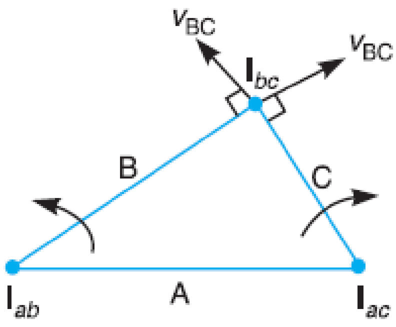


Fig. 3.15 Aronhold Kennedy's theorem

– We have already discussed that the velocity of the instantaneous centre is same whether it is regarded as a point on the first link or as a point on the second link. Therefore, the velocity of the point Ibc cannot be perpendicular to both lines Iab Ibc and Iac Ibc unless the point Ibc lies on the line joining the points Iab and Iac. Thus the three instantaneous centres (Iab, Iac and Ibc) must lie on the same straight line. The exact location of Ibc on line Iab Iac depends upon the directions and magnitudes of the angular velocities of B and C relative to A.

3.13 Acceleration Diagram for a Link

– Consider two points A and B on a rigid link as shown in Fig. (a). Let the point B moves with respect to A, with an angular velocity of **ω** rad/s and let **α** rad/s2 be the angular acceleration of the link AB.

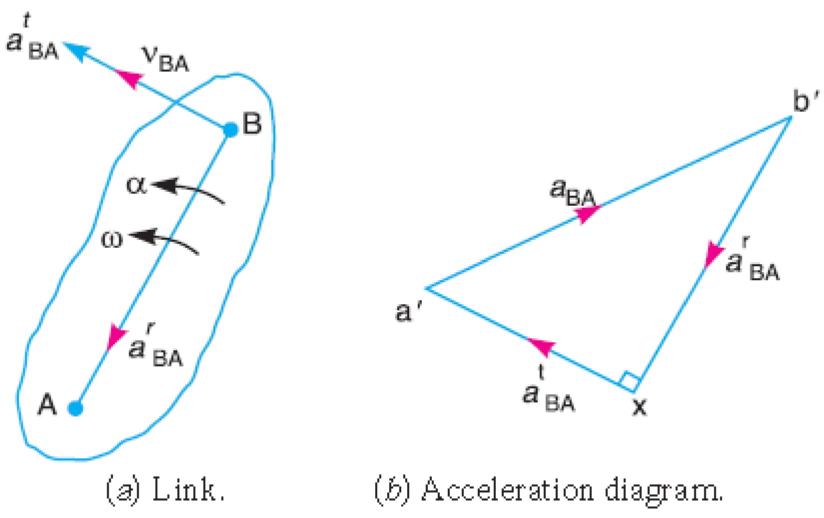


Fig. 3.16 Acceleration of a link

– We have already discussed that acceleration of a particle whose velocity changes both in magnitude and direction at any instant has the following two components .

1 The centripetal or radial component, which is perpendicular to the velocity of the particle at the given instant.

2 The tangential component, which is parallel to the velocity of the particle at the given instant.

– Thus for a link A B, the velocity of point B with respect to A (i.e. vBA) is perpendicular to the link A B as shown in Fig.(a). Since the point B moves with respect to A with an angular velocity of ω rad/s, therefore centripetal or radial component of the
acceleration of B with respect to A

$$a^r_{BA} = \omega^2 \times Length\ of\ link\ AB = \omega^2 \times AB = {v^2_{BA}}/{AB}$$

– This radial component of acceleration acts perpendicular to the velocity vBA, In other words, it acts parallel to the link AB.

We know that tangential component of the acceleration of B with respect to A ,

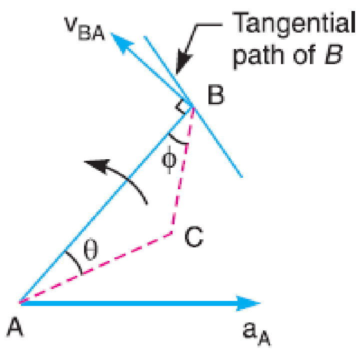
$$a^t_{BA} = \alpha \times Length\ of\ link\ AB = \alpha \times AB$$

– This tangential component of acceleration acts parallel to the velocity vBA. In other words, it acts perpendicular to the link AB.

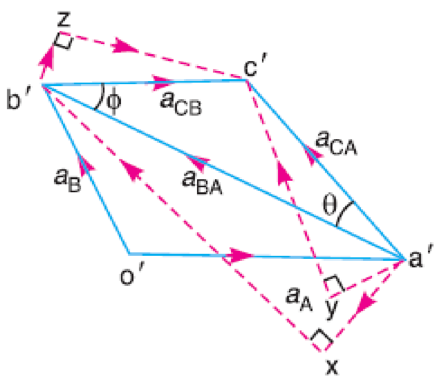
– In order to draw the acceleration diagram for a link A B, as shown in Fig. 8.1 (b), from any point b', draw vector b'x parallel to BA to represent the radial component of acceleration of B with respect to A.

3.14 Acceleration of a Point on a Link

– Consider two points A and B on the rigid link, as shown in Fig. (a). Let the acceleration of the point A i.e. aA is known in magnitude and direction and the direction of path of B is given. The acceleration of the point B is determined in magnitude and direction by drawing the acceleration diagram as discussed below.



(a) Points on a Link.



(b) Acceleration diagram.

Fig. 3.17 acceleration of a point on a link

– From any point o', draw vector o'a' parallel to the direction of absolute acceleration at point A i.e. a_A , to some suitable scale, as shown in Fig. 8.2 (b).

– We know that the acceleration of B with respect to A i.e. aBA has the following two components:

1 Radial component of the acceleration of B with respect to A i.e. a^r_{BA}

2 Tangential component of the acceleration B with respect to A i.e. a^t_{BA}

- Draw vector a’x parallel to the link AB such that,

$$\boldsymbol{vector\ a'x = a^r_{BA} = v^2_{BA}/AB}$$

- From point x, draw vector xb’ perpendicular to AB or vector a’x and through o’ draw a line parallel to the path of B to represent the absolute acceleration of B i.e. a_B
- By joining the points a' and b' we may determine the total acceleration of B with respect to A i.e. a_{BA}. The vector a' b' is known as acceleration image of the link AB.
- For any other point C on the link, draw triangle a' b' c' similar to triangle ABC. Now vector b' c' represents the acceleration of C with respect to B i.e. a_{CB}, and vector a' c' represents the acceleration of C with respect to A i.e. a_{CA}. As discussed above, a_{CB} and a_{CA} will each have two components as follows :
  a. a_{CB} has two components; $\boldsymbol{a^r_{CB}}$ and $\boldsymbol{a^t_{CB}}$ as shown by triangle b’zc’ in fig.b
  b. a_{CA} has two components; $\boldsymbol{a^r_{CA}}$ and $\boldsymbol{a^t_{CA}}$ as shown by triangle a’yc’
- The angular acceleration of the link AB is obtained by dividing the tangential component of acceleration of B with respect to A to the length of the link.

$$\boldsymbol{\alpha_{AB} = a^t_{BA}/AB}$$

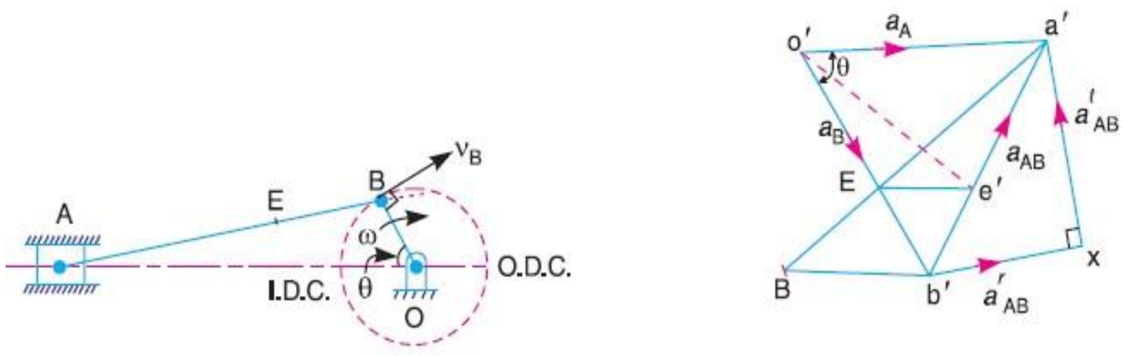
3.15 Acceleration in Slider Crank Mechanism

- A slider crank mechanism is shown in Fig. 8.3 (a). Let the crank OB makes an angle **θ** with the inner dead centre (I.D.C) and rotates in a clockwise direction about the fixed point O with uniform angular velocity $\boldsymbol{\omega}_{BO}$ rad/s
- Velocity of B with respect to O or velocity of B (because O is a fixed point),

$$\boldsymbol{v_{BO} = v_B = \omega_{BO} \times OB\ acting\ tangentially\ at\ B}$$

- We know that centripetal or radial acceleration of B with respect to O or acceleration of B (Because O is a fixed point)

$$\boldsymbol{a^r_{BO} = a_B = \omega^2_{BO} \times OB = \frac{v^2_{BO}}{BO}}$$



(a) Slider crank mechanism. (b) Acceleration diagram.

Fig. 3.18 acceleration in the slider crank mechanism

- The acceleration diagram, as shown in Fig. 8.3 (b), may now be drawn as discussed below:

1 Draw vector o' b' parallel to BO and set off equal in magnitude of a=a, to some BO suitable scale.

2 From point b', draw vector b'x parallel to BA. The vector b'x represents the radial component of the acceleration of A with respect to B whose magnitude is given by :

$$a^r_{AB} = v^2_{AB}/BA$$

3 From point x, draw vector xa’ perpendicular to b’x. The vector xa’ represents the tangential components of the acceleration of A with respect to B.

4 Since the point A reciprocates along AO, therefore the acceleration must be parallel to velocity. Therefore from o', draw o' a' parallel to A O, intersecting the vector xa' at a'.

5 The vector b' a', which is the sum of the vectors b' x and x a', represents the total acceleration of A with respect to B i.e. a_{AB}. The vector b'a' represents the acceleration of the connecting rod AB.

6 The acceleration of any other point on A B such as E may be obtained by dividing the vector b' a' at e' in the same ratio as E divides A B in Fig. 8.3 (a). In other words

$$a'e'/a'b' = AE/AB$$

7 The angular acceleration of the connecting rod A B may be obtained by dividing the tangential component of the acceleration of A with respect to B to the length of AB. In other words, angular acceleration of AB,

$$\alpha_{AB} = a^t_{AB}/AB$$

3.16 Examples Based on Acceleration

3.16.1 The crank of the slider crank mechanism rotates clockwise at a constant speed of 300 r.p.m. The crank is 150 mm and the connecting rod is 600 mm long. Determine :

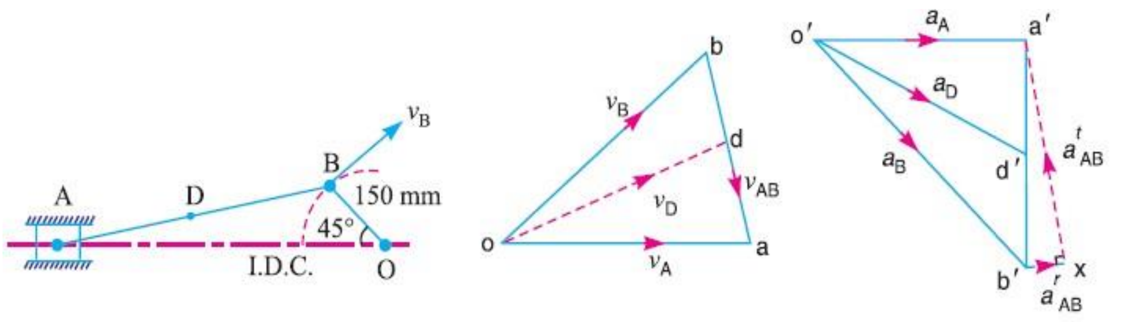
1. **Linear velocity and acceleration of the midpoint of the connecting rod, and**
2. **Angular velocity and angular acceleration of the connecting rod, at a crank angle of 45° from inner dead centre position**

- **Given:**
- N_{BO} = 300 r.p.m. or ω_{BO} = 2 π × 300/60 = 31.42 rad/s; OB = 150 mm = 0.15 m ; B A = 600 mm = 0.6 m
- We know that linear velocity of B with respect to O or velocity of B,

$$v_{BO} = v_B = \omega_{BO} \times OB = 31.42 \times 0.15 = 4.713\ m/s$$

- Draw vector ob perpendicular to BO, to some suitable scale, to represent the velocity of B with respect to O or simply velocity of B i.e. v_{BO} or vB, such that

vector ob = v_{BO} = v_B = 4.713 m/s



(a) Space diagram. (b) Velocity diagram. (c) Acceleration diagram.

Fig. 3.19

- From point b, draw vector ba perpendicular to BA to represent the velocity of A with respect to B i.e. v_{AB} , and from point o draw vector oa parallel to the motion of A (which is along AO) to represent the velocity of A i.e. v_A. The vectors ba and oa intersect at a.
- By measurement we find the velocity A with respect to B,

$$v_{AB} = vector\ ba = 3.4\ m/s$$
$$v_A = \text{vector oa} = 4\ \text{m/s}$$

- In order to find the velocity of the midpoint D of the connecting rod A B, divide the vector ba at d in the same ratio as D divides A B, in the space diagram. In other words,

$$bd/ba = BD/BA$$

- By measurement, we find that

$$\text{v}_\text{D} = \text{vector od} = 4.1\ \text{m/s}$$

- We know that the radial component of the acceleration of B with respect to O or the acceleration of B,

$$a^2_{BO} = a_B = \frac{v^2_{BO}}{OB} = \frac{(4.713)^2}{0.15} = 148.1\ m/s^2$$

- And the radial component of the acceleration of A with respect to B,

$$a^r_{\frac{A}{B}} = \frac{v^2_{AB}}{BA} = \frac{(3.4)^2}{0.6} = 19.3\ m/s^2$$
$$vector\ o'b' = a^r_{BO} = a_B = 148.1\ m/s^2$$

- By measurement, we find that

$$a = vector\ o'd' = 117\ m/s^2$$

- We know that angular velocity of the connecting rod AB,

$$\omega_{AB} = \frac{v_{AB}}{BA} = \frac{3.4}{0.6} = 5.67\ rad/s^2$$

- From the acceleration diagram, we find that

$$\text{a}^\text{t}_\text{AB} = 103\ \text{m/s}^2$$

- We know that angular acceleration of the connecting rod AB,

$$\alpha_{AB} = \frac{a_{AB}^t}{BA} = \frac{103}{0.6} = 171.67\ rad/s^2$$

3.16.2 An engine mechanism is shown in Fig. 8.5. The crank CB = 100 mm and the connecting rod BA = 300 mm with centre of gravity G, 100 mm from B. In the position shown, the crankshaft has a speed of 75 rad/s and an angular acceleration of 1200 rad/s². Find:

1. Velocity of G and angular velocity of AB, and

2. Acceleration of G and angular acceleration of AB.

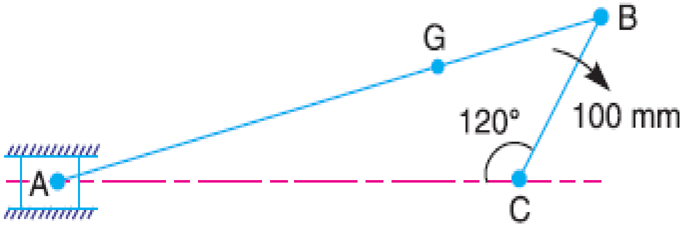


Fig. 3.20

- **Given :**
- ω_{BC} = 75 rad/s ; α_{BC} = 1200 rad/s², CB = 100 mm = 0.1 m; B A = 300 mm = 0.3 m
- We know that velocity of B with respect to C or velocity of B

$$v_{BC} = v_B = \omega_{BC} \times CB = 75 \times 0.1 = 7.5\ m/s$$

- Since the angular acceleration of the crankshaft, α_{BC} = 1200 rad/s², therefore tangential component of the acceleration of B with respect to C,

$$a_{BC}^t = \alpha_{BC} \times CB = 1200 \times 0.1 = 120\ m/s^2$$

$$vector\ cb = v_{BC} = v_B = 7.5\ \frac{m}{s}$$

- By measurement, we find that velocity of G,

$$v_G = ector\ cg = 6.8\ m/s$$

- From velocity diagram, we find that the velocity of A with respect to B,

$$v_{AB} = vector\ ba = 4\ m/s$$

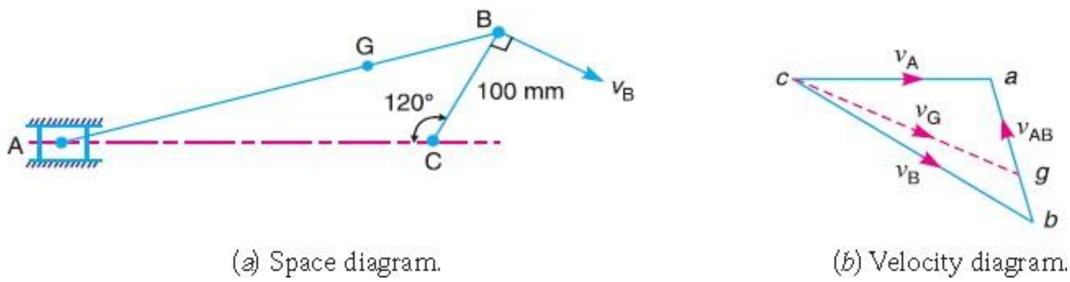
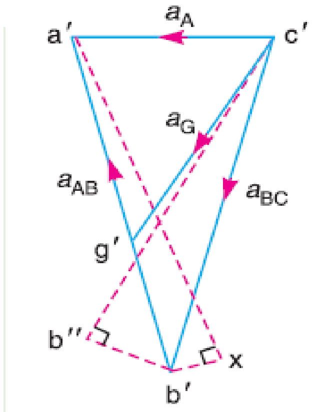


Fig. 3.21

- We know that angular velocity of AB,

$$\omega_{AB} = \frac{v_{AB}}{BA} = \frac{4}{0.3} = 13.3\ rad/s$$



(c) Acceleration diagram.

Fig. 3.22

- We know that radial component of the acceleration of B with respect to C

$$a^r_{BC} = \frac{v^2_{BC}}{CB} = \frac{(7.5)^2}{0.1} = 562.5\ m/s^2$$

- And radial component of the acceleration of A with respect to B,

$$a^r_{AB} = \frac{v^2_A}{CB} = \frac{(4)^2}{0.3} = 53.3\ m/s^2$$

$$vector\ c'b'' = {}^r\ {}_{BC} = 562.5\ m/s^2$$

$$vecto\ ''b' = a^t\ {}_{BC} = 120\ m/s^2$$

$$vecto\ 'x = a^r\ {}_{AB} = 53.3\ m/s^2$$

- By measurement we find that acceleration of G,

$$a_G = vector\ xa' = 414\ m/s^2$$

- From acceleration diagram, we find that tangential component of the acceleration of A with respect to B,

$$a^t_{AB} = ector\ xa' = 546 m/s^2$$

- Angular acceleration of AB

$$\alpha_{AB} = \frac{a^t_{AB}}{BA} = \frac{546}{0.3} = 1820\ rad/s^2$$

3.16.3 In the mechanism shown in Fig. 8.7, the slider C is moving to the right with a velocity of 1 m/s and an acceleration of 2.5 m/s2.The dimensions of various links are AB = 3 m inclined at 45° with the vertical and BC = 1.5 m inclined at 45° with the horizontal. Determine: 1. the magnitude of vertical and horizontal component of the acceleration of the point B, and 2. the angular acceleration of the links AB and BC.

- **Given:**
- v_C = 1 m/s ; a_C = 2.5 m/s2; AB = 3 m ; BC = 1.5 m
- Here,

$$vector\ d = v_{CD} = v_c = 1 m/s$$

- By measurement, we find that velocity of B with respect to A

$$v_{BA} = vector\ ab = 0.72\ m/s$$

– Velocity of B with respect to C

$$v_{BC} = vector\ cb = 0.72\ m/s$$

– We know that radial component of acceleration of B with respect to C,

$$a^r_{BC} = \frac{v^2_{BC}}{CB} = \frac{(0.72)^2}{1.5} = 0.346\ m/s^2$$

– And radial component of acceleration of B with respect to A,

$$a^r_{BA} = \frac{v^2_{BA}}{AB} = \frac{(0.72)^2}{3} = 0.173\ m/s^2$$

$$vectro\ d'c' = a_{cd} = a_c = 2.5\ m/s^2$$

$$vecto\ 'x = a^r_{BC} = 0.346\ \frac{m}{s^2}$$

$$vecto\ 'y = a^r_{BA} = 0.173\ m/s^2$$

– By measurement,

$$vector\ b'b'' = 1.13\ m/s^2$$

– By measurement from acceleration diagram, we find that tangential component of acceleration of the point B with respect to A

$$a^t_{BA} = ector\ yb' = 1.41\ m/s^2$$

– And tangential component of acceleration of the point B with respect to C,

$$a^t_{BC} = vector\ xb' = 1.94\ m/s^2$$

– we know that angular velocity of AB,

$$\alpha_{AB} = \frac{v^t_{BA}}{AB} = 0.47\ rad/s^2$$

– And aglular acceleration of BC,

$$\alpha_{BC} = \frac{a^t_{BC}}{CB} = \frac{1.94}{1.5}\ rad/s^2$$