SNS COLLEGE OF ENGINEERING

Kurumbapalayam (Po), Coimbatore – 641 107

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Redesigning Common Mind & Business Towards Excellence

Build an Entrepreneurial Mindset Through Our Design Thinking FrameWork

DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

COURSE NAME : 23EET206 CONTROL SYSTEMS AND INSTRUMENTATION

II YEAR ECE /III SEMESTER

Unit 1- Control System Modelling

Topic 5 : Modeling of Physical Systems – Rotational System (Mechanical)

MODELING OF PHYSICAL SYSTEMS

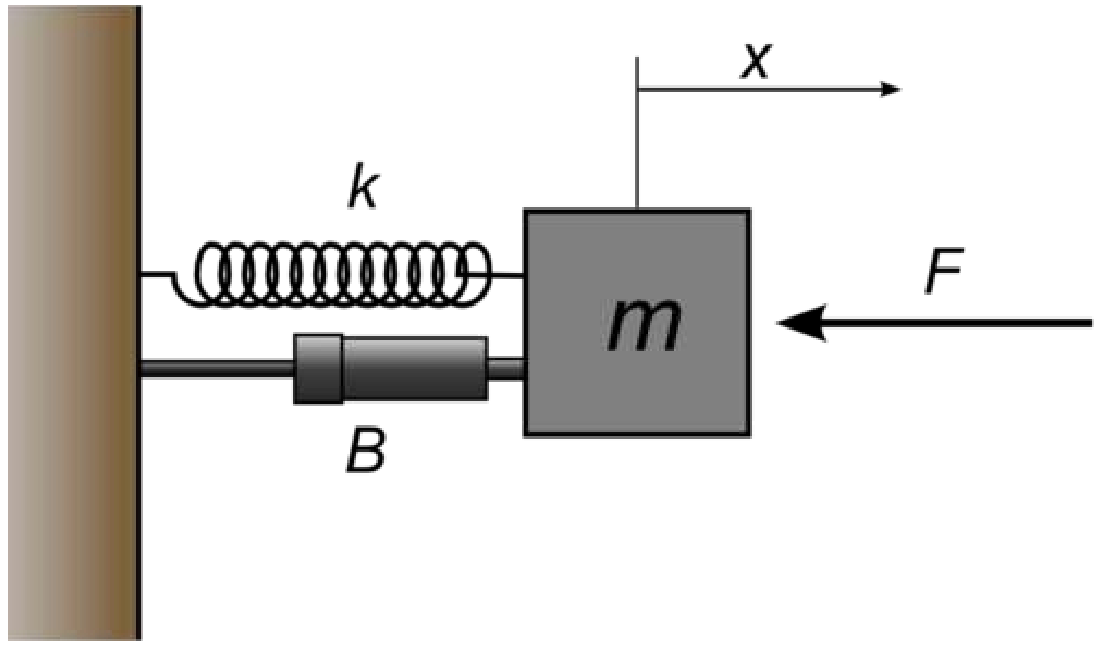
- The control systems can be represented with a set of mathematical equations known as **mathematical model.**
- **Mathematical Models are obtained by using**
  - Differential equation model
  - Transfer function model
  - State space model
- These models are useful for analysis and design of control systems. Analysis of control system means finding the output when we know the input and mathematical model.
- Design of control system means finding the mathematical model when we know the input and the output.

MATHEMATICAL MODEL

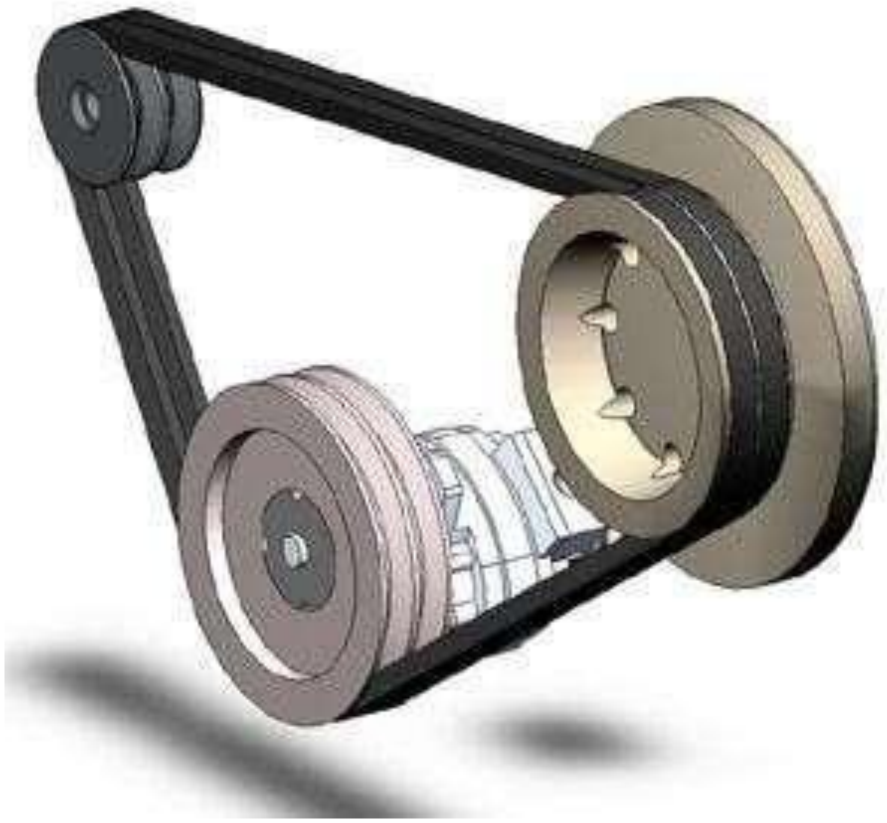
- A mathematicalmodel is a set of equations (usually differential equations) that represents the dynamics of systems.
- In practice, the complexity of the system requires some assumptions in the determination model.
- How do we obtain the equations?
  - Physical law of the process
  - Examples:
    - Mechanical system (Newton's laws)
    - Electrical system (Kirchhoff's laws)

BASIC TYPES OF MECHANICAL SYSTEMS

- ❑ Translational System



- ❑ Rotational System



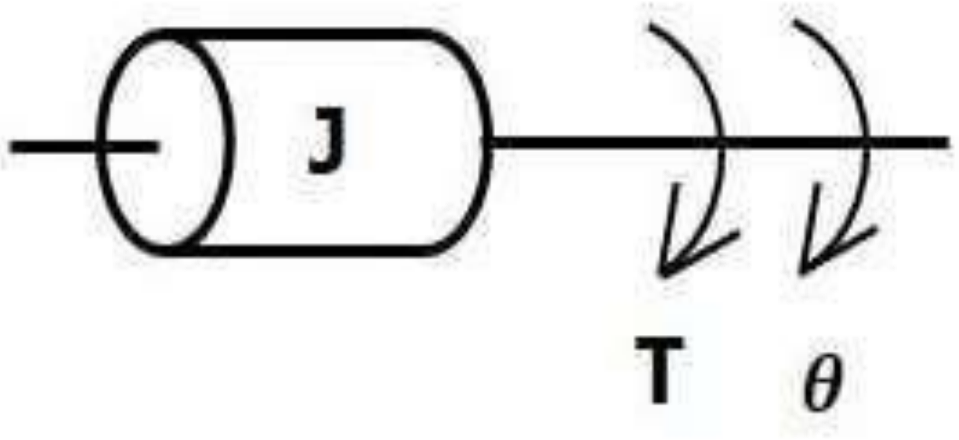
ROTATIONAL MECHANICAL SYSTEMS

- These systems mainly consist of three basic elements. Those are **moment of inertia, torsional spring** and **dashpot**.

- **Moment of Inertia**

In translational mechanical system, mass stores kinetic energy

Similarly, in rotational mechanical system, moment of inertia stores **kinetic energy**.



ROTATIONAL MECHANICAL SYSTEMS

$T_j \alpha \alpha$ => $J\alpha = T_j = J\,\dfrac{d^2\theta}{dt^2}$

=> $T = T_j = J\dfrac{d^2\theta}{dt^2}$

- Where,
  - **T** is the applied torque
  - $\mathbf{T_j}$ is the opposing torque due to moment of inertia
  - **J** is moment of inertia
  - **α** is angular acceleration
  - **θ** is angular displacement

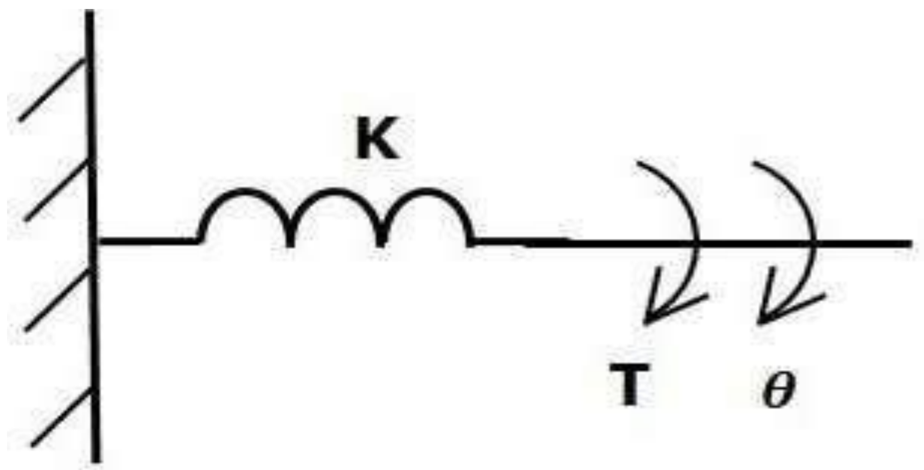
ROTATIONAL MECHANICAL SYSTEMS

➢ **Torsional Spring**

In translation mechanical system, spring stores potential energy. In rotational system, torsional spring stores energy in the form of potential energy.

$$T_k \alpha \, \theta \quad \Rightarrow T_k = K\theta$$

$$\Rightarrow T = T_k = K\theta$$



➢ Where,

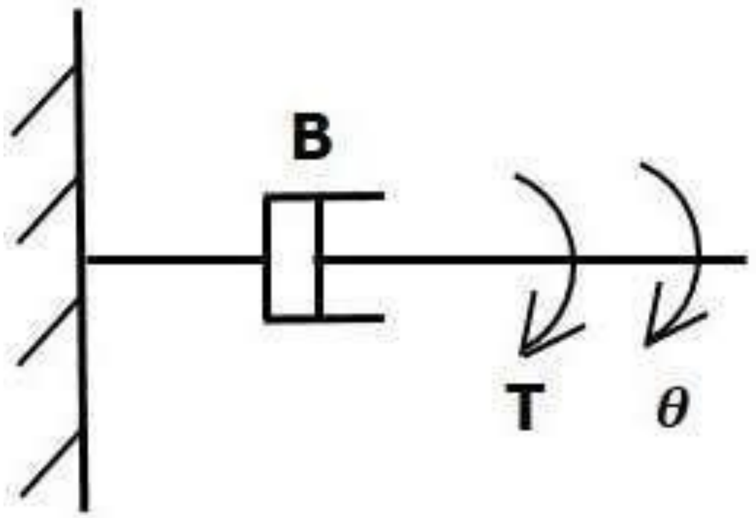
- **T** is the applied torque
- $\mathbf{T_k}$ is the opposing torque due to elasticity of torsional spring
- **K** is the torsional spring constant
- **θ** is angular displacement

➢ Dashpot

If a torque is applied on dashpot **B**, then it is opposed by an opposing torque due to the **rotational friction** of the dashpot.

$T_b \alpha\, \omega$ => $T_b = B\omega = B\,\frac{d\theta}{dt}$

=> $T = T_b = B\,\frac{d\theta}{dt}$



➢ Where,

- **T_b** is the opposing torque due to the rotational friction of the dashpot
- **B** is the rotational friction coefficient
- **ω** is the angular velocity
- **θ** is the angular displacement

References

1. Nagrath, J., Gopal, M., "Control System Engineering", New Age International Publishers, 7th Edition, 2021 (Unit I-III).
2. Benjamin.C.Kuo.,"Automatic Control Systems", Prentice Hall of India, New Delhi, 9th Edition,2007 (Unit I-III).
3. Richard C. Dorf and Robert H. Bishop, "Modern Control Systems", Addison, 12th Edition, 2010. (Unit I-III).
4. Katsuhiko Ogata, "Modern Control Engineering", Prentice Hall of India, New Delhi, 5th Edition, 2009(Unit I-III).

Thank You