $\ddot{\theta}_{12}$ be the input:

$$\begin{bmatrix} -a_3sin\theta_{13} & a_4sin\theta_{14} \\ a_3cos\theta_{13} & -a_4cos\theta_{14} \end{bmatrix} \begin{bmatrix} \dot{\theta}_{13} \\ \dot{\theta}_{14} \end{bmatrix} = \begin{bmatrix} a_2sin\theta_{12} \\ -a_2cos\theta_{12} \end{bmatrix} \dot{\theta}_{12}$$

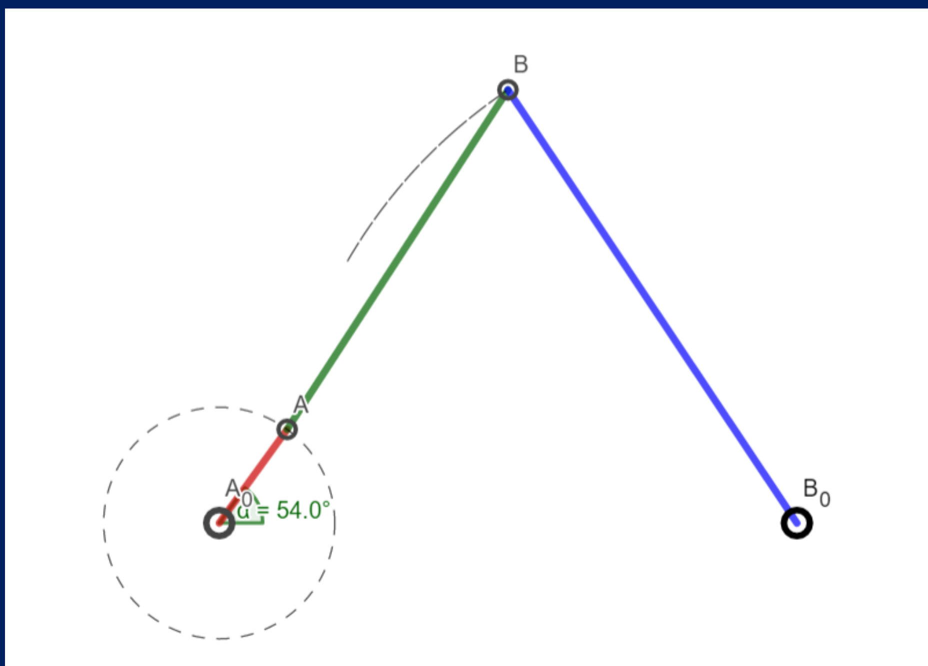
$$\dot{\theta}_{14} = \frac{a_2sin(\theta_{12} - \theta_{13})}{a_4sin(\theta_{14} - \theta_{13})}\dot{\theta}_{12} = g_{24}\dot{\theta}_{12}$$

$$\dot{\theta}_{14} = 0 \text{ when } sin(\theta_{12} - \theta_{13}) = 0 \rightarrow \begin{cases} \theta_{12} - \theta_{13} = 0, \text{Extended dead center} \\ \theta_{12} - \theta_{13} = \pi, \text{Folded dead center} \end{cases}$$

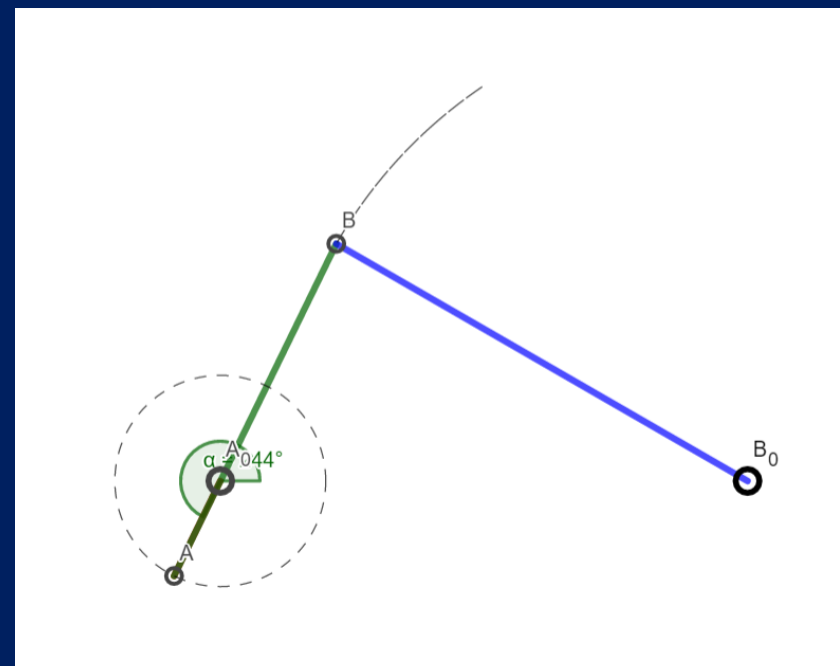
5. Four Bar Mechanism

Dead-Center Positions of a Crank Rocker Mechanism:

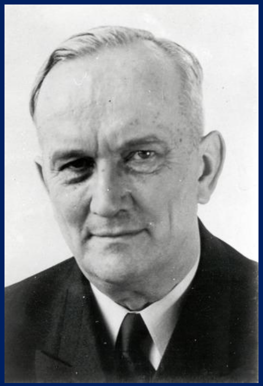
$\theta_{12} - \theta_{13} = 0$, Extended dead center



$\theta_{12} - \theta_{13} = \pi$, Folded dead center



5. Four Bar Mechanism

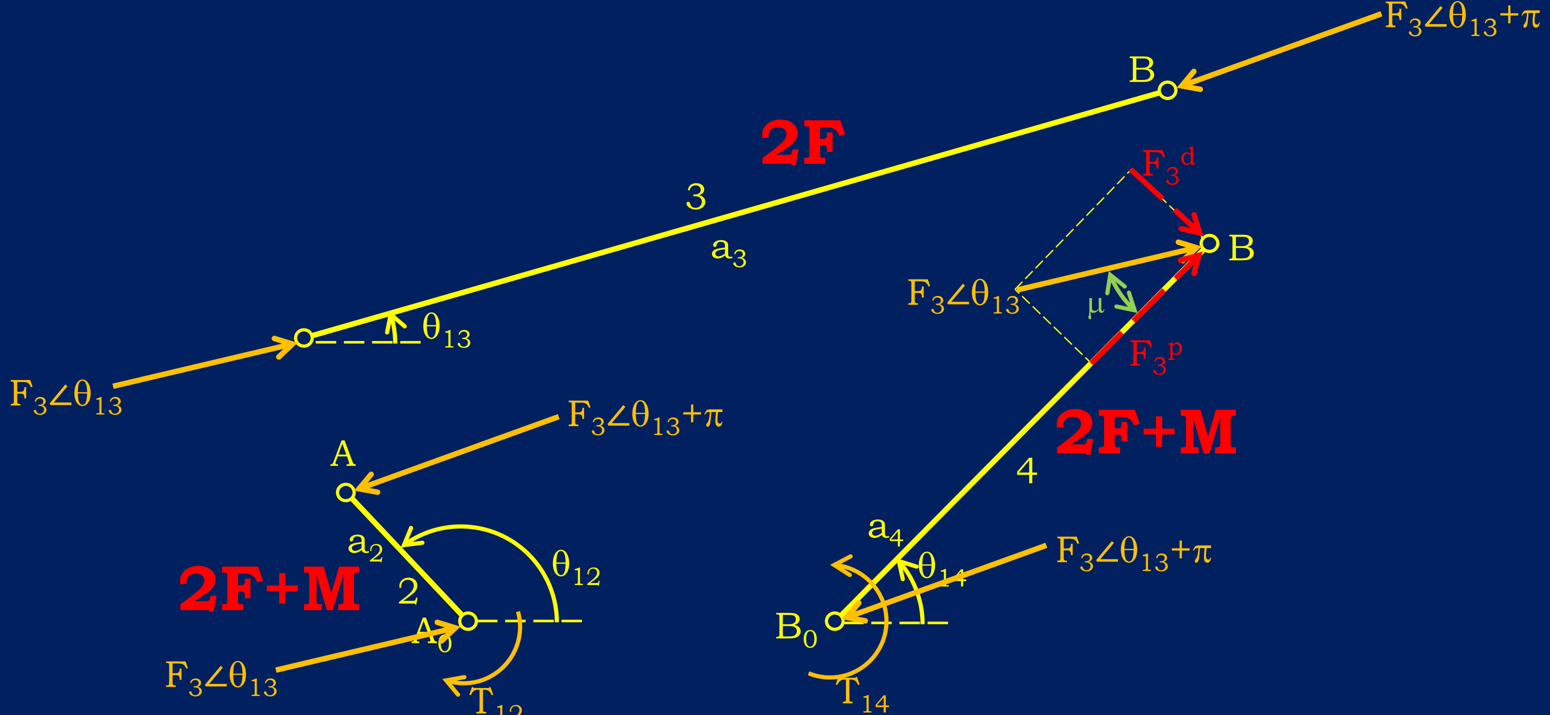


Transmission Angle:

Alt[1] defined the transmission angle as:

$$tan\mu = \frac{F_3{}^d}{F_3{}^p} \text{ or } sin\mu = \frac{F_3{}^d}{F_3}$$

[1] Alt, Hermann (1889 - 1954). Der Übertragungswinkel und seine Bedeutung für das Konstruieren periodischer Getriebe (*The transmission angle and its importance for designing periodic mechanisms*). Werkstattstechnik 26 (1932) 61–64.



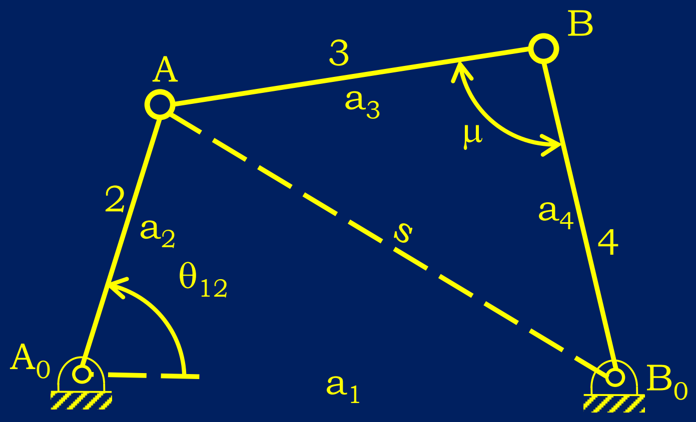
2. Kinematic Analysis

2. Stepwise Solution of Loop Closure Equations

Law of cosines:

$$s^2 = a_1{}^2 + a_2{}^2 - 2a_1a_1cos\theta_{12}$$

$$s^2 = a_3{}^2 + a_4{}^2 - 2a_3a_4cos\mu$$



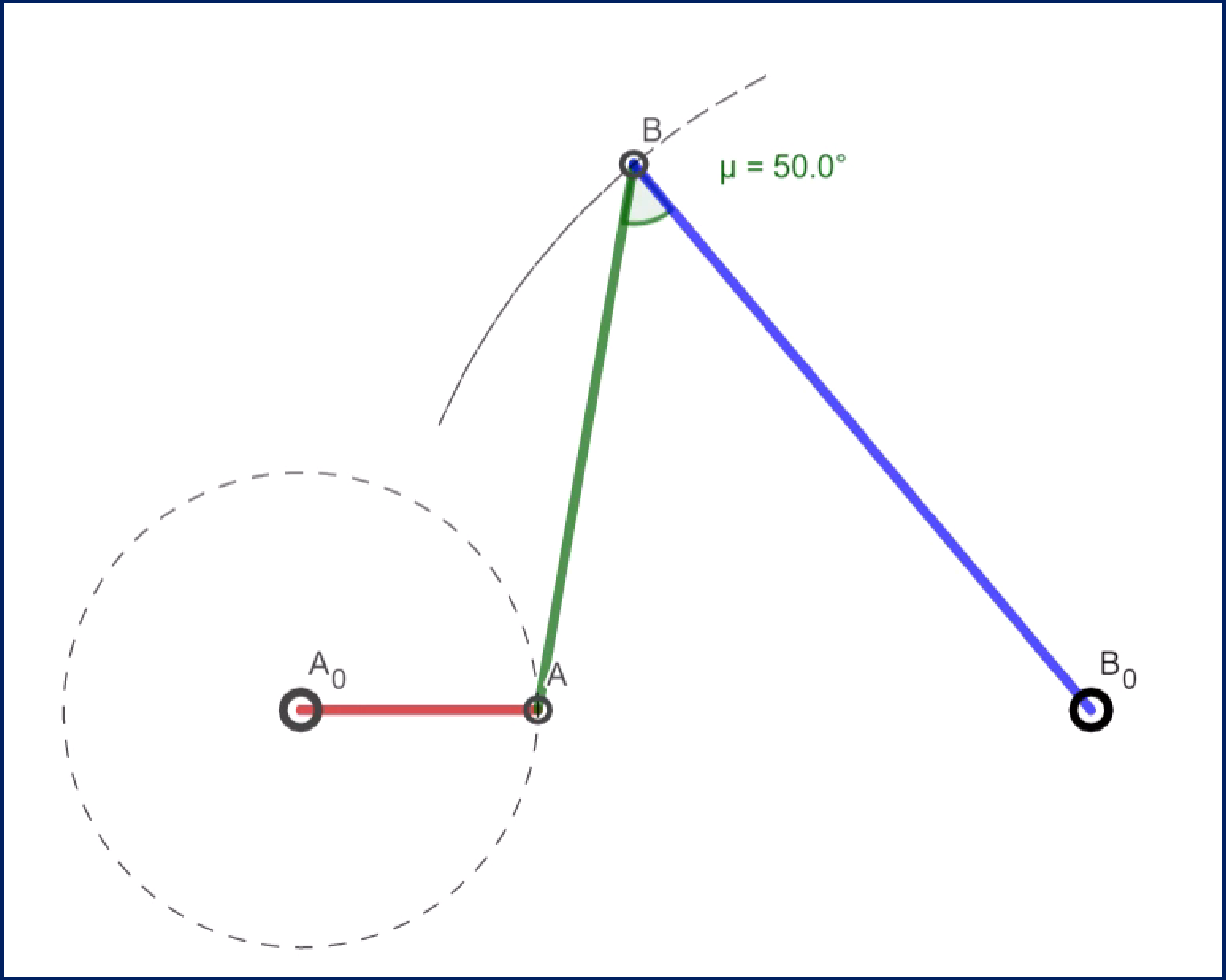
$$cos\mu = \frac{a_3{}^2 + a_4{}^2 - a_1{}^2 - a_2{}^2 + 2a_1a_2cos\theta_{12}}{2a_3a_4}$$

The extremums of the transmission angle is

$$\frac{d\mu}{d\theta_{12}} = sin\theta_{12} = 0 \rightarrow \begin{cases} \theta_{12} = 0 \\ \theta_{12} = \pi \end{cases}$$

5. Four Bar Mechanism

Transmission Angle:



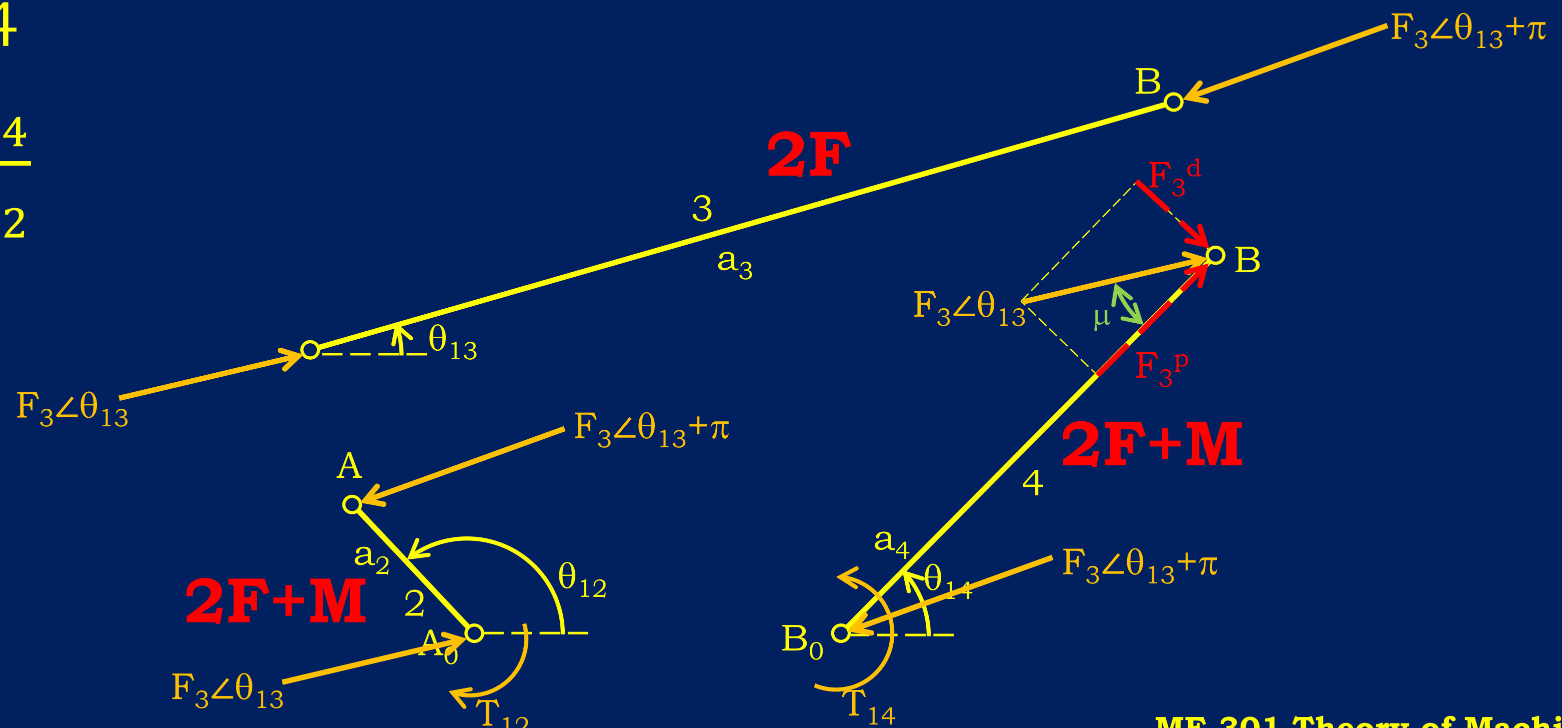
5. Four Bar Mechanism

Mechanical Advantage:

<u>Definition</u>: The mechanical advantage of a mechanism is the instantaneous ratio of output torque (force) to input torque (force).

For a four bar mechanism where input is link 2 and output is link 4

$$MA = \frac{T_{14}}{T_{12}}$$



5. Four Bar Mechanism

Mechanical Advantage:

$$MA = \frac{T_{14}}{T_{12}}$$

Neglecting friction, kinetic and gravitational potential energy changes of the links

$$\mathbb{P}_{in} = \mathbb{P}_{out}$$

$$-T_{12}\omega_{12} = T_{14}\omega_{14}$$

$$MA = \frac{T_{14}}{T_{12}} = -\frac{\omega_{12}}{\omega_{14}}$$

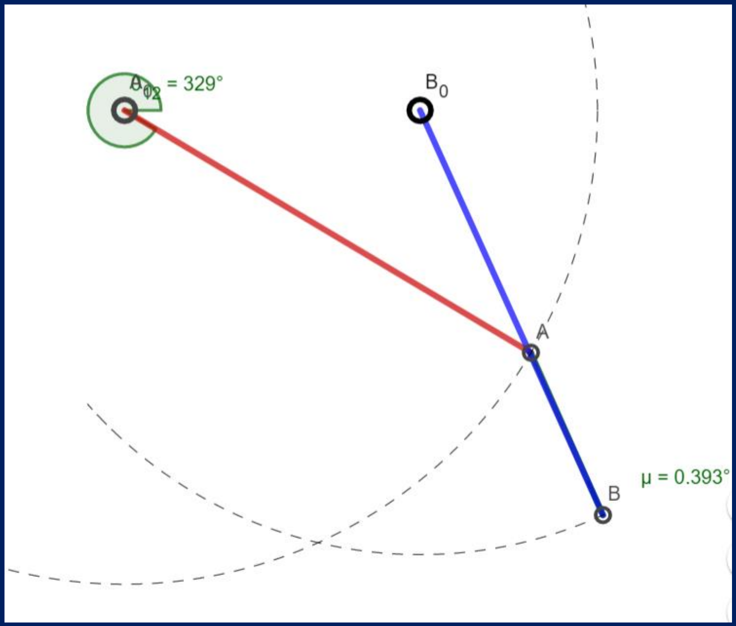
5. Four Bar Mechanism

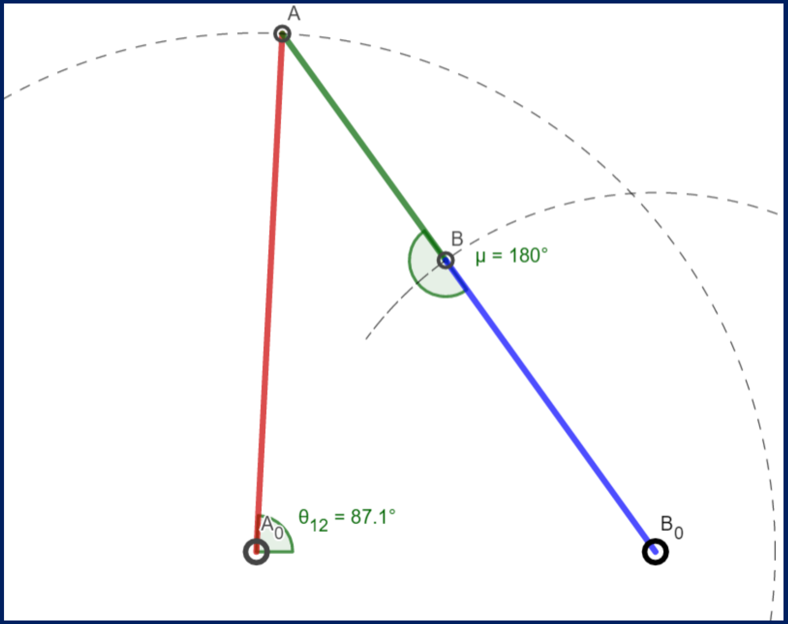
Mechanical Advantage:

$$MA = \frac{T_{14}}{T_{12}} = -\frac{\omega_{12}}{\omega_{14}} = -\frac{\dot{\theta}_{12}}{\dot{\theta}_{14}} = \frac{a_4 sin(\theta_{14} - \theta_{13})}{a_2 sin(\theta_{12} - \theta_{13})}$$

$sin(\theta_{12} - \theta_{13}) = 0, MA \rightarrow \infty$ Dead centers!

$sin(\theta_{14} - \theta_{13}) = 0, MA = 0, \mu = 0$





5. Four Bar Mechanism

Body Guidance-Two Position Synthesis:

Chasles Theorem: The motion of a rigid body from one position to another in plane motion occurs most simply by a rotation about the pole P_{12} which is located at the perpendicular bisectors of two pairs of *homologous points*.

Please note that when the separation between the two positions of the plane diminishes the homologous points define the velocity and the pole P_{12} boils down to instant center of zero velocity (ICZV).

5. Four Bar Mechanism

Body Guidance-Two Position Synthesis:

<u>Chasles Theorem</u>: