19EE701 – AI TECHNIQUES IN ELECTRICAL ENGINEERING

UNIT 3 – FUZZY LOGIC & ANFIS

Topic : Fuzzification and Arithmetic Operations of Fuzzy Numbers

1. Fuzzification

1.1. Definition

- **Fuzzification:** The process of converting crisp, precise values into fuzzy values by associating them with degrees of membership in fuzzy sets. It translates quantitative inputs into fuzzy variables that can be processed by a fuzzy inference system.

1.2. Steps in Fuzzification

- **Determine the Input Variables:** Identify the crisp variables that need to be converted into fuzzy values.
- **Define Fuzzy Sets:** Specify the fuzzy sets that represent the possible values of each input variable. Each fuzzy set is characterized by a membership function.
- **Apply Membership Functions:** Compute the degree of membership of each crisp input value in the defined fuzzy sets using their membership functions.
- **Resulting Fuzzy Values:** Represent the crisp input values as fuzzy sets with associated membership values.

1.3. Example

- **Crisp Input:** Temperature = 25°C
- **Fuzzy Sets:**
  - Cold: Triangular membership function with parameters (0, 5, 15)
  - Warm: Triangular membership function with parameters (15, 25, 35)
  - Hot: Triangular membership function with parameters (30, 40, 50)

- Fuzzification:
  - Compute membership values for 25°C:
    - Cold: $\mu_{cold}(25) = 0$
    - Warm: $\mu_{warm}(25) = 1$
    - Hot: $\mu_{hot}(25) = 0$

2. Arithmetic Operations on Fuzzy Numbers

2.1. Definition

- **Fuzzy Numbers:** Special types of fuzzy sets where each element represents a number with a membership function that defines the degree of certainty or vagueness about that number. Fuzzy numbers are used in fuzzy arithmetic operations.

2.2. Basic Arithmetic Operations

2.2.1. Addition of Fuzzy Numbers

- **Definition:** The addition of two fuzzy numbers A and B is another fuzzy number C, where the membership function of C is defined as:

$$\mu_C(x) = \sup\{\min(\mu_A(x-y), \mu_B(y)) \mid y \in \mathbb{R}\}$$

- **Example:** Let A and B be fuzzy numbers with membership functions μ_A and μ_B. For a crisp value x, the membership function of the sum is calculated by:

$$\mu_{A+B}(x) = \sup\{\min(\mu_A(x-y), \mu_B(y)) \mid y \in \mathbb{R}\}$$

2.2.2. Subtraction of Fuzzy Numbers

- **Definition:** The subtraction of two fuzzy numbers A and B is another fuzzy number C, where the membership function of C is:

$$\mu_C(x) = \sup\{\min(\mu_A(x+y), \mu_B(y)) \mid y \in \mathbb{R}\}$$

- **Example:** For fuzzy numbers A and B, the membership function for subtraction is:

$$\mu_{A-B}(x) = \sup\{\min(\mu_A(x+y), \mu_B(y)) \mid y \in \mathbb{R}\}$$

2.2.3. Multiplication of Fuzzy Numbers

- **Definition:** The multiplication of two fuzzy numbers A and B is a fuzzy number C, where the membership function of C is:

$$\mu_C(x) = \sup\{\min(\mu_A(y), \mu_B(z)) \mid y \cdot z = x \text{ and } y, z \in \mathbb{R}\}$$

- **Example:** For fuzzy numbers A and B, the membership function for multiplication is:

$$\mu_{A \cdot B}(x) = \sup\{\min(\mu_A(y), \mu_B(z)) \mid y \cdot z = x \text{ and } y, z \in \mathbb{R}\}$$

2.2.4. Division of Fuzzy Numbers

- **Definition:** The division of two fuzzy numbers A and B is a fuzzy number C, where the membership function of C is:

$$\mu_C(x) = \sup\{\min(\mu_A(y), \mu_B(z)) \mid y/z = x \text{ and } z \neq 0 \text{ and } y, z \in \mathbb{R}\}$$

- **Example:** For fuzzy numbers A and B, the membership function for division is:

$$\mu_{A/B}(x) = \sup\{\min(\mu_A(y), \mu_B(z)) \mid y/z = x \text{ and } z \neq 0 \text{ and } y, z \in \mathbb{R}\}$$

2.3. **Properties of Fuzzy Arithmetic Operations**

- **Commutativity:**
  - Addition: $A + B = B + A$
  - Multiplication: $A \cdot B = B \cdot A$

- **Associativity:**
  - Addition: $(A + B) + C = A + (B + C)$
  - Multiplication: $(A \cdot B) \cdot C = A \cdot (B \cdot C)$
- **Distributivity:**
  - Over Addition: $A \cdot (B + C) = (A \cdot B) + (A \cdot C)$
- **Non-Negativity:**
  - The result of arithmetic operations on fuzzy numbers is always a fuzzy number with membership values in the range [0, 1].