SNS COLLEGE OF ENGINEERING

Kurumbapalayam (Po), Coimbatore – 641 107

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Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

COURSE NAME : 23EET206 CONTROL SYSTEMS AND INSTRUMENTATION

II YEAR ECE /III SEMESTER

Unit 1- Control System Modelling

Topic 4 : Modeling of Physical Systems – Mechanical Systems

MODELING OF PHYSICAL SYSTEMS

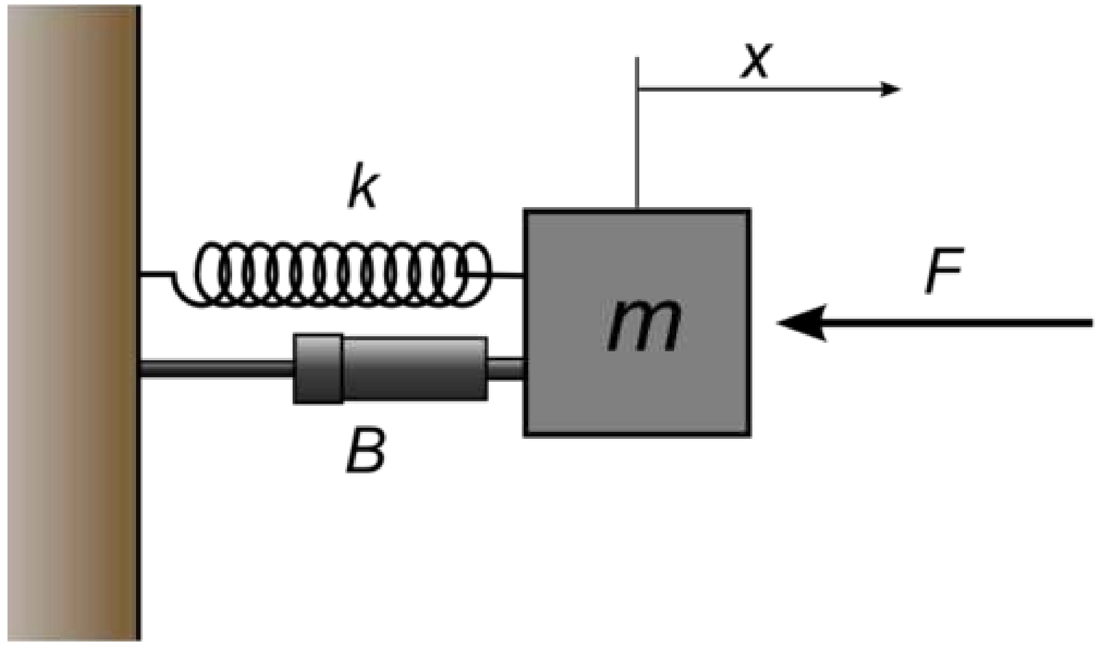
- The control systems can be represented with a set of mathematical equations known as **mathematical model.**
- **Mathematical Models are obtained by using**
    - Differential equation model
    - Transfer function model
    - State space model
- These models are useful for analysis and design of control systems. Analysis of control system means finding the output when we know the input and mathematical model.
- Design of control system means finding the mathematical model when we know the input and the output.

MATHEMATICAL MODEL

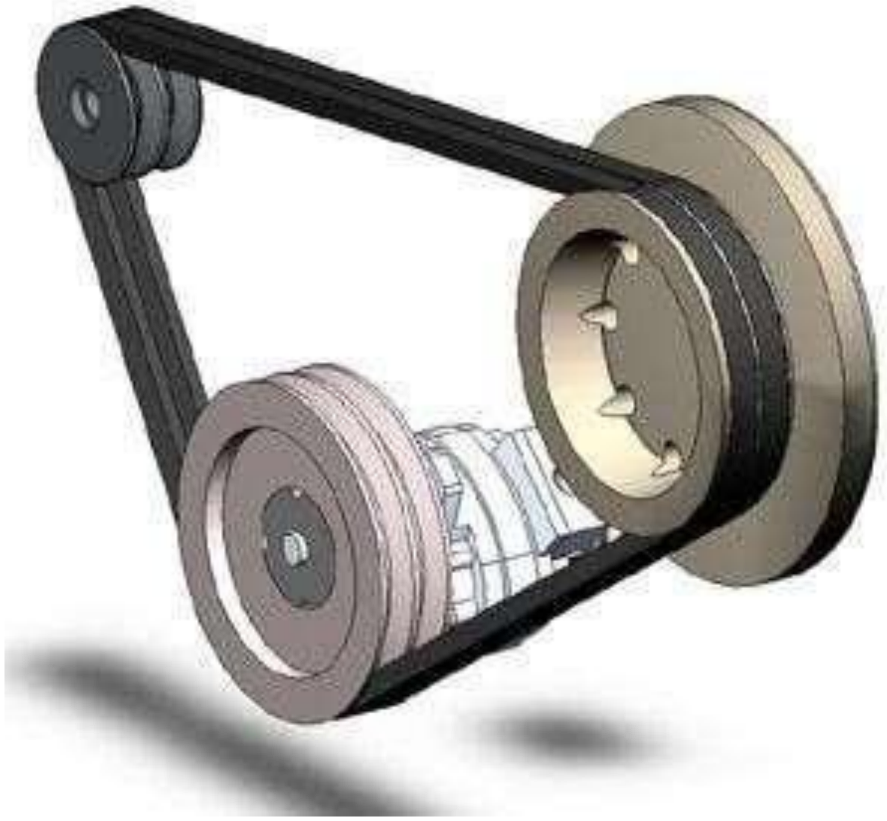
- A mathematicalmodel is a set of equations (usually differential equations) that represents the dynamics of systems.
- In practice, the complexity of the system requires some assumptions in the determination model.
- How do we obtain the equations?
  - Physical law of the process
  - Examples:
    - Mechanical system (Newton's laws)
    - Electrical system (Kirchhoff's laws)

BASIC TYPES OF MECHANICAL SYSTEMS

- Translational System

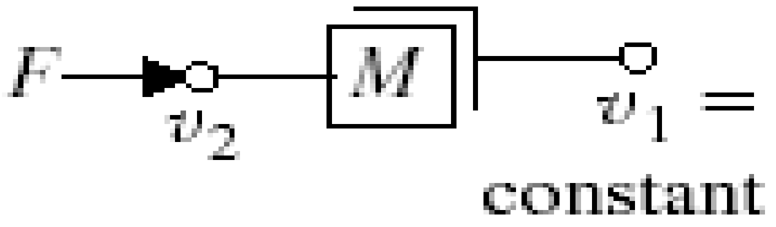


- Rotational System

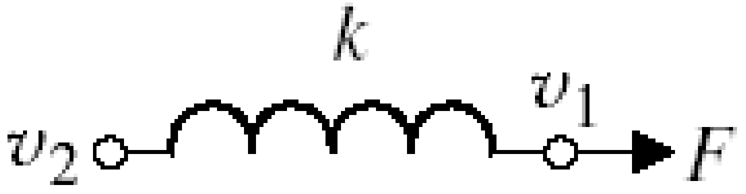


TRANSLATIONAL MECHANICAL SYSTEMS

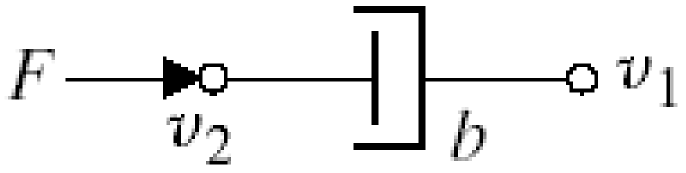
These systems mainly consist of three basic elements. Mass, spring and dashpot or damper.



Translational Mass



Translational Spring

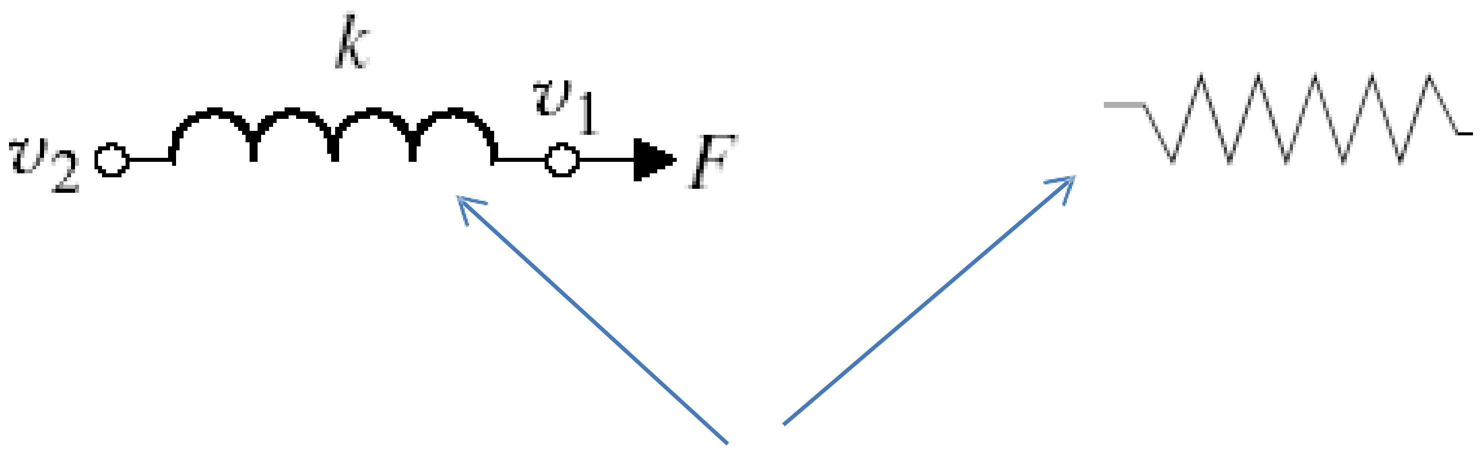


Translational Damper

TRANSLATIONAL MECHANICAL SYSTEMS

- A translational spring is a mechanical element that can be deformed by an external force such that the deformation is directly proportional to the force applied to it.

Translational Spring

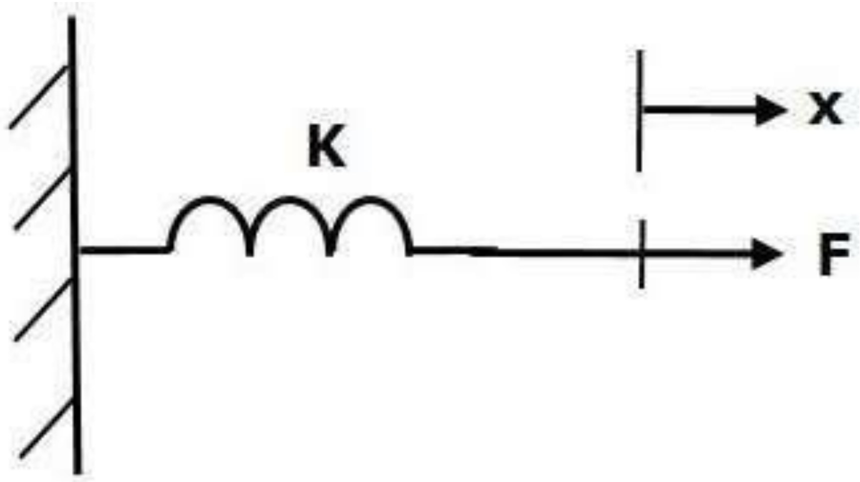


Circuit Symbols

Translational Spring

TRANSLATIONAL MECHANICAL SYSTEMS

- Spring is an element, which stores **potential energy**.



$F \alpha x$ => $F_k = K\,x$

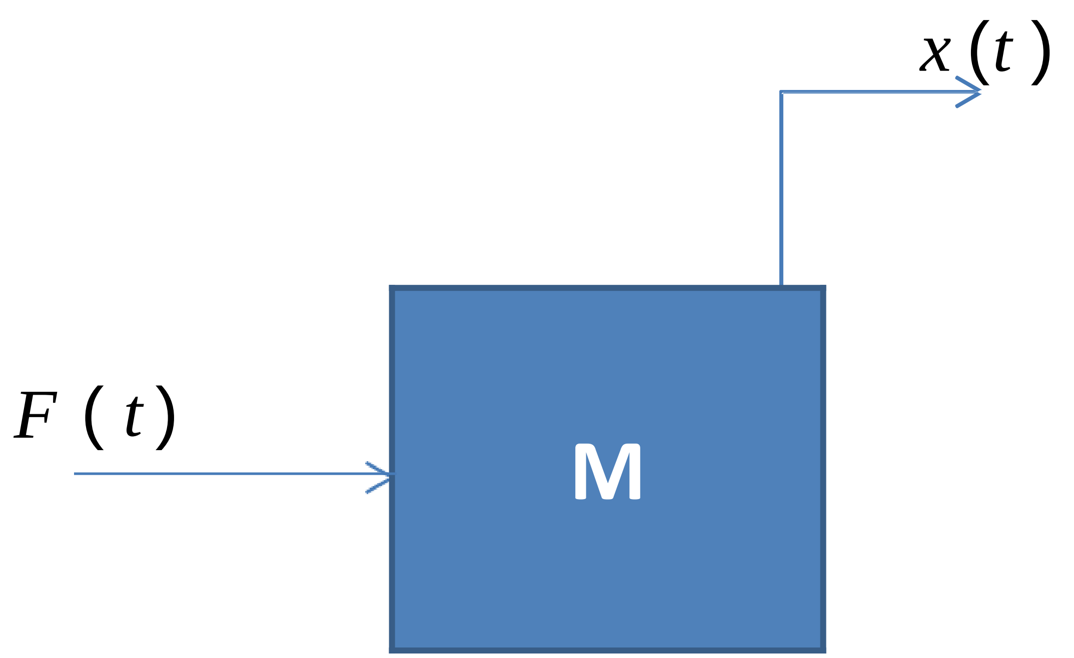
=> $F = F_k = K\,x$

- Where,
  - **F** is the applied force
  - $\mathbf{F_k}$ is the opposing force due to elasticity of spring
  - **K** is spring constant
  - **x** is displacement

TRANSLATIONAL MECHANICAL SYSTEMS

- Translational Mass is an inertia element.
- A mechanical system without mass does not exist.
- If a force F is applied to a mass and it is displaced to x meters then the relation b/w force and displacements is given by Newton's law.

Translational Mass

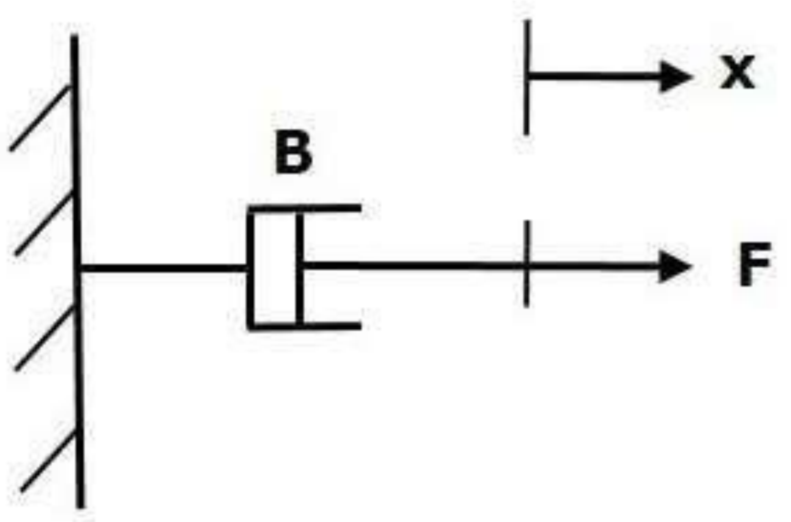


$F_m \alpha\, a$ $\quad F_m = Ma \quad$ =>

$$=> F = F_m = M\frac{d^2x}{dt^2}$$

TRANSLATIONAL MECHANICAL SYSTEMS

- **Dash Pot:** If a force is applied on dashpot **B**, then it is opposed by an opposing force due to **friction** of the dashpot. This opposing force is proportional to the velocity of the body. Assume mass and elasticity are negligible.



$$F_b \alpha v \quad \Rightarrow F_b = Bv = B\frac{dx}{dt}$$

$$\Rightarrow \quad F = F_b = B\frac{dx}{dt}$$

TRANSFER FUNCTION OF TRANSLATIONAL MECHANICAL SYSTEMS

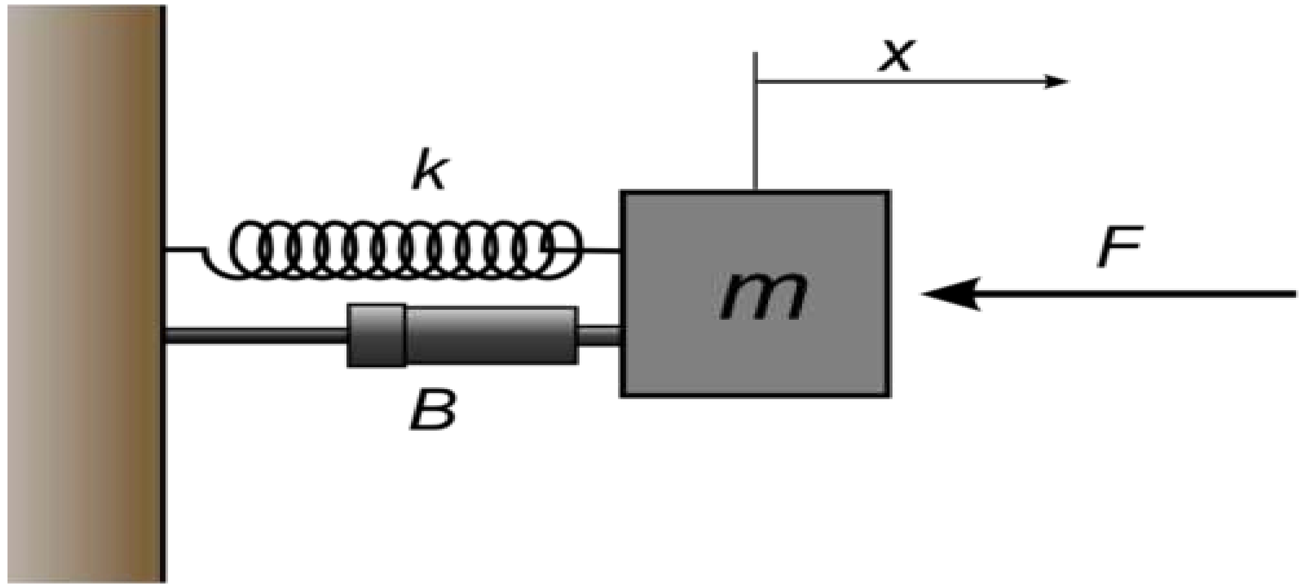
- **First,** draw a free-body diagram, placing on the body all forces that act on the body either in the direction of motion or oppositeto it.

- **Second,** use Newton's law to form a differential equation of motion by summing the forces and setting the sum equal to zero.

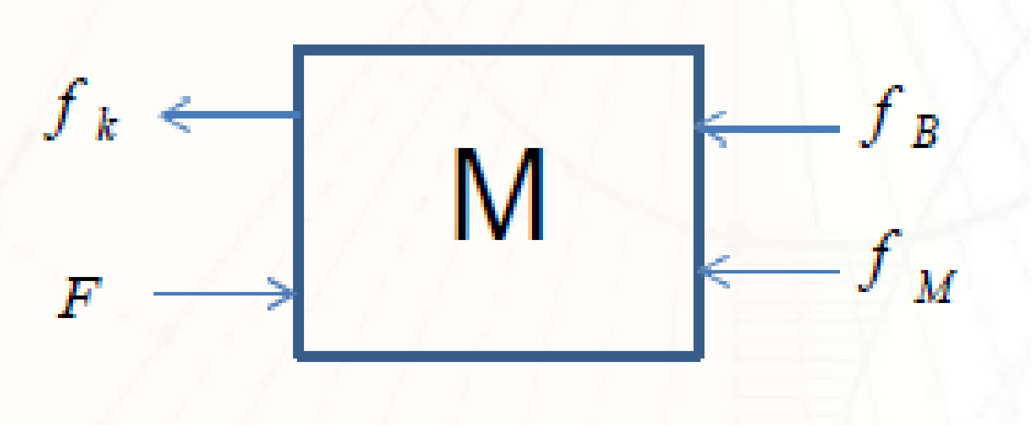
- **Finally,** assuming zero initial conditions, we take the Laplace transform of the differential equation, separate the variables, and arrive at the transfer function.

MECHANICAL TRANSLATIONAL SYSTEM

- Consider the following system



- Free Body Diagram

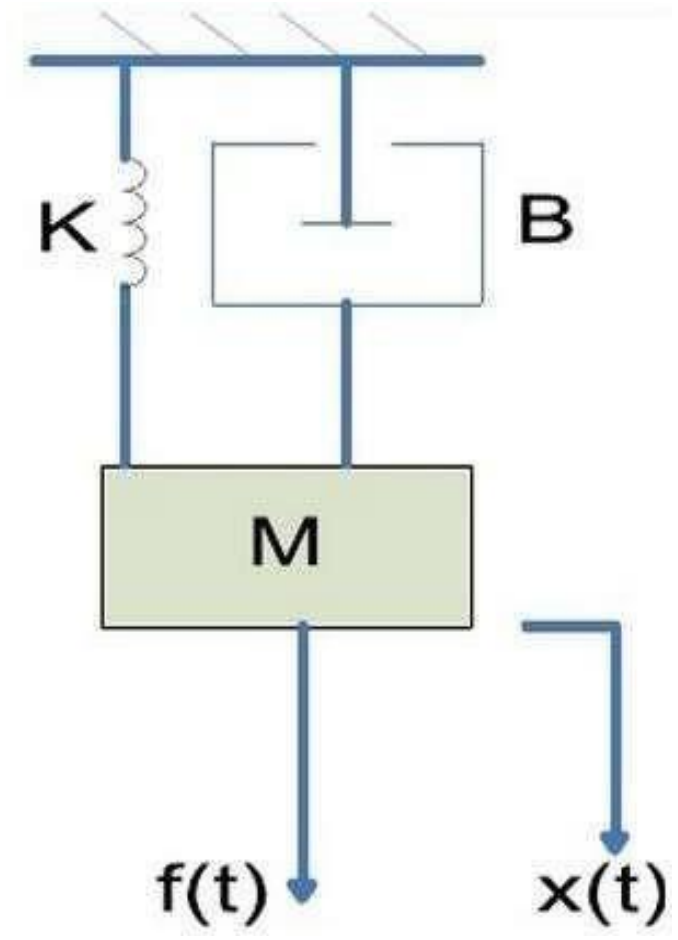


$$\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + Bs + k}$$

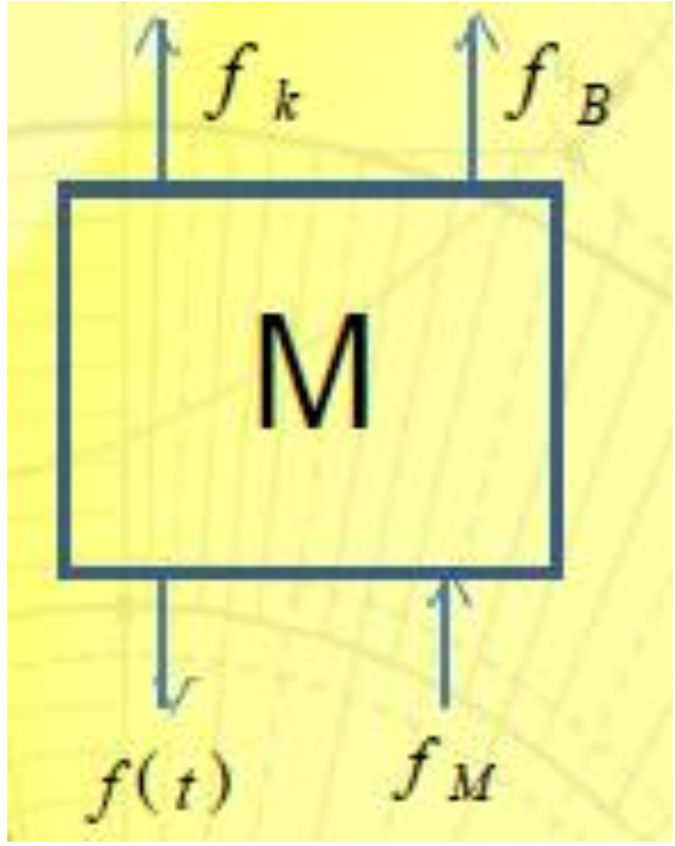
$$F = f_k + f_M + f_B$$

TRANSFER FUNCTION OF MECHANICAL TRANSLATION SYSTEM

Find the transfer function of the mechanicaltranslationalsystem given in Figure.



Free Body Diagram

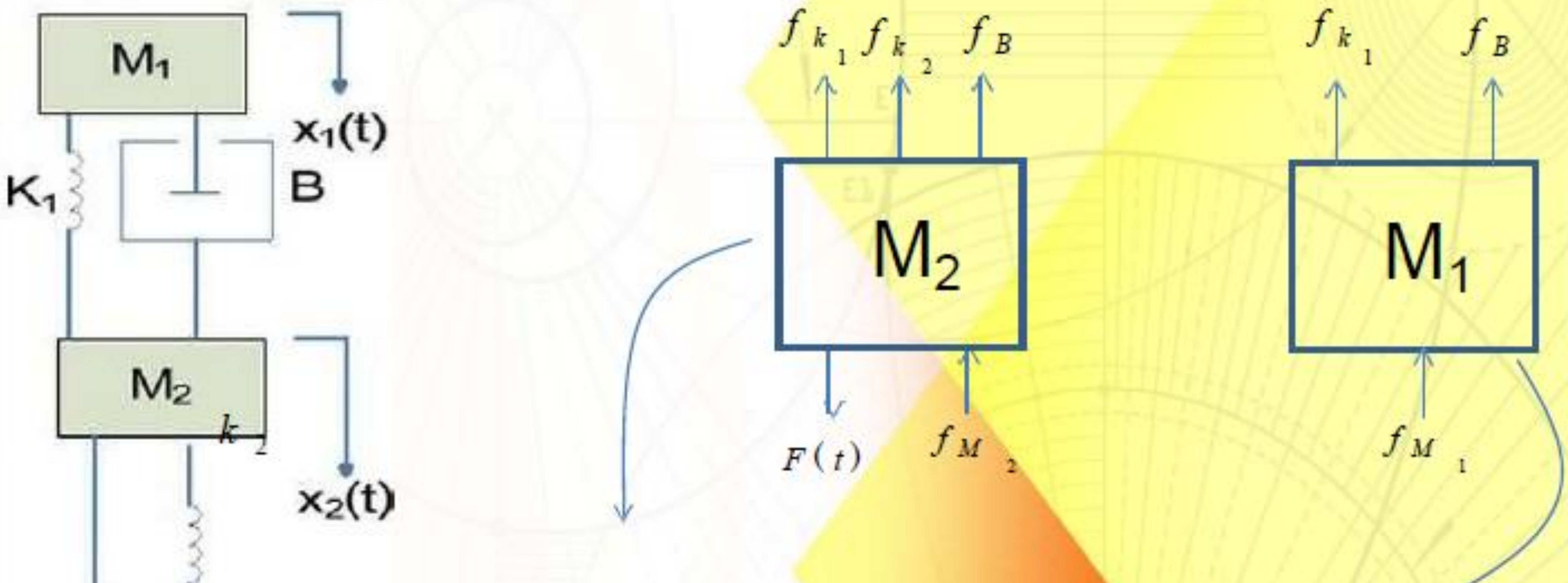


$$f(t) = f_k + f_M + f_B$$

$$\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + Bs + k}$$

MODELING OF A MECHANICAL SYSTEM

Draw the free body diagram for the mechanical system



$$F(t) = f_{k_1} + f_{k_2} + f_{M_2} + f_B$$

$$0 = f_{k_1} + f_{M_1} + f_B$$

ANALOGOUS SYSTEMS

Electrical Analogous of mechanical Translational System:

As the electrical systemshas two types of inputs either voltage or current source. There are two types of analogies .

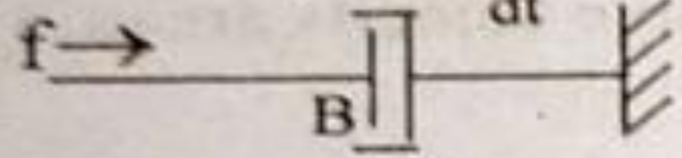
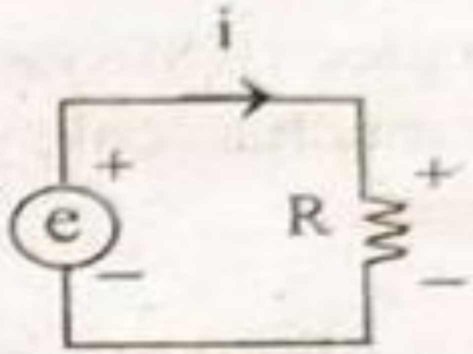
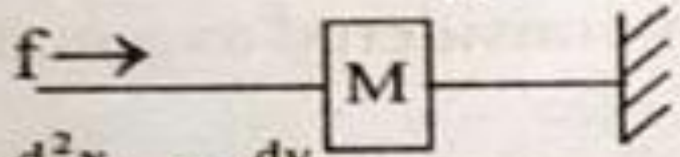
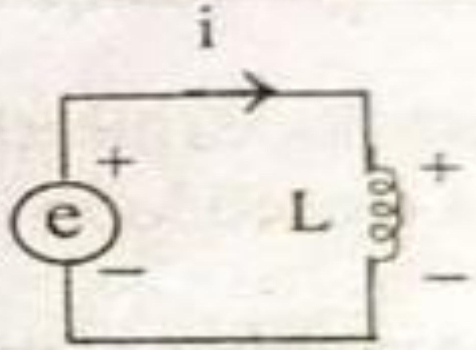
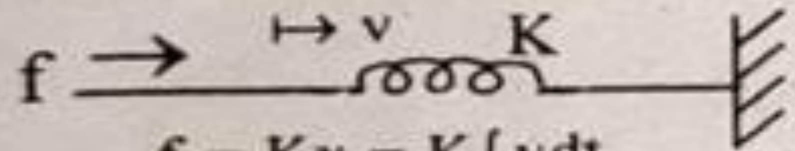
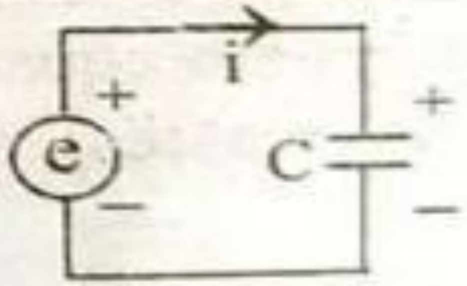
- Force- Voltage analogy
- Force- Current analogy

Force- Voltage Analogy:

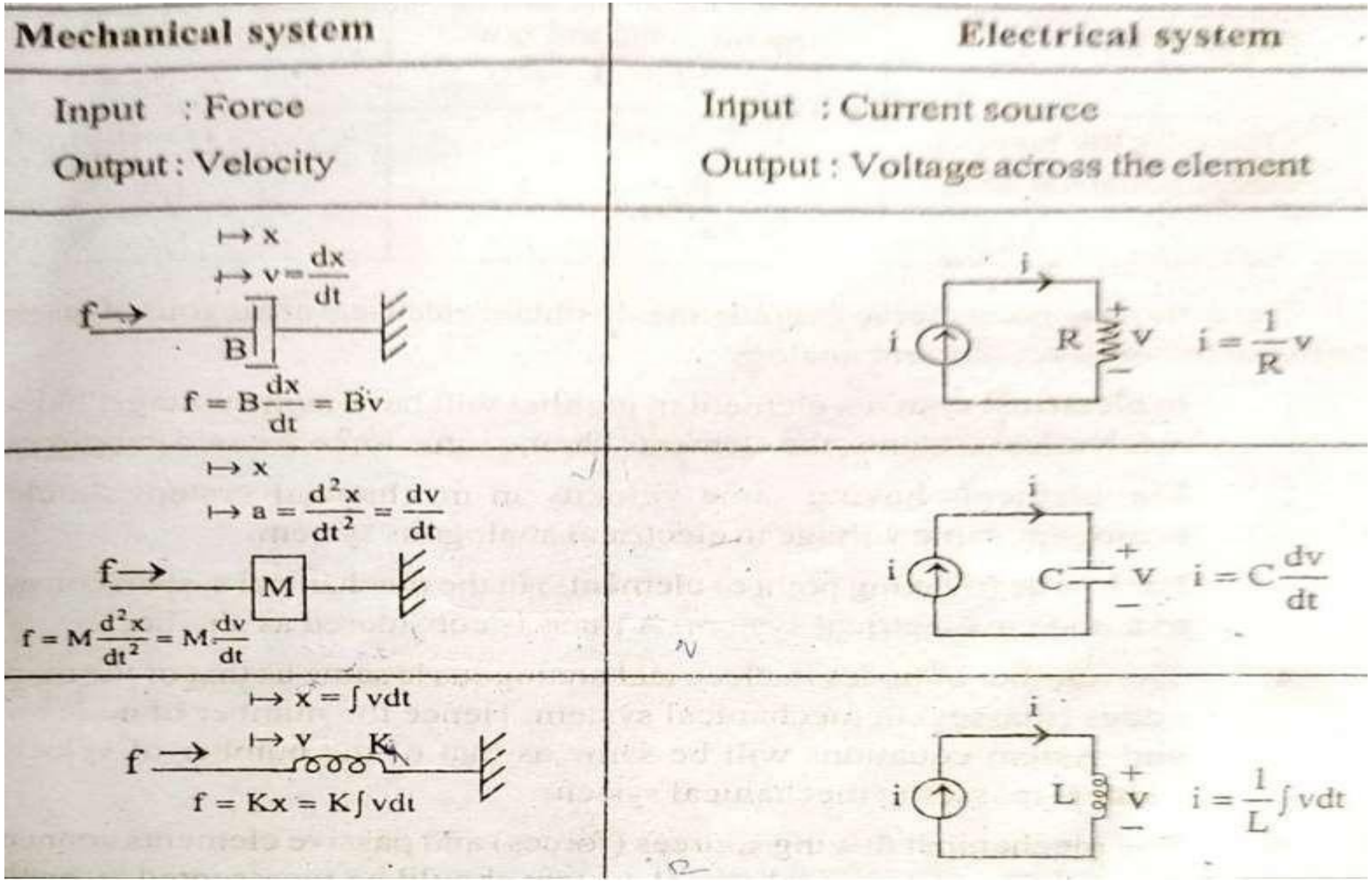
TABLE 3

Item	Mechanical system	Electrical system (mesh basis system)
Independent variable (input)	Force, f	Voltage, e
Dependent variable (output)	Velocity, v	Current, i
	Displacement, x	Charge, q
Dissipative element	Frictional coefficient of dashpot, B	Resistance, R
Storage element	Mass, M	Inductance, L
	Stiffness of spring, K	Inverse of capacitance, 1/C
Physical law	Newton's second law $\Sigma F = 0$	Kirchoff 's voltage law $\Sigma V = 0$
Changing the level of independent variable	lever $\frac{f_1}{f_2} = \frac{l_1}{l_2}$	Transformer $\frac{e_1}{e_2} = \frac{N_1}{N_2}$

Force- Voltage Analogy:

Mechanical system	Electrical system
Input : Force Output : Velocity	Input : Voltage source Output : Current through the element
$\mapsto x$ $\mapsto v = \frac{dx}{dt}$  $f = B\frac{dx}{dt} = Bv$	 $e = Ri$
$\mapsto x$ $\mapsto a = \frac{d^2x}{dt^2} = \frac{dv}{dt}$  $f = M\frac{d^2x}{dt^2} = M\frac{dv}{dt}$	 $e = L\frac{di}{dt}$
$\mapsto x = \int v dt$ $\mapsto v$  $f = Kx = K\int v dt$	 $e = \frac{1}{C}\int i dt$

Force- Current Analogy:



Mechanical system	Electrical system
Input : Force Output : Velocity	Input : Current source Output : Voltage across the element
$\mapsto x$, $\mapsto v = \frac{dx}{dt}$, f, B $f = B\frac{dx}{dt} = Bv$	i, R, v $i = \frac{1}{R}v$
$\mapsto x$, $\mapsto a = \frac{d^2x}{dt^2} = \frac{dv}{dt}$, f, M $f = M\frac{d^2x}{dt^2} = M\frac{dv}{dt}$	i, C, v $i = C\frac{dv}{dt}$
$\mapsto x = \int v dt$, $\mapsto v$, f, K $f = Kx = K\int v dt$	i, L, v $i = \frac{1}{L}\int v dt$

References

1. Nagrath, J., Gopal, M., “Control System Engineering”, New Age International Publishers, 7th Edition, 2021 (Unit I-III).
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3. Richard C. Dorf and Robert H. Bishop, “Modern Control Systems”, Addison, 12th Edition, 2010. (Unit I-III).
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Thank You