SNS COLLEGE OF ENGINEERING

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AN AUTONOMOUS INSTITUTION

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DISCRETE MATHEMATICS

UNIT I - LOGIC & PROOF

PART-A

1. **Express the statement, "The crop will be destroyed if there is a flood," in symbolic form.**
   Solution:
   Let C: The crop will be destroyed.
   F: There is a flood
   Symbolic form: F→C

2. **State the truth table of "If tigers have wings then the earth travels round the sun."**
   Solution: Let P: Tigers have wings. "F"
   Q: The earth travels round the sun. "F"
   Therefore, given statement is P→Q, has the truth value "T".

P	Q	P→Q
F	F	T

3. **Give the converse and contrapositive of the implication " If it is raining, then I get wet".**
   Solution: P: It is raining.
   Q: I get wet.
   Converse: (Q→P) If I get wet, then it is raining.
   Contrapositive:(⅂Q→⅂P) If I do not get wet, then it is not raining.

4. **What are the contrapositive, the converse and the inverse of the conditional statement. "If you work hard then you will be rewarded."**
   Solution:

P:You work hard	⅂P: You will not work hard
Q: You will be rewarded	⅂Q: You will not be rewarded.

   Converse: (Q→P) You will be rewarded only if you work hard.
   Contrapositive:(⅂Q→⅂P) If you will not be rewarded then you will not work hard.
   Inverse: (⅂P→⅂Q)If you will not work hard then you will not be rewarded.

5. **Define Tautology with an example.**

Solution: A statement that is true for all possible values of propositional variables is called a tautology or universally valid formula or a logical truth.

   Example: P∨ꓶP is a tautology.

6. **Using truth table, show that the proposition p∨ꓶ(p∧q) is a tautology.**
   Solution:

p	q	p∧q	ꓶ(p∧q)	p∨ꓶ(p∧q)
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

Therefore, p∨ꓶ(p∧q) is a tautology.

7. **Show that the proposition P→Q and ꓶP∨Q are logically equivalent.**
   Solution:

   We should prove that (P→Q) ⇔ꓶP∨Q.

   i.e., To prove (P→Q) ↔ꓶP∨Q is tautology.

P	Q	P→Q	ꓶP	ꓶP∨Q	(P→Q) ↔ꓶP∨Q
T	T	T	F	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

(P→Q) ⇔ꓶP∨Q.

8. **State the rules of inference theory.**
   Solution:
   Rule P: A premise may be introduced at any point in the derivation.
   Rule T: A formula S may be introduced in a derivation of s is tautologically implied by any one or more of the preceding formulas in the derivation.
   Rule CP: If we can derive S from R and a set of premises, then we can derive R→S from the set of premises alone.

9.**Determine whether the conculsion C follows logically from the premises H_1 and H_2 or not.**
   H_1: P→Q, H_2:P, C:Q.
   Solution:

P	Q	P→Q	(P→Q)∧P	[(P→Q)∧P]→Q
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Therefore, conclusion is valid.

9. **Write the negation of the statement (∃x) (∀y) p(x,y).**
   Solution:

Given : $(\exists x)(\forall y)\ p(x, y)$

Negation : $(\forall x)(\exists y)\ \overline{p(x, y)}.$

10. Write the scope of the quantifiers in the formula (x) (P(x)→(∃y)R(x, y))
Solution:

The scope of (x) is P(x)→(∃y)R(x,y).

The scope of (∃y) is R(x,y).

11. "Every parrot is ugly"-Express using quantifiers.
Solution: P(x): x is a parrot;

Q(x): x is ugly.

Symbolic form: (x)(P(x)→Q(x)).

12. Symbolize the statement "Some men are genius"
Solution:

Let M(x): x is a man

G(x): x is genius.

Symbolic form: (∃x)(M(x)∧G(x)).

13. Symbolize the statement "All men are giants."
Solution:

Case(i): Using G(x): x is a giant.

M(x): x is a man.

The given statement can be symbolized as (∀x)[(M(x)→G(x)]

Case(ii): However, if we restrict the variable x to the universe which is the class of men, then the statement is (∀x)G(x).

14. Symbolize the statement "x is the father of the mother of y".
Solution:

P(x): x is a person

F(x,y): x is father of y.

M(x,y): x is mother of y.

Symbolic form: (∃z)[P(z)∧ F(x,z)∧ M(z,x)].

15. Explain the two types of quantifiers through example.
Solution:

(i)Universal Quantifiers: The quantifier "all" is called the universal Quantifier and we shall denote it by ∀x.

Eg: For all x, x is an integer is written as (∀x) I(x).

(ii)Existential Quantifiers: The quantifier "some" is called the existential quantifier and we shall denote it by ∃x.

Eg: There exists an x such that x is a man is written as (∃x) M(x)

PART-B

Problems based on Logical Equivalence:

1. Show that p∨(q∧r) and (p∨q)∧(p∨r) are logically equivalent.
2. Without using the truth table, prove that ⌐p→(q→r)⇔ (q→(p∨r).
3. Show that ⌐(p∧⌐(p∧q)) and (⌐p ∧⌐q) are logically equivalent.
4. Show that (⌐p∧(⌐q∧r) ∨(q∧r) ∨(p∧r)⇔ r

Problems based on Normal forms:

5. Without using truth table find the PCNF and PDNF of P→(Q∧P)∧(⌐P→(⌐Q∧⌐R)).
   [Ans: PCNF: (⌐P∨Q∨R)∧ (⌐P∨Q∨⌐R)∧ (⌐P∨⌐Q∨R)∧ (P∨⌐Q∨R)∧ (P∨⌐Q∨⌐R)∧ (P∨Q∨⌐R)
   PDNF: (P∧Q∧R)∨(⌐P∧⌐Q∧⌐R)].
6. Obtain the product of sum canonical form of the formula (⌐P→R) ∧ (Q↔P)
7. Find the PCNF of (P∨R) ∧ (P∨⌐Q). Also find its PDNF, without using truth table.
8. Obtain the PCNF and PDNF of (P∧Q) ∨(⌐P∧R)

Problems based on Inference theory of Statement Calculus:

9. Show that R∨S follows logically from the premises C∨D, (C∨D)→⌐H, ⌐H→(A∧⌐B) and (A∧⌐B)→(R∨S).
10. Show that (P→Q)∧ (R→S), (Q∧M)∧ (S→N), ⌐(M∧N) and (P→R) ⇒ ⌐P
11. Show that R∧(P∨Q) is a valid conclusion from the premises P∨Q, Q→R, P→M, ⌐M.
12. Prove that the premises a→(b→c), d→(b∧⌐c) and (a∧d) are inconsistent.
13. Show that R→S can be derived from the premises P→(Q→S), ⌐R∨P & Q
14. Using conditional proof prove that ⌐P∨Q, ⌐Q∨R, R→S ⇒ P→S
15. Show that S∨R is tautologically implied by (P∨Q) ∧(P→R) and (Q→S).
16. Show that the following set of premises are inconsistent
    (i) If Jack misses many classes through illness, then he fails high school
    (ii) If Jack fails high school, then he is uneducated
    (iii) If Jack reads a lot of books, then he is not uneducated.
    (iv) Jack misses many classes through illness and reads a lot of books.
17. Show that the following implication by using indirect method. (R→⌐Q), R∨S, S→⌐Q, P→Q ⇒⌐P.
18. Using Indirect method of proof, derive p→⌐s from the premises p→(q∨r), q→⌐p, s→⌐r and p.

Problems based on Predicate Calculus:

19. Prove that (∀x) (P(x)→ Q(x)), (∀x) (R(x)→⌐Q(x)) ⇒ (∀x) (R(x)→ ⌐P(x)).
20. Show that the conclusion (∀x) (F(x)→ ⌐S(x)) follows from the premises (∃x)(F(x)∧S(x))→(y) (M(y)→ W(y)) and (∃y) (M(y)∧⌐W(y)).
21. Show that (∀x) (P(x)∨Q(x)) ⇒ (∀x) (P(x)∨ (∃x) Q(x)) by indirect method of proof.
22. Show that (x) (P(x) →Q(x) ∧ (x) (Q(x) → R(x)) ⇒ (x) (P(x) →R(x)

UNIT II- COMBINATORICS
PART-A

23. Use mathematical induction to show that $n! \geq 2^{n-1}, n = 1, 2, 3$ ....
Solution:

Let $p(n): n! \geq 2^{n-1}, n = 1,2,3 \ldots$

Step 1: **To prove p(1) is true.**
$$1! \geq 2^{1-1}$$
$$1! \geq 2^0$$
$$1 \geq 1 \Rightarrow p(1) \text{ is true.}$$

Step 2: Assume that p(k) is true.
$$k! \geq 2^{k-1}, k = 1,2, \ldots.$$

Step 3: To prove p(k+1) is true.
$$\text{i.e. } (k+1)! \geq 2^{(k+1)-1}$$
$$(k+1)! = (k+1)(k!)$$
$$\geq (k+1)2^{k-1}$$
$$\geq 2 \, 2^{k-1} \quad [\text{Since } (k+1) \geq 2]$$
$$\geq 2^{(k+1)-1}$$

Hence p(k+1) is true.

24.State the pigeon hole principle.

Solution:

If n pigeons are assigned to m pigeon holes, and m<n, then at least one pigeonhole contains two or more pigeons.

24. How many students must be in a class to guarantee that atleast two students receive the same score on the final exam if the exam is graded on a scale from 0 to 100 points.
Solution:

There are 101 possible scores 0,1,2………100

By Pigeonhole principle, we have assuming 102 students there must be atleast 2 students with the same score.

Therefore 102 students must be in a class to guarantee that atleast two students reveive the same score.

25.**If seven colours are used to paint 50 bicycles, then show that atleast 8 bicycles will be the same colour.**

Solution:

Here, Number of Pigeon = n = Number of bicycle = 50

Number of Holes = m = Number of colours = 7

By generalized Pigeon hole principle, we get

$$\left[\frac{n-1}{m}\right] + 1 = \left[\frac{50-1}{7}\right] + 1 = 8$$

Therefore, Atleast 8 bicycles will have the same colour.

<u>Problems based on Mathematical Induction:</u>

25. **Prove, by mathematical induction, that for all n≥1, n³+2n is a multiple of 3.**

26. **Prove by the principle of mathematical induction for a positive integer,**

$$1^2 + 2^2 + 3^2 + \ldots\ldots\ldots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

27. **Use mathematical induction to show that** $\dfrac{1}{\sqrt{1}} + \dfrac{1}{\sqrt{2}} + \dfrac{1}{\sqrt{3}} + \ldots\ldots\ldots + \dfrac{1}{\sqrt{n}} > \sqrt{n}, n \geq 2.$

28. **Use the mathematical induction to prove the inequality n<2ⁿ for all positive integer n.**

29. **Use mathematical induction to show that 1+2+2²+……………2ⁿ=2ⁿ⁺¹-1 for all non negative integers n.**

30. **Using mathematical induction prove that**

$$1^2 + 3^2 + 5^2 + \ldots\ldots\ldots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}.$$