



Consistency and Inconsistency of premises

A set of formulae H_1, H_2, \dots, H_m is said to be inconsistent if their conjunction implies contradiction.

$$i) H_1 \wedge H_2 \wedge \dots \wedge H_m \Leftrightarrow F$$

A set of formulae H_1, H_2, \dots, H_m is said to be consistent if it is not inconsistent.

① Prove that $P \rightarrow Q, Q \rightarrow R, S \rightarrow \neg R, P \wedge S$ are inconsistent.



$\{1\}$	1) $P \rightarrow Q$	Rule P
$\{2\}$	2) $Q \rightarrow R$	Rule P
$\{1,2\}$	3) $P \rightarrow R$	Rule T ($P \rightarrow Q, Q \rightarrow R \Rightarrow P \rightarrow R$)
$\{4\}$	4) $S \rightarrow \neg R$	Rule P
$\{4\}$	5) $R \rightarrow \neg S$	Rule T ($P \rightarrow Q \Leftrightarrow \neg Q \rightarrow \neg P$)
$\{1,2,4\}$	6) $P \rightarrow \neg S$	Rule T ($P \rightarrow Q, Q \rightarrow R \Rightarrow P \rightarrow R$)
$\{1,2,4\}$	7) $\neg P \vee \neg S$	Rule T ($P \rightarrow Q \Leftrightarrow \neg P \vee Q$)
$\{1,2,4\}$	8) $\neg(P \wedge S)$	Rule T (Demorgan's)
$\{9\}$	9) $P \wedge S$	Rule P
$\{1,2,4,9\}$	10) $(P \wedge S) \wedge \neg(P \wedge S)$	Rule T ($P, Q \Rightarrow P \wedge Q$)

which is nothing but false value. Therefore given premises are inconsistent.



② Prove that $P \rightarrow Q, Q \rightarrow R, R \rightarrow S, S \rightarrow \neg R$ and $P \wedge S$ are inconsistent.

$\{1\}$	1) $P \rightarrow Q$	Rule P
$\{2\}$	2) $Q \rightarrow R$	Rule P
$\{1,2\}$	3) $P \rightarrow R$	Rule T ($P \rightarrow Q, Q \rightarrow R \Rightarrow P \rightarrow R$)
$\{4\}$	4) $R \rightarrow S$	Rule P
$\{1,2,4\}$	5) $P \rightarrow S$	Rule T ($P \rightarrow Q, Q \rightarrow R \Rightarrow P \rightarrow R$)
$\{6\}$	6) $S \rightarrow \neg R$	Rule P
$\{6\}$	7) $R \rightarrow \neg S$	Rule T ($P \rightarrow Q \Leftrightarrow \neg Q \rightarrow \neg P$)
$\{6\}$	8) $\neg R \vee \neg S$	Rule T ($P \rightarrow Q \Leftrightarrow \neg P \vee Q$)
$\{6\}$	9) $\neg S$	Rule T ($P \vee Q \Rightarrow Q$)
$\{1,2,4,6\}$	10) $\neg P$	Rule T ($P \rightarrow Q, \neg Q \Rightarrow \neg P$)
$\{1,2,4,6\}$	11) $\neg P \vee \neg S$	Rule T ($P, Q \Rightarrow P \vee Q$)
$\{12\}$	12) $P \wedge S$	Rule P
$\{1,2,4,6\}$	13) $\neg(P \wedge S)$	Rule T (Demorgan's)
$\{1,2,4,6,12\}$	14) $(P \wedge S) \wedge \neg(P \wedge S)$	Rule T ($P, Q \Rightarrow P \wedge Q$)

which is nothing but false value. Therefore given premises are inconsistent.



Indirect Method of Proof

In order to show that a conclusion C follows logically from the premises H_1, H_2, \dots, H_m , we assume C is FALSE and consider $\neg C$ as an additional premises. If $H_1 \wedge H_2 \wedge \dots \wedge H_m \wedge \neg C$ is a contradiction, then C follows logically from H_1, H_2, \dots, H_m .

1. Using indirect method of proof, derive $p \rightarrow \neg s$ from the premises $p \rightarrow (q \vee r)$, $q \rightarrow \neg p$, $s \rightarrow \neg r$ and P .

we consider $\neg (p \rightarrow \neg s)$ as an additional premises.
 $= \neg (\neg p \vee \neg s) = p \wedge s$

{1}	1) $p \wedge s$	Assumed premises
{2}	2) $p \rightarrow (q \vee r)$	Rule P
{3}	3) p	Rule P
{2, 3}	4) $q \vee r$	Rule T ($P, P \rightarrow Q \Rightarrow Q$)
{1}	5) s	Rule T ($P \wedge Q \Rightarrow Q$)
{6}	6) $s \rightarrow \neg r$	Rule P
{1, 6}	7) $\neg r$	Rule T ($P, P \rightarrow Q \Rightarrow Q$)



$\{2,3\}$	8) $\neg q \rightarrow r$	Rule T ($P \rightarrow Q \Leftrightarrow \neg P \vee Q$)
$\{2,3\}$	9) $\neg r \rightarrow q$	Rule T (contrapositive)
$\{1,2,3,6\}$	10) q	Rule T ($P, P \rightarrow Q \Rightarrow Q$)
$\{11\}$	11) $q \rightarrow \neg p$	Rule P
$\{1,2,3,6,11\}$	12) $\neg p$	Rule T ($P, P \rightarrow Q \Rightarrow Q$)
$\{1,2,3,6,11\}$	13) $p \wedge \neg p$	Rule T ($P, Q \Rightarrow P \wedge Q$)

which is nothing but false value.

. By method of contradiction, $p \rightarrow \neg s$



2) Show that the following argument is valid.

"My father praises me only if I can be proud of myself. Either I do well in sports or I cannot be proud of ^{my}self. If study hard, then I cannot do well in sports. Therefore, if father praises me, then I do not study well."

Let A : My father praises me

B : I can be proud of myself

C : I do well in sports

D : I study hard

Then, the premises are

$$A \rightarrow B, C \vee \neg B, D \rightarrow \neg C$$

Conclusion is $A \rightarrow \neg D$



$\{1\}$	1) A	Assumed premises
$\{2\}$	2) $A \rightarrow B$	Rule P
$\{1,2\}$	3) B	Rule T ($P, P \rightarrow Q \Rightarrow Q$)
$\{4\}$	4) $C \vee \neg B$	Rule P
$\{4\}$	5) $B \rightarrow C$	Rule T ($P \rightarrow Q \Leftrightarrow \neg P \vee Q$)
$\{1,2,4\}$	6) C	Rule T ($P, P \rightarrow Q \Rightarrow Q$)
$\{7\}$	7) $D \rightarrow \neg C$	Rule P
$\{7\}$	8) $C \rightarrow \neg D$	Rule T
$\{1,2,4,7\}$	9) $\neg D$	Rule T ($P, P \rightarrow Q \Rightarrow Q$)
	10) $A \rightarrow \neg D$	Rule CP