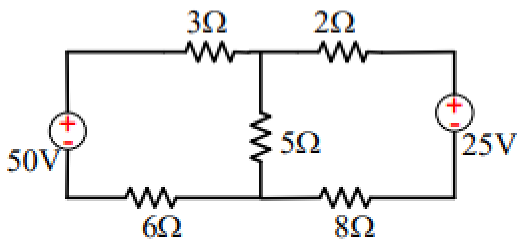
Mesh Analysis

Steps to find a current flowing in a circuit using Mesh Analysis

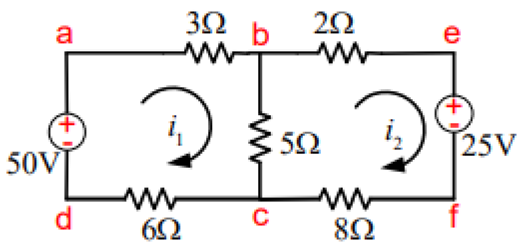
1. Identify loops or meshes in a circuit and Label a mesh current to N meshes
2. Apply KVL to each mesh with the corresponding mesh current to generate N equations.
3. Solve the resulting simultaneous linear equations for the unknown mesh currents using Cramer's Rule.

Problem:

In the circuit shown in Fig and determine all branch currents and the voltage across the 5 Ω resistor by loop current analysis.



Solution:



Loop 1:

$$\begin{aligned} 3i_1 + 5(i_1 - i_2) + 6i_1 - 50 &= 0 \\ 14i_1 - 5i_2 &= 50 \end{aligned}$$

Loop 2:

$$\begin{aligned} 2i_2 + 8i_2 + 5(i_2 - i_1) + 25 &= 0 \\ -5i_1 + 15i_2 &= -25 \end{aligned}$$

$$14i_1 - 5i_2 = 50 \qquad (1)$$

$$-5i_1 + 15i_2 = -25 \qquad (2)$$

Multiply eqn 1 by 3 and adding with equation 2

$$\begin{aligned} 42i_1 - 15i_2 &= 150 \\ -5i_1 + 15i_2 &= -25 \\ ------ &= -- \\ 37i_1 &= 125 \end{aligned}$$

$i_1 = 3.3784A \quad i_2 = -0.541A$
$i_{ab} = 3.3784A \quad i_{eb} = -i_2 = 0.541A$
$i_{bc} = i_1 - i_2 = 3.3784 - (-0.541) = 3.9194A$
voltage across the 5 Ω resistor is $5i_{bc} = 19.597V$

Problem:

In the circuit shown in Figure 3 determine the mesh currents i1, i2, i3

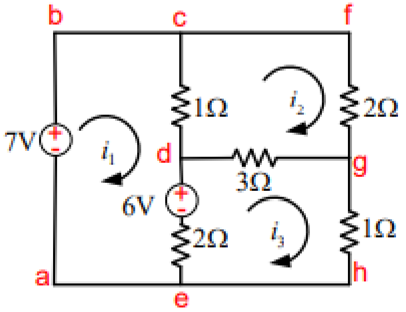


Figure 3

Solution:

Applying the KVL for the loop abcdea

$$\begin{aligned} 1(i_1 - i_2) + 2(i_1 - i_3) + 6 - 7 &= 0 \\ 3i_1 - i_2 - 2i_3 &= 1 \end{aligned}$$

For the loop cfgdc

$$\begin{aligned} 2i_2 + 3(i_2 - i_3) + 1(i_2 - i_1) &= 0 \\ -i_1 + 6i_2 - 3i_3 &= 0 \end{aligned}$$

For the loop dghed

$$\begin{aligned} 3(i_3 - i_2) + 2(i_3 - i_1) + i_3 - 6 &= 0 \\ -2i_1 - 3i_2 + 6i_3 &= 6 \end{aligned}$$

The three mesh equations are,

$$\begin{aligned} 3i_1 - i_2 - 2i_3 &= 1 \\ -i_1 + 6i_2 - 3i_3 &= 0 \\ -2i_1 - 3i_2 + 6i_3 &= 6 \end{aligned}$$

Solving these equations Using Cramer's rule

$$\begin{bmatrix} 3 & -1 & -2 \\ -1 & 6 & -3 \\ -2 & -3 & 6 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 6 \end{bmatrix}$$

$$ZI = V$$

$$I = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}$$

$$V = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

$$Z = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{21} & Z_{22} & Z_{23} \\ Z_{31} & Z_{32} & Z_{33} \end{bmatrix}$$

$$i_1 = \frac{\begin{bmatrix} v_1 & -1 & -2 \\ v_2 & 6 & -3 \\ v_3 & -3 & 6 \end{bmatrix}}{\Delta}$$

where Δ is

$$\Delta = \begin{vmatrix} 3 & -1 & -2 \\ -1 & 6 & -3 \\ -2 & -3 & 6 \end{vmatrix}$$

$3[6 \times 6 - (-3 \times -3)] + 1[-1 \times 6 - (-2 \times -3)]$
$-2[-1 \times -3 - (-2 \times 6)]$
$=3(36\text{-}9)+1(\text{-}6\text{-}6)\text{-}2(3+12)=81\text{-}12\text{-}30=39$

$$i_1 = \frac{\begin{vmatrix} 1 & -1 & -2 \\ 0 & 6 & -3 \\ 6 & -3 & 6 \end{vmatrix}}{\Delta} = \frac{1(36-9) + 1(18) - 2(-36)}{39}$$

$$\frac{27 + 18 + 72}{39} = 3A$$

$$i_2 = \frac{\begin{vmatrix} 3 & 1 & -2 \\ -1 & 0 & -3 \\ -2 & 6 & 6 \end{vmatrix}}{\Delta} = \frac{3(18) - 1(-6-6) - 2(-6)}{39}$$

$$\frac{54 + 12 + 12}{39} = 2A$$

$$i_3 = \frac{\begin{vmatrix} 3 & -1 & 1 \\ -1 & 6 & 0 \\ -2 & -3 & 6 \end{vmatrix}}{\Delta} = \frac{3(36) + 1(-6) + 1(3+12)}{39}$$

$$\frac{108 - 6 + 15}{39} = 3A$$

$i_1 = 3A \quad i_2 = 2A \quad i_3 = 3A$