SNS COLLEGE OF ENGINEERING

(An Autonomous Institution)

CO-ORDINATE SYSTEMS

Coordinate systems:

- Coordinates systems are often used to specify the position of a point, but they may also be used to specify the position of more complex figures such as lines, planes, circles or spheres.

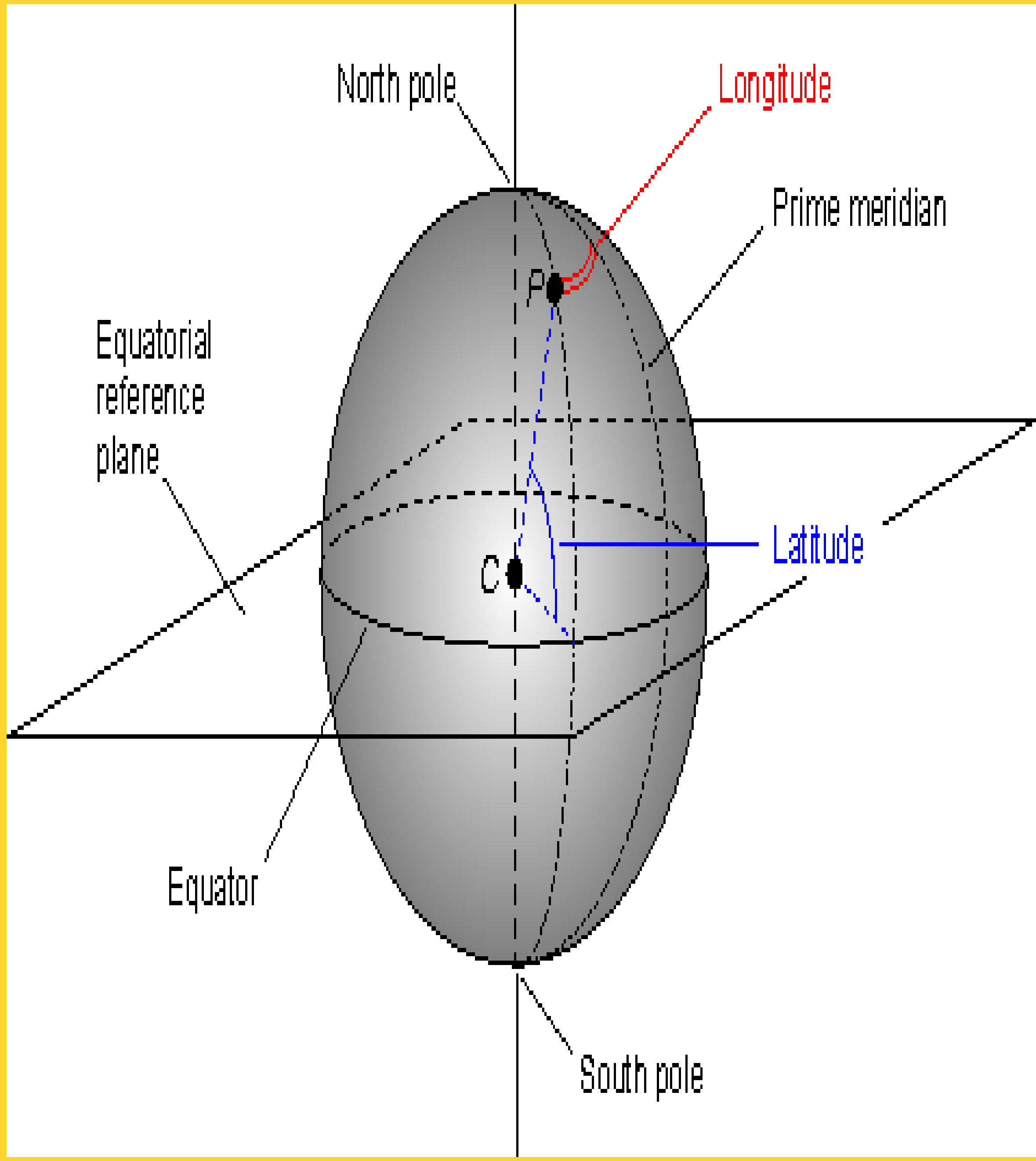
- The choice of the coordinate system is based on the problem one is studying.
- Certain problems are solved easily by using rectangular coordinate systems whereas certain others are not.

- Some coordinate systems make more sense, make it easier to describe a system.

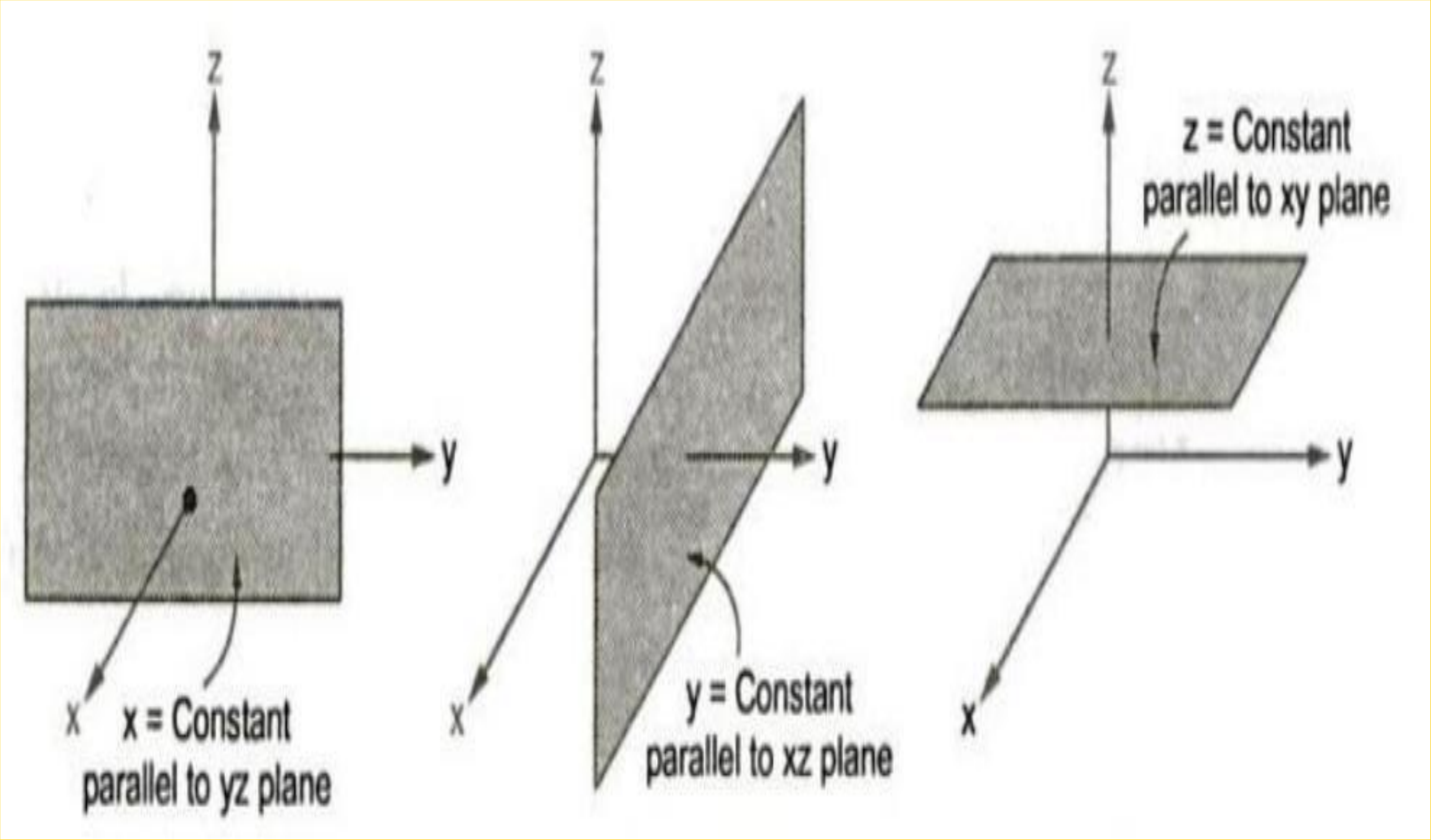
- Coordinates give you a systematic way of naming the points in a space.

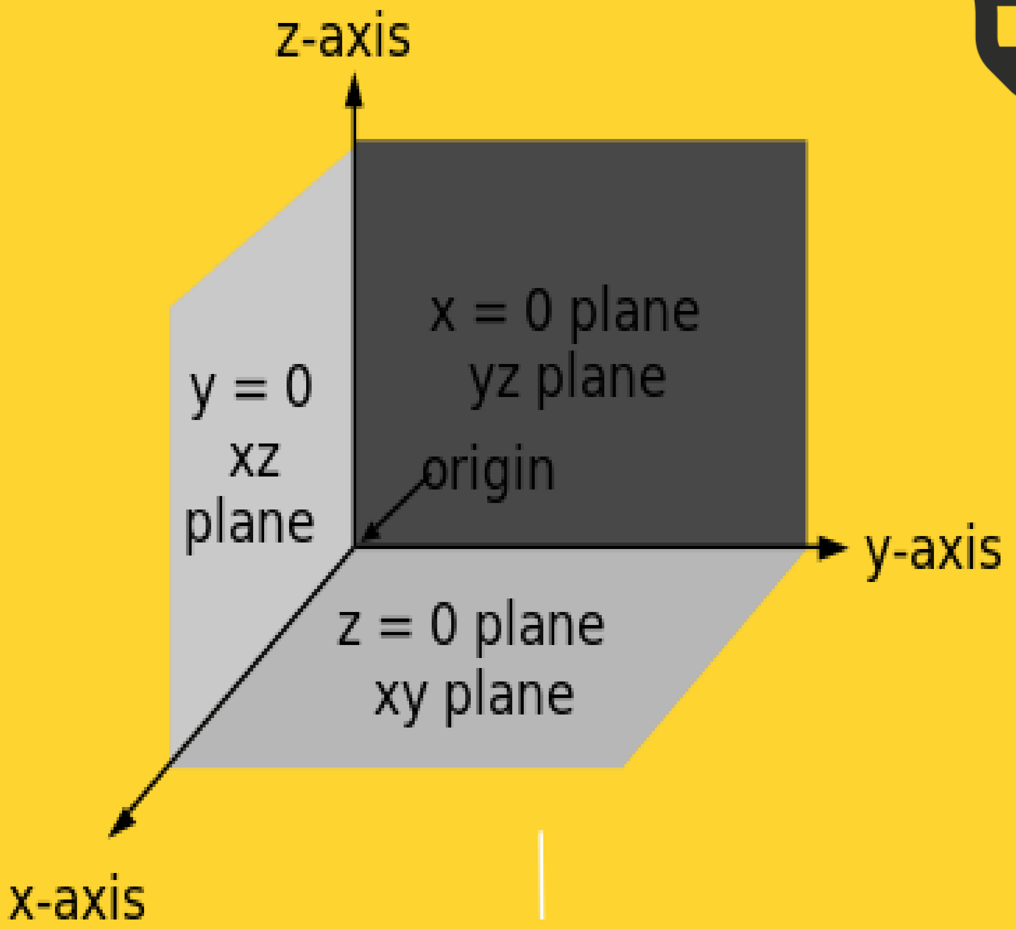
Cont…

- Consider the set of locations in your room. Each point has a unique identity, but they **don't** come with names.
- We can use descriptions, like **"the** point at the corner of the desk**"**, or **"the** set of points exactly three inches from the top of the **lamp"**, but that sort of thing is **ad hoc**. If we can name them systematically, we can start reasoning about the whole space.
- A simple way to systematically name every point, called a Cartesian coordinate system, is to give its perpendicular distance from the floor and two adjacent walls—each point gets a unique name in this system.
- If the room is circular, **you'd** have to make an imaginary wall, or you could use the height from the floor, the distance from the center, and the angle between a line from the center through the point and a line from the center through another fixed point, like the door. This is an example of cylindrical coordinates.
- On the globe, we systematically name locations by giving their latitude, longitude, and altitude. **you're** using a spherical coordinate plane in real life.



Cartesian/Rectangular coordinate system:





Ranges

x	$-\infty$ to $+\infty$
y	$-\infty$ to $+\infty$
z	$-\infty$ to $+\infty$

Cartesian/Rectangular coordinate system:

dx = Differential length in x direction

dy = Differential length in y direction

dz = Differential length in z direction

$$\overline{dl} = dx\,\bar{a}_x + dy\,\bar{a}_y + dz\,\bar{a}_z$$

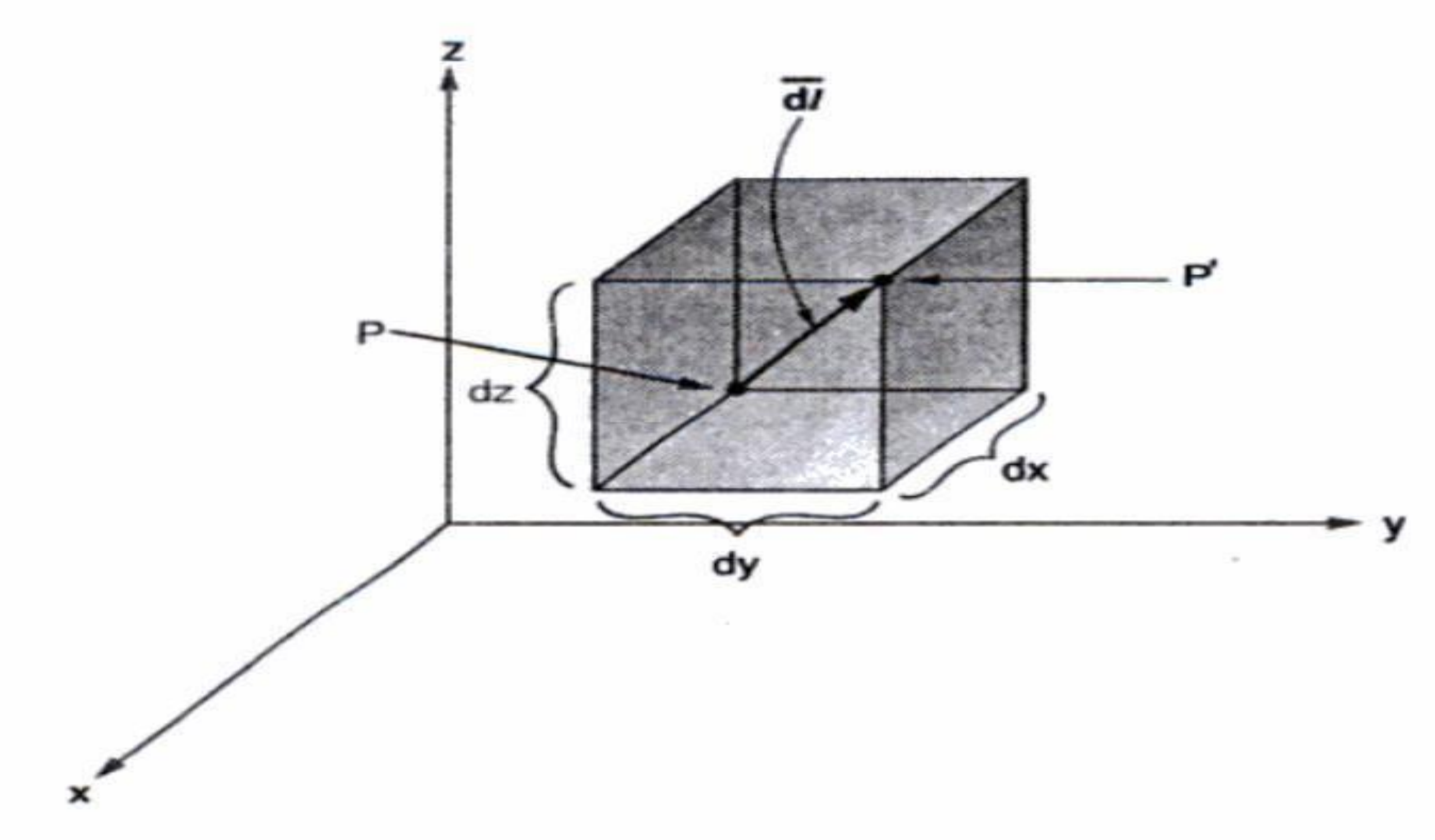
$$|\overline{dl}| = \sqrt{(dx)^2 + (dy)^2 + (dz)^2}$$

$$dv = dx\,dy\,dz$$

$$d\bar{S} = dS\,\bar{a}_n$$

where dS = Differential surface area of the element

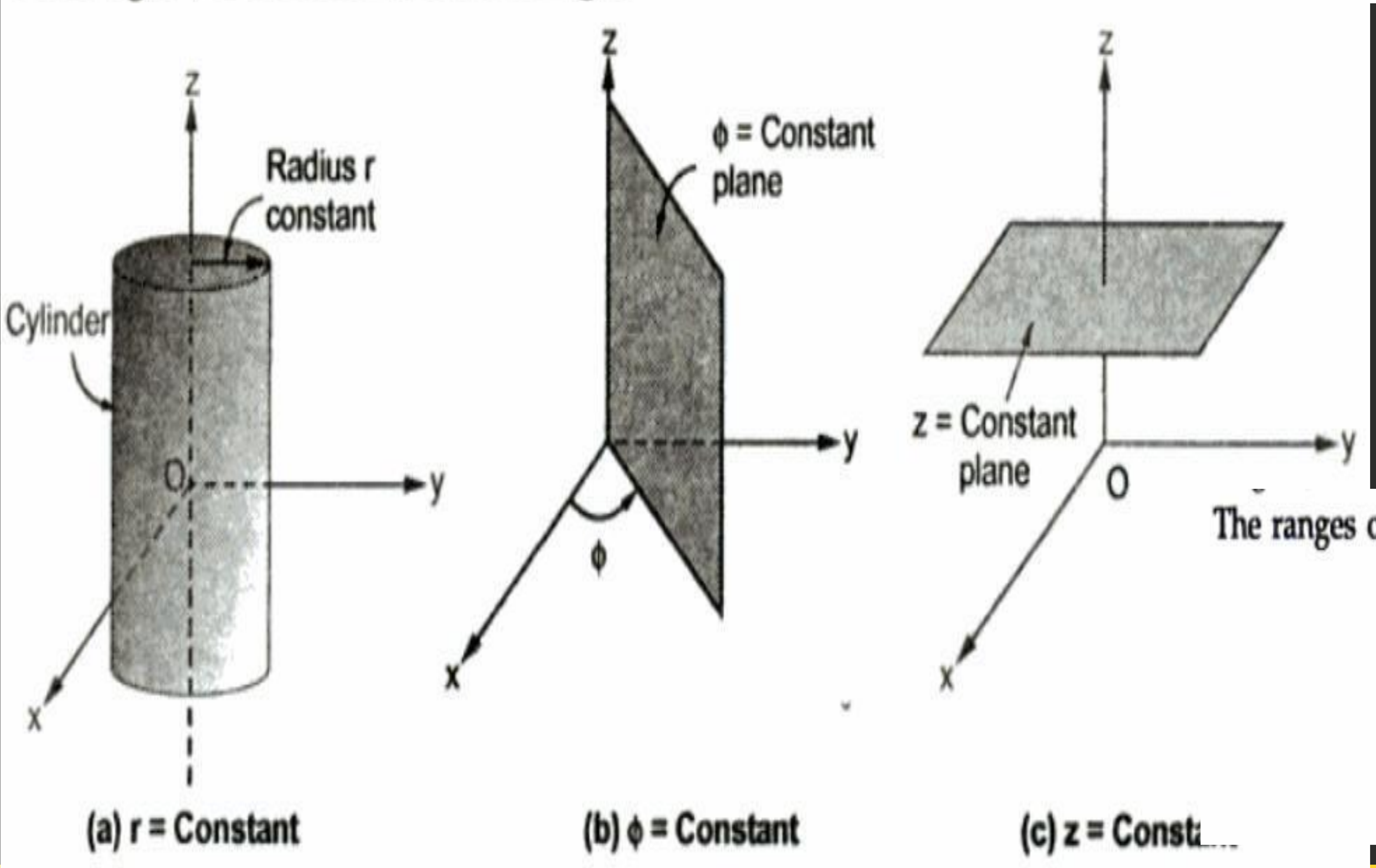
$\bar{a}_n$ = Unit vector normal to the surface dS



Cylindrical coordinate system:

1. Plane of constant z which is parallel to xy plane.
2. A cylinder of radius r with z axis as the axis of the cylinder.
3. A half plane perpendicular to xy plane and at an angle ϕ with respect to xz plane.

The angle ϕ is called **azimuthal angle.**



The ranges of the variables are,

$$0 \le r \le \infty$$

$$0 \le \phi \le 2\pi$$

$$-\infty < z \le \infty$$

Cylindrical coordinate system:

dr = Differential length in r direction

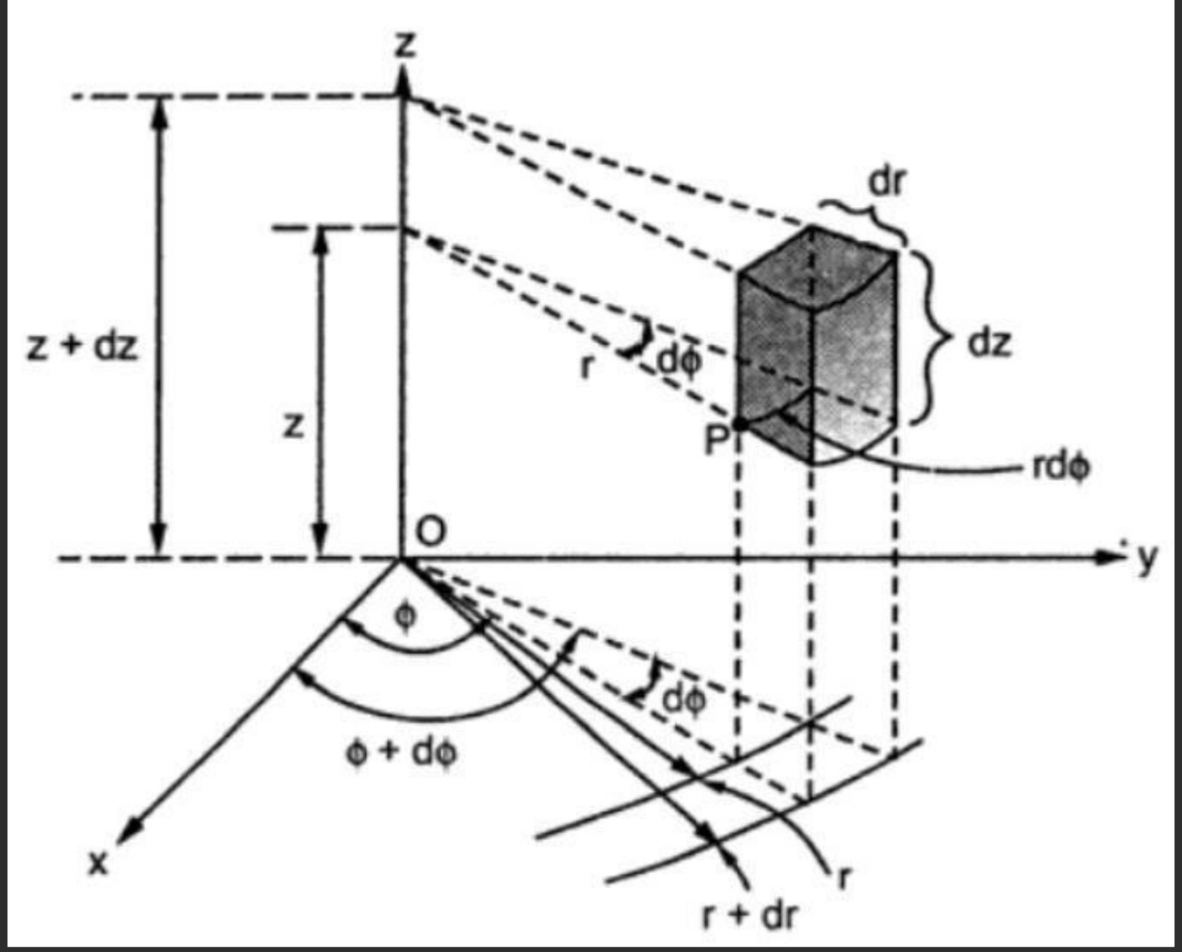
r dϕ = Differential length in ϕ direction

dz = Differential length in z direction

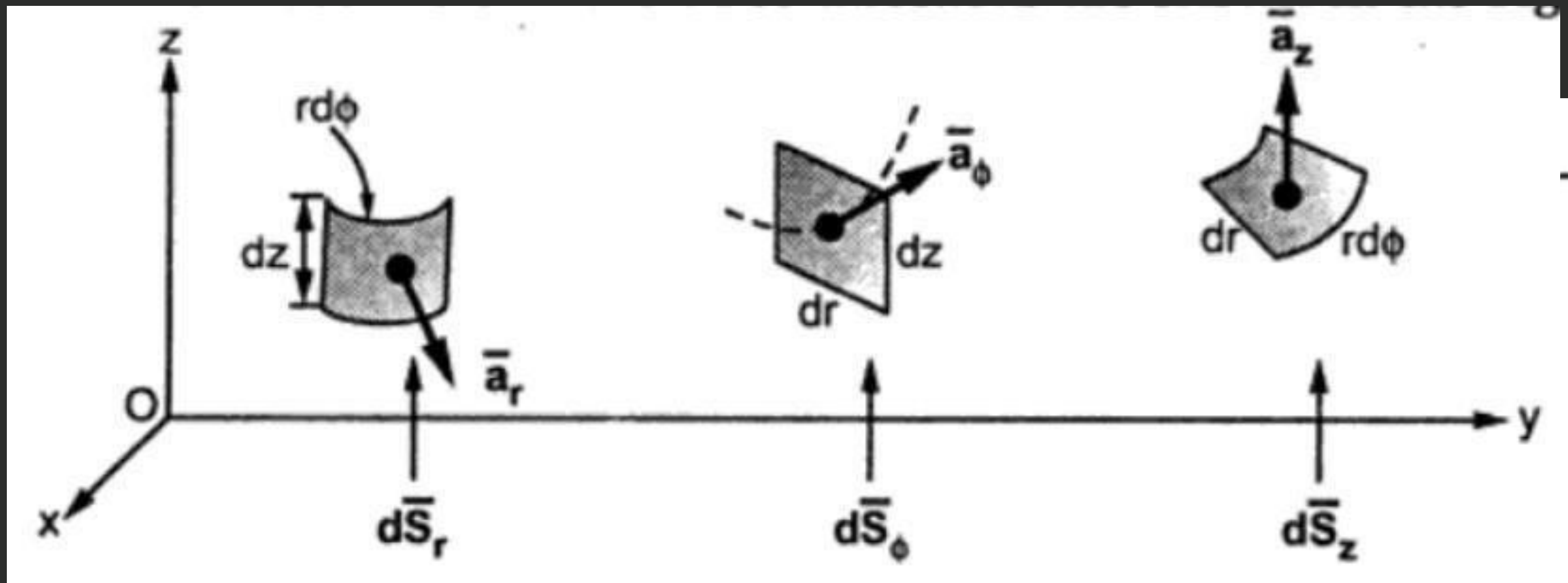
$$\overline{dl} = dr\,\bar{a}_r + r\,d\phi\,\bar{a}_\phi + dz\,\bar{a}_z$$

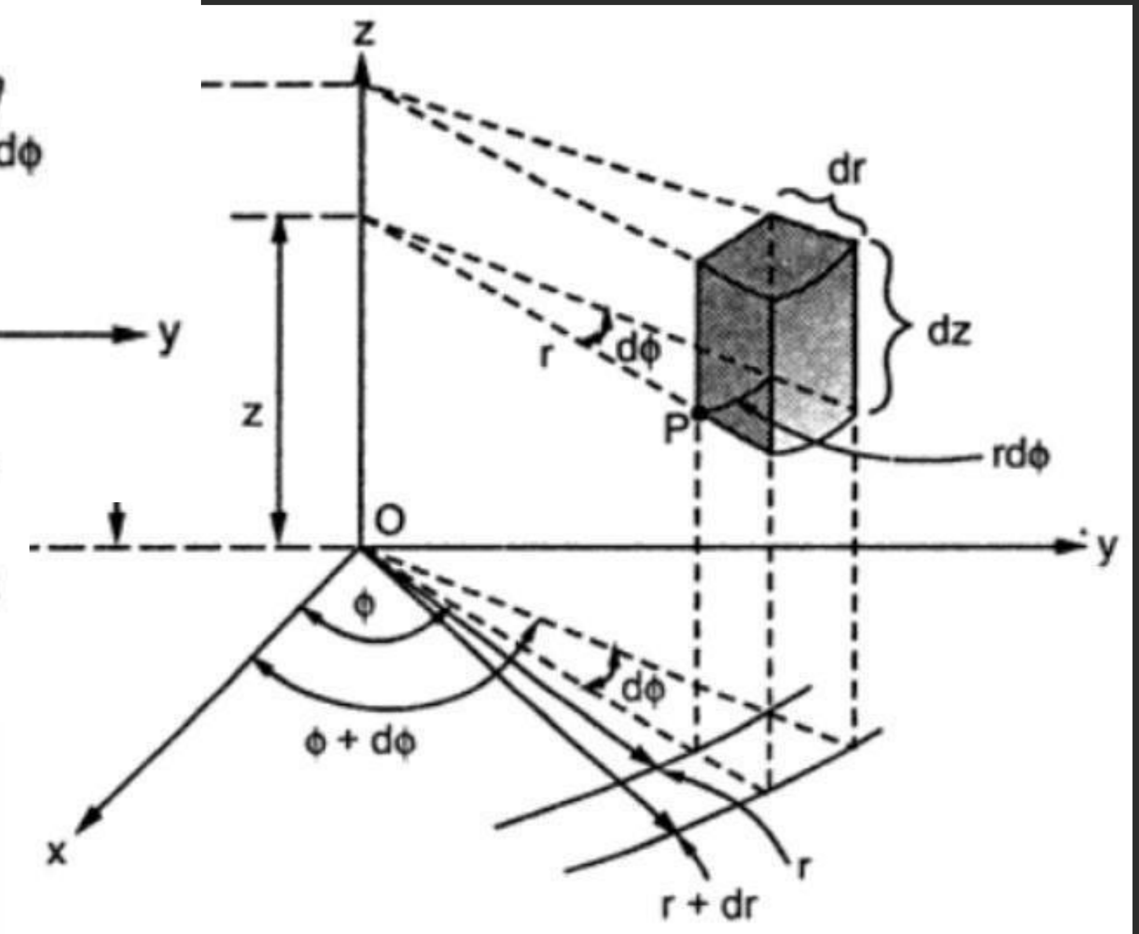
$$|\overline{dl}| = \sqrt{(dr)^2 + (r\,d\phi)^2 + (dz)^2}$$

$$dv = r\,dr\,d\phi\,dz$$



Cylindrical coordinate system:





$d\bar{S}_r$ = Differential vector surface area normal to r direction

$= r\,d\phi\,dz\,\bar{a}_r$

$d\bar{S}_\phi$ = Differential vector surface area normal to ϕ direction

$= dr\,dz\,\bar{a}_\phi$

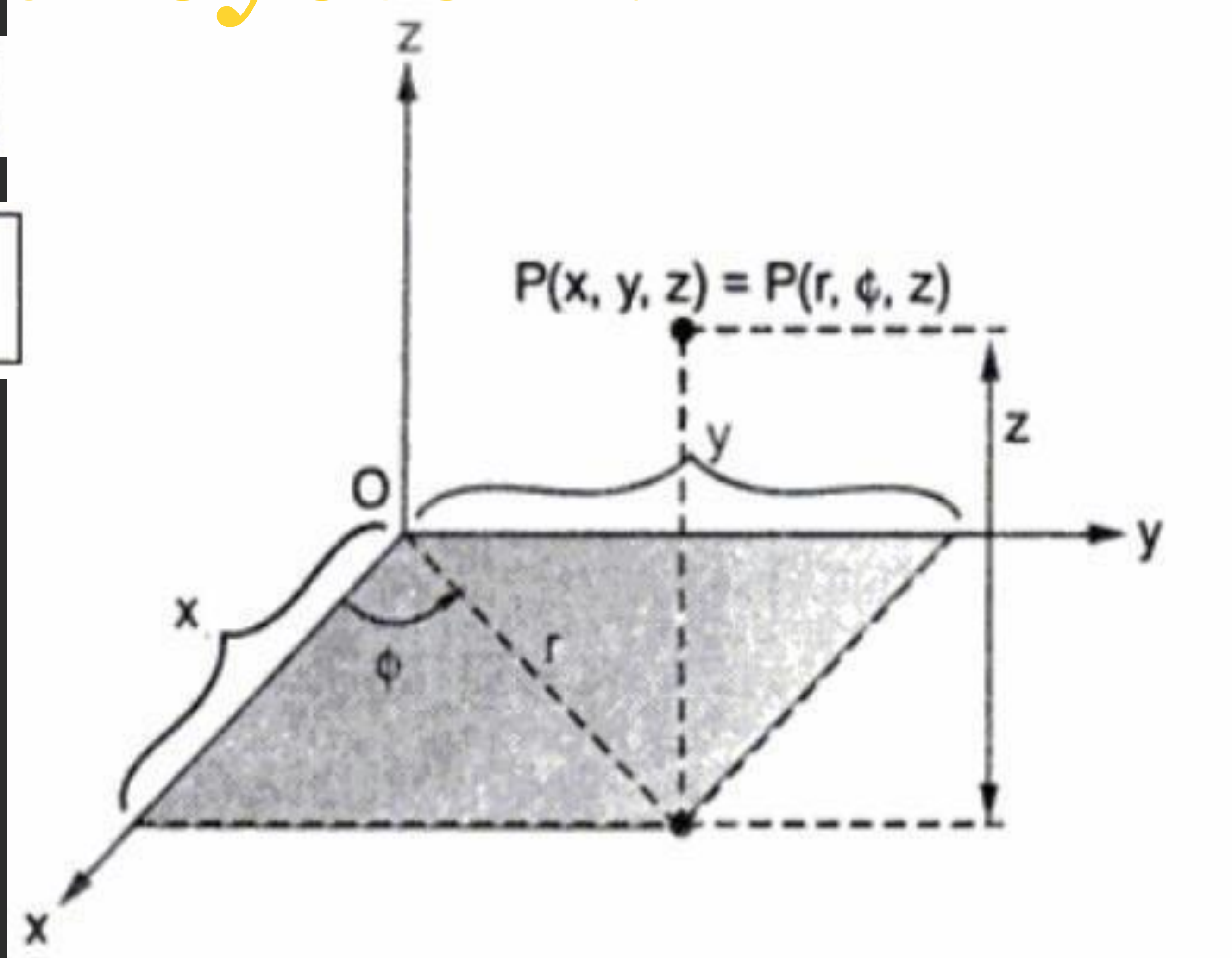
$d\bar{S}_z$ = Differential vector surface area normal to z direction

$= r\,dr\,d\phi\,\bar{a}_z$

Relation between Cartesian and cylindrical system:

$$x = r\cos\phi,\quad y = r\sin\phi \quad\text{and}\quad z = z$$

$$r = \sqrt{x^2 + y^2},\quad \phi = \tan^{-1}\frac{y}{x} \quad\text{and}\quad z = z$$

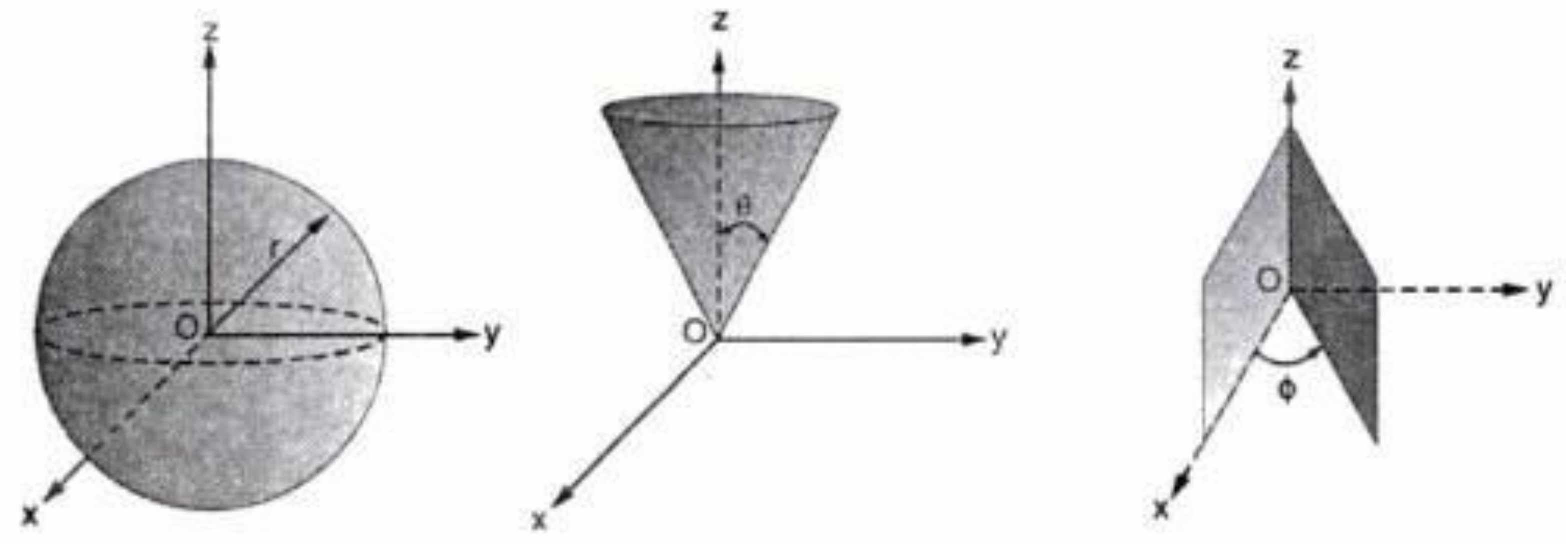


Spherical coordinate system

The surfaces which are used to define the spherical co-ordinate system on the three cartesian axes are,

1. Sphere of radius r, origin as the centre of the sphere.
2. A right circular cone with its apex at the origin and its axis as z axis. Its half angle is θ. It rotates about z axis and θ varies from 0 to 180°.
3. A half plane perpendicular to xy plane containing z axis, making an angle ϕ with the xz plane.

Thus the three co-ordinates of a point P in the spherical co-ordinate system are (r, θ, ϕ).



(a) Sphere of radius r with centre as origin

(b) Right circular cone with apex at origin

(c) Half plane perpendicular to xy plane

$0 \le r < \infty$

$0 \le \phi \le 2\pi$

$0 \le \theta \le \pi$ as half angle

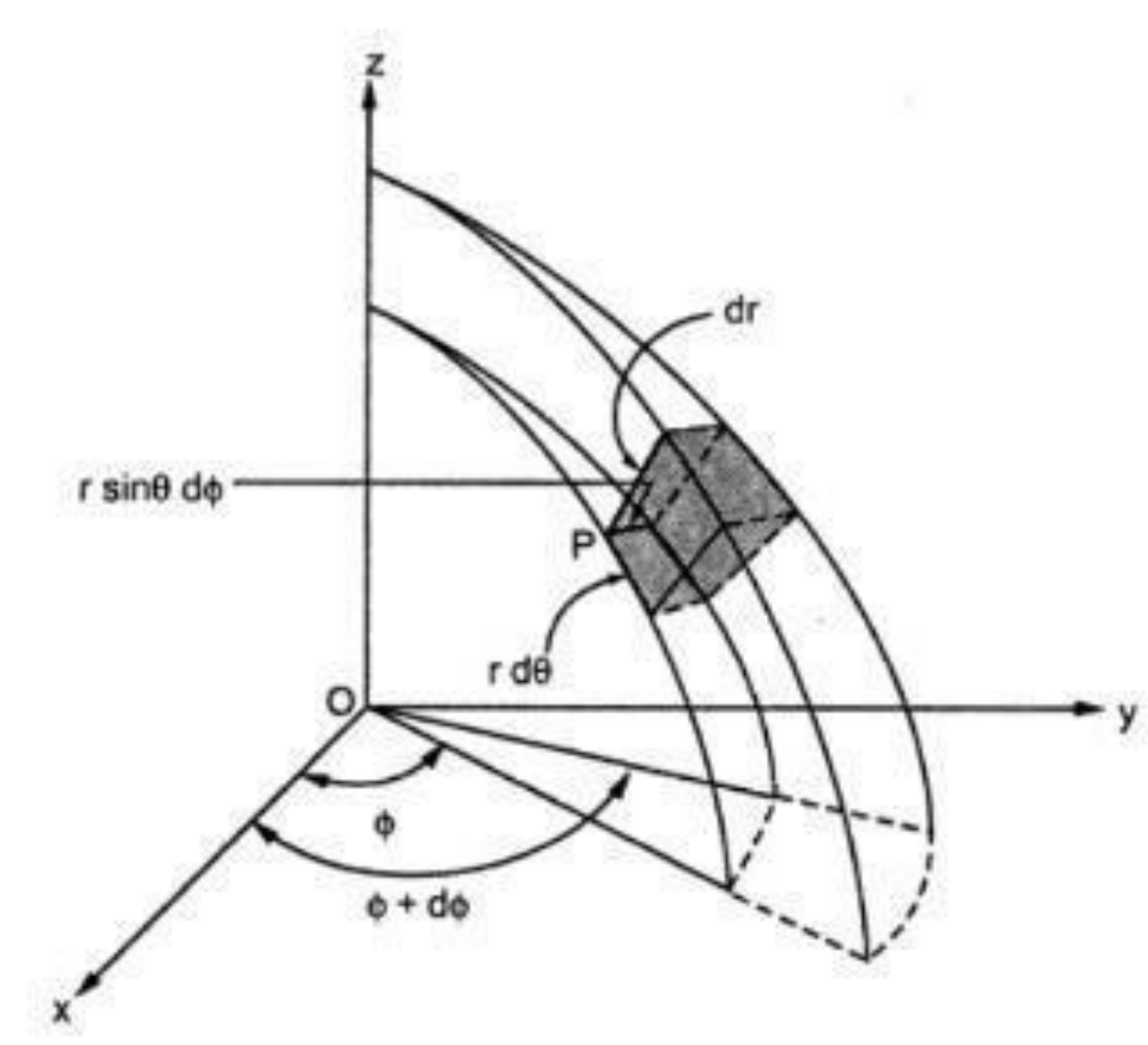
Spherical coordinate system

dr	=	Differential length in r direction
$r\,d\theta$	=	Differential length in θ direction
$r\sin\theta\,d\phi$	=	Differential length in φ direction

$$\overline{dl} = dr\,\bar{a}_r + r\,d\theta\,\bar{a}_\theta + r\sin\theta\,d\phi\,\bar{a}_\phi$$

$$\left|\overline{dl}\right| = \sqrt{(dr)^2 + (r\,d\theta)^2 + (r\sin\theta\,d\phi)^2}$$

$$dv = r^2 \sin\theta\,dr\,d\theta\,d\phi$$



Spherical coordinate system

$d\bar{S}_r$ = Differential vector surface area normal to r direction

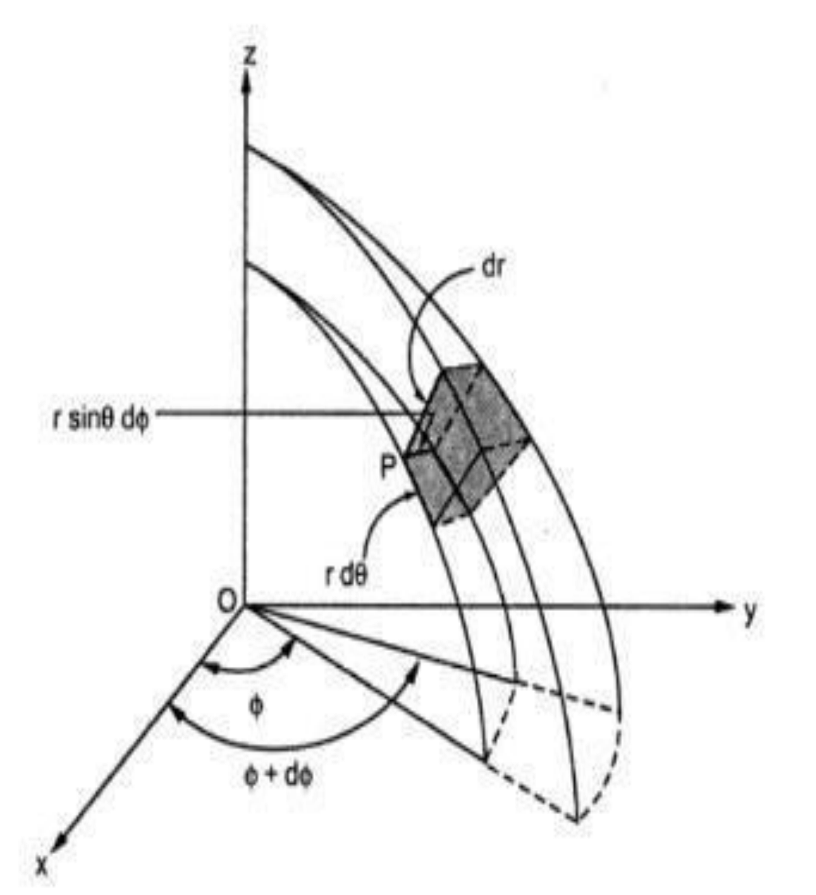
$= r^2 \sin\theta \, d\theta \, d\phi$

$d\bar{S}_\theta$ = Differential vector surface area normal to θ direction

$= r \sin\theta \, dr \, d\phi$

$d\bar{S}_\phi$ = Differential vector surface area normal to φ direction

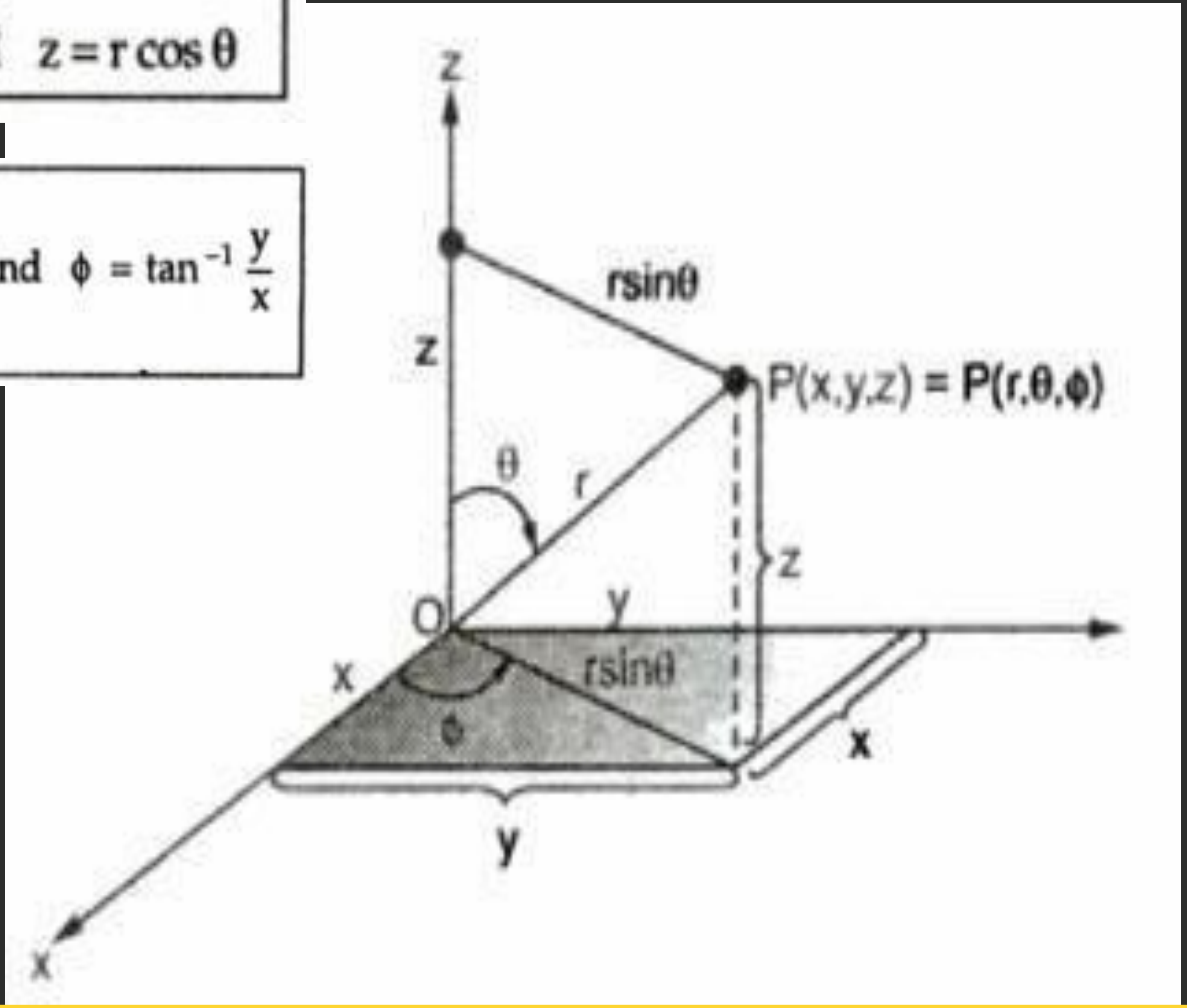
$= r \, dr \, d\theta$



Relation between Cartesian and spherical system

$$x = r\sin\theta\cos\phi,\quad y = r\sin\theta\sin\phi \quad \text{and} \quad z = r\cos\theta$$

$$r = \sqrt{x^2+y^2+z^2},\quad \theta = \cos^{-1}\left[\frac{z}{\sqrt{x^2+y^2+z^2}}\right] \quad \text{and} \quad \phi = \tan^{-1}\frac{y}{x}$$



THANK YOU