SNS COLLEGE OF ENGINEERING

Kurumbapalayam (Po), Coimbatore – 641 107

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Redesigning Common Mind & Business Towards Excellence

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DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

COURSE NAME : 23EET206 CONTROL SYSTEMS AND INSTRUMENTATION

II YEAR ECE /III SEMESTER

Unit 1- Control System Modelling

Topic 2 : Transfer Function

TRANSFER FUNCTION

The transfer function of a control system is defined as the ratio of the Laplace transform of the output variable to Laplace transform of the input variable assuming all **initial conditions** to be **zero.**

$$Transfer function = \frac{Laplace\ transform\ of\ output\ variable}{Laplace\ transform\ of\ input\ variable}$$

$$T(S) = \frac{C(S)}{R(S)} = G(S)$$

T(S) = Transfer function of the system.
C(S) = output.
R(S) = Reference output.
G(S) = Gain

Negative Feedback TF=G/1+GH

Positive Feedback TF=G/1-GH

METHODS OF OBTAINING A TRANSFER FUNCTION

Block Diagram Method: It is not convenient to derive a complete transfer function for a complex control system. Therefore the transfer function of each element of a control system is represented by a block diagram. Block diagram reduction techniques are applied to obtain the desired transfer function.

Signal Flow Graphs: The modified form of a block diagram is a signal flow graph. Block diagrams visually outline a control system, while signal flow graphs provide a more compact representation.

POLES AND ZEROS

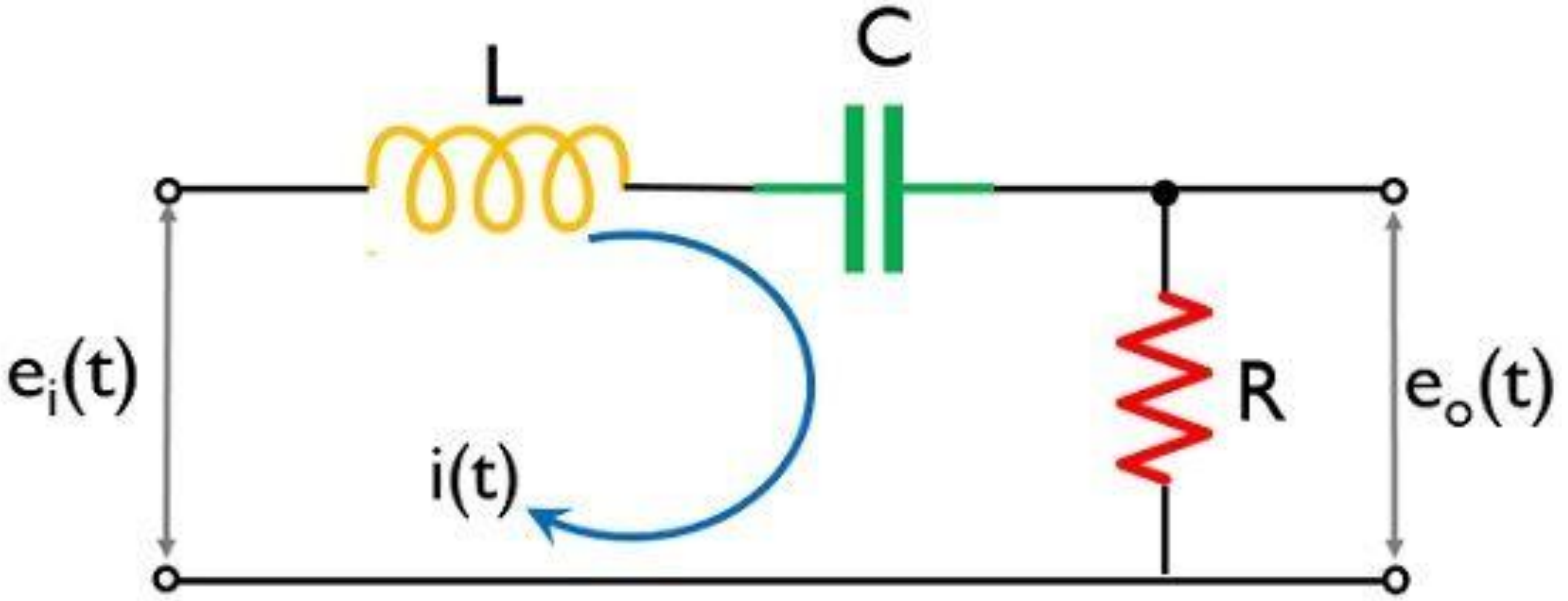
$$T(s) = \frac{Y(s)}{X(s)}$$

$$T(s) = \frac{a_0 s^m + a_1 s^{m-1} + - - - - - a_m}{b_0 s^n + a_1 s^{n-1} + - - - - - b_n}$$

$$T(s) = \frac{K(s - s_a)\,(s - s_b) - - - -(s - s_m)}{(s - s_1)\,(s - s_2) - - - -(s - s_n)}$$

- $S_1, S_2, S_3, \ldots., S_n$ are the roots of Poles of the System
- $S_a, S_b, S_c, \ldots., S_m$ are the roots of Zeros of the System

TRANSFER FUNCTION OF AN ELECTRIC CIRCUIT - EXAMPLE



Element	Time domain expression for voltage	Laplace domain expression for voltage	Laplace domain behaviour
Resistance R	$i(t)*R$	$I(s)\,R$	R
Capacitance C	$\frac{1}{C}\int i(t)dt$	$\frac{1}{sC}I(s)$	$\frac{1}{sC}$
Inductance L	$L\frac{d\,i(t)}{dt}$	$sL\,I(s)$	sL

TRANSFER FUNCTION OF AN ELECTRIC CIRCUIT - EXAMPLE

Let $e_i(t)$ and $e_o(t)$ be the input applied and output of the circuit respectively.

On applying KVL in the above circuit,

$$e_i(t) = L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)dt + i(t)R \quad \text{------ eq 1}$$

$$e_o(t) = i(t)R \quad \text{------- eq 2}$$

$$E_i(s) = sL\,I(s) + \frac{1}{C}\frac{I(s)}{s} + R\,I(s) \quad \text{------ eq 3}$$

$$E_o(s) = R\,I(s) \quad \text{------- eq 4}$$

$$E_i(s) = I(s)\,[\,sL + R + \frac{1}{sC}] \quad \text{------- eq 5}$$

$$I(s) = \frac{E_o(s)}{R}$$

TRANSFER FUNCTION OF AN ELECTRIC CIRCUIT - EXAMPLE

$$E_i(s) = \frac{E_o(s)}{R}\left[sL + R + \frac{1}{sC}\right]$$

$$E_i(s) = \frac{E_o(s)}{R}\left[\frac{s^2LC+sRC+1}{sC}\right]$$

$$\frac{E_i(s)}{E_o(s)} = \frac{s^2LC+sRC+1}{sRC}$$

$$\frac{E_o(s)}{E_i(s)} = \frac{sRC}{s^2LC+sRC+1}$$

ADVANTAGES AND DISADVANTAGES OF TRANSFER FUNCTION

Advantages

- The complex time-domain equations can be converted into simple algebraic form using Laplace transform.
- It provides the mathematical model of the overall system along with each system component.
- For a known transfer function, the output response is easy to determine for any reference input.
- It helps to determine important parameters of the system like poles, zeros, etc.
- The stability of the system can be easily analyzed using the transfer function.
- It helps to relate output with input.

Disadvantages

- It is not applicable to non-linear systems.
- The initial conditions are not considered as the effects generated by them are neglected.

References

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3. Richard C. Dorf and Robert H. Bishop, “Modern Control Systems”, Addison, 12th Edition, 2010. (Unit I-III).
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Thank You