SNS COLLEGE OF ENGINEERING

(An Autonomous Institution)

COIMBATORE-35

Accredited by NAAC – UGC with A Grade Approved by AICTE, New Delhi
& Affiliated to Anna University, Chennai

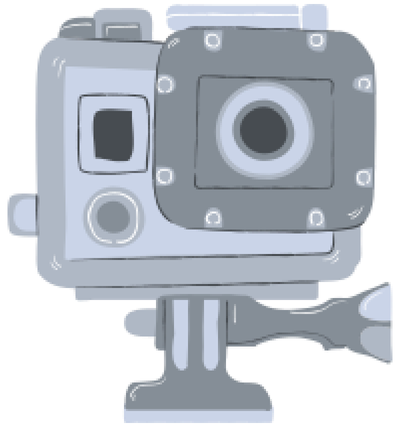
DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

COURSE NAME: 23EET202/ FIELD THEORY

II YEAR / III SEMESTER

Unit 1 – INTRODUCTION

Topic : DIVERGENCE THEOREM



Contents:

- Divergence Theorem
- Problems on divergence theorem

DIVERGENCE THEOREM

It states that the total outward flux of a vector field **A** at the closed surface **S** is the same as volume integral of divergence of **A**.

$$\oint_V \mathbf{A} \bullet dS = \int_V \nabla \bullet \mathbf{A} dV$$

PROBLEMS…

A vector field $\vec{\mathbf{D}} = \rho^3 \mathbf{a}_\rho$ exists in the region between two concentric cylindrical surfaces defined by ρ = 1 and ρ = 2, with both cylinders extending between z = 0 and z = 5. Verify the divergence theorem by evaluating:

(a) $\oint_S D \bullet ds$

(b) $\int_V \nabla \bullet D dV$

SOLUTION...

(a) For two concentric cylinder, the left side:

$$\oint_S \mathbf{D} \bullet d\mathbf{S} = \mathbf{D}_{inner} + \mathbf{D}_{outer} + \mathbf{D}_{bottom} + \mathbf{D}_{top}$$

Where,

$$D_{inner} = \int_{\phi=0}^{2\pi} \int_{z=0}^{5} \rho^3 \mathbf{a}_\rho \bullet \rho d\phi dz(-\mathbf{a}_\rho)\Big|_{\rho=1}$$

$$= \int_{\phi=0}^{2\pi} \int_{z=0}^{5} -\rho^4 \mathbf{a}_\rho \bullet d\phi dz(\mathbf{a}_\rho)\Big|_{\rho=1} = -10\pi$$

Cont…

$$D_{outer} = \int_{\phi=0}^{2\pi} \int_{z=0}^{5} \rho^3 \mathbf{a}_\rho \bullet \rho \, d\phi \, dz (\mathbf{a}_\rho) \Big|_{\rho=2}$$

$$= \int_{\phi=0}^{2\pi} \int_{z=0}^{5} \rho^4 \mathbf{a}_\rho \bullet d\phi \, dz (\mathbf{a}_\rho) \Big|_{\rho=2} = 160\pi$$

$$D_{bottom} = \int_{\rho=1}^{2} \int_{\phi=0}^{2\pi} \rho^3 \mathbf{a}_\rho \bullet \rho \, d\phi \, d\rho (-\mathbf{a}_z) \Big|_{z=0} = 0$$

$$D_{top} = \int_{\rho=1}^{2} \int_{\phi=0}^{2\pi} \rho^3 \mathbf{a}_\rho \bullet \rho \, d\phi \, d\rho (\mathbf{a}_z) \Big|_{z=5} = 0$$

Cont…

Therefore

$$\oint_S \mathbf{D} \bullet d\mathbf{S} = -10\pi + 160\pi + 0 + 0$$

$$= 150\pi$$

(b) For the right side of Divergence Theorem,

evaluate divergence of **D**

$$\nabla \bullet \mathbf{D} = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\rho^3\right) = 4\rho^2$$

So, $$\iiint \nabla \bullet \mathbf{D} dV = \int_{z=0}^{5}\int_{\phi=0}^{2\pi}\int_{\rho=1}^{2} 4\rho^2 \rho d\rho d\phi dz$$

$$= \left(\left(\left(\rho^4 \Big|_{r=1}^{2} \right)\Big|_{\phi=0}^{2\pi} \right)\Big|_{z=0}^{5} \right) = 150\pi$$

Hence, proved

Thank You!