



Coimbatore - 641 107

TOPIC : 7 – HALF RANGE COSINE SERIES

Half range cosine series in the interval (0, TD, 10, 1) Formula: (0, TD) fix) = ao to an cosnix $a_0 = \frac{2}{b-a} \int_{a}^{b} f(x) dx$ $a_n = \frac{2}{b-a} \int_{b-a}^{b} f(x) \cos nx dx$ Formula: (0, l) fini= ao to an cosmix de $a_0 = \frac{2}{L} \int f(x) dx$ an = 2 / fras cosnonda () Find the half range cosine series of fix)=2 in (0, TT). Sol: f(x) = x $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$ $a_0 = \frac{2}{\pi} \int x \, dx$ $\frac{2}{2} \left(\frac{\chi^2}{2}\right)^{U}$ 0/ 502

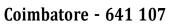




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 $a_n = \frac{2}{\pi} \int x \cos nx \, dx$ $= \frac{2}{\pi} \left[\frac{\chi}{n} \frac{\sin n \chi}{n} + \frac{\cos n \chi}{n^2} \right]_0^{\pi}$ $= \frac{2}{\pi} \left[\frac{(-1)^n}{n^2} - \frac{1}{n^2} \right]$ $a_n = \frac{2}{n^2 \pi} \left[-1 + (-1)^n \right]$ $a_n = \begin{cases} \frac{-4}{m^2 \pi} & \text{if } n \text{ is odd} \\ 0 & \text{if } n \text{ is even} \end{cases}$ fix)= TI + 5 2 [1- (-D)] cosnx $f(n) = \frac{\pi}{2} + \frac{2}{n-n} d = \frac{-4}{n^2 \pi} cosnal$ 2. Find the half range cosine series of the function $f(x) = x (\pi - x)$ in $(0, \pi)$. Sol: $f(x) = x \pi - x^2$ \$121= ao + 2° an cosha $ao = \frac{2}{\pi} \int (2\pi - x^2) dx$ $=\frac{2}{2}\left[\frac{\chi^2}{2}\Pi-\frac{\chi^2}{3}\right]_0^{\Pi}$ $=\frac{2}{3}\left[\frac{71^{3}}{3}-\frac{71^{3}}{3}\right]$







$$A_{n} = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx \, dx$$

$$= \frac{2}{\pi} \left[-\frac{x}{n} \frac{\sin nx}{n} + \frac{\cos nx}{nz} \int_{0}^{\pi} \right]$$

$$= \frac{2}{\pi} \left[\frac{(-1)^{n}}{n^{2}} - \frac{1}{n^{2}} \right]$$

$$a_{n} = \frac{2}{n^{2} \pi} \left[-1 + (-1)^{n} \right]$$

$$a_{n} = \int_{1}^{-\frac{n}{2}} \frac{1}{n^{2} \pi} \quad \text{if } n \text{ is odd}$$

$$a_{n} = \int_{1}^{-\frac{n}{2}} \frac{1}{n^{2} \pi} \quad \text{if } n \text{ is odd}$$

$$f(x) = \frac{\pi}{2} + \frac{x}{n^{2} \pi} \left[\frac{1}{n^{2} \pi} - \frac{1}{(n^{2} \pi)^{2}} - \frac{1}{(n^{2} \pi)^{2}} - \frac{1}{(n^{2} \pi)^{2}} \right]$$
2. Find the half range cosine series of the function
$$f(x) = x (\pi - x) \text{ in } (0, \pi).$$

$$\frac{Sol!}{5o!} \quad f_{1}(x) = x(\pi - x^{2}) \text{ dx}$$

$$= \frac{n}{\pi} \int_{1}^{\pi} (x\pi - x^{2}) \text{ dx}$$

$$= \frac{2}{\pi} \left[-\frac{\pi^{2}}{3} - \frac{\pi^{2}}{3} \right]^{\pi}$$



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 $a_{n} = \frac{2}{\pi} \int_{0}^{\pi} (x \pi - x^{2}) \cos nx \, dx$ $u = x\pi - x^{2} \qquad \int dv = \int \cos nx \, dx$ $u_{1} = \pi - 2x \qquad v = \frac{\sin nx}{n}$ $U_2 = -2 \qquad \qquad V_1 = -\frac{\cos n^{\mathcal{H}}}{n^2}$ V2 = - Binnx 48=0 $a_{n} = \frac{2}{\pi} \left[(\pi - x^{2}) \frac{\sin nx}{n} + (\pi - 2x) \frac{\cos nx}{n^{2}} + 2 \frac{\sin nx}{n^{3}} \right]_{0}^{\pi}$ $=\frac{2}{\pi}\left[-\pi\frac{C^{-1}}{n^2}-\frac{\pi}{n^2}\right]$ $= \frac{2\pi}{\pi b^2} \left[- (-1)^n - 1 \right]$ $= -\frac{2}{n^2} \left[\left[\left[+ \left(-1 \right)^n \right] \right] \right]$ $a_n = \begin{cases} 0 & \text{if nisodd} \\ -\frac{4}{n^2} & \text{if niseren} \end{cases}$ $f(\varkappa) = \frac{\overline{m^2}}{6} + \frac{\overline{z}}{n=\text{edden}} \frac{-4}{n^2} \cos n\varkappa$ 3 Find the half range cosine series of fin) = (x-1)2, OLXLI.





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$$a_{0} = \frac{2}{\lambda} \int_{0}^{1} f(x) dx$$

$$= 2 \int_{0}^{1} (x-1)^{2} dx$$

$$= 2 \left[\frac{(x-1)^{2}}{3} \right]_{0}^{1}$$

$$= \frac{2}{3} \left[\frac{b+1}{2} \right]$$

$$a_{0} = \frac{2}{3}$$

$$a_{1} = \frac{2}{3} \int_{0}^{1} f(x) \cos \pi \pi x dx$$

$$a_{2} = 2 \int_{0}^{1} (x-1)^{2} \cos \pi \pi x dx$$

$$u = (x-1)^{2} \qquad \left[dv = \int \cos \pi \pi x dx \right]$$

$$u_{1} = 2(x-1) \qquad V = \frac{\sin \pi \pi x}{n\pi}$$

$$u_{2} = 2 \qquad V_{1} = -\frac{\cos n\pi x}{n^{2}\pi^{2}}$$

$$V_{2} = -\frac{\sin n\pi x}{n^{2}\pi^{2}}$$

$$a_{n} = 2 \left[(x-1)^{2} \frac{\sin n\pi x}{n\pi} + 2(x-1) \frac{\cos n\pi x}{n^{2}\pi^{2}} - 2\frac{\sin n\pi x}{n^{2}\pi^{2}} \right]$$

$$= 2 \left[\frac{1}{n^{2}\pi^{2}} \right]$$