SIGNALS AND SYSTEMS

Energy and power signals

- **Definition of an energy signal**

A signal is said to be an energy signal if its normalized energy is nonzero and finite. i.e.,

For an energy signal, $0 < E < \infty$

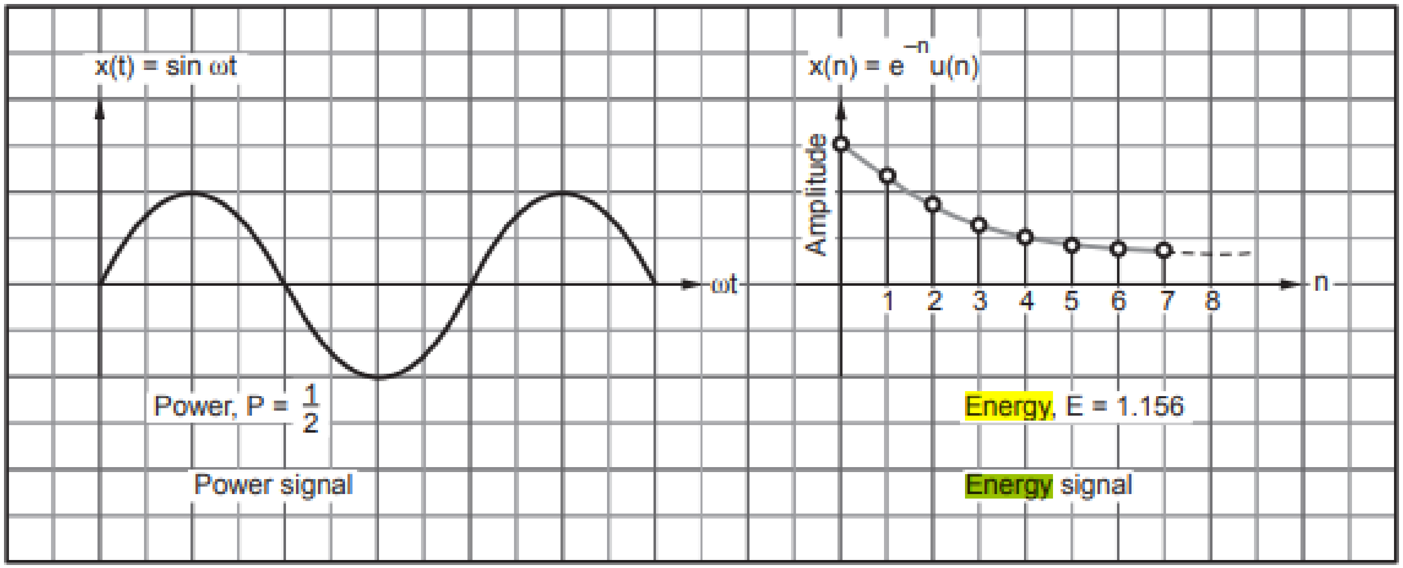
- **Definition of power signal**

A signal is said to be power signal if its normalized power is non zero and finite. i.e.,

For power signal, $0 < P < \infty$

Energy and power signals:

- Examples of Energy signal and Power signal



Energy and power signals:

CT signal x(t):

Energy: E = $$\int_{-\infty}^{\infty} x^2(t)dt$$

Power: P = $$\lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x^2(t)dt$$

DT signal x[n]:

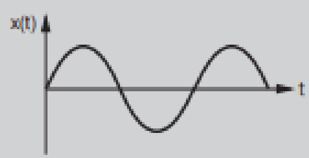
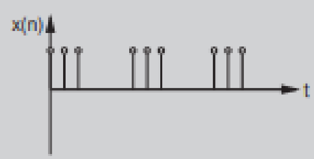
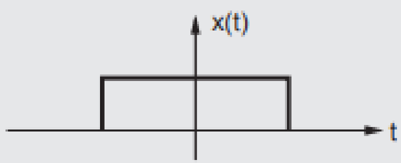
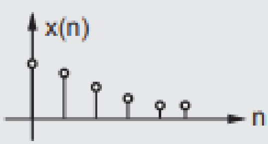
Energy: E = $\sum_{-\infty}^{\infty} x^2[n]$

Power: P = $\lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} x^2[n]$

Energy signal: if $0 < E < \infty$

Power signal: if $0 < P < \infty$

Comparison between Power and Energy Signal

Sr. No.	Parameter	Power signal	Energy signal
1.	Definition	$0 < P < \infty$	$0 < E < \infty$
2.	Equation	$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)dt$ $\lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \lvert x(n) \rvert^2$	$E = \int_{-\infty}^{\infty} \lvert x(t) \rvert^2 dt$ $= \sum_{n=-\infty}^{\infty} \lvert x(n) \rvert^2$
3.	Periodicity	Most of periodic signals are power signals	Most of the non-periodic signals are energy signals.
4.	Energy and power	Energy of the power signal is infinite.	Power of the energy signal is zero.
5.	Examples	 	 

Points to remember

Observe the signal carefully. If it is periodic and infinite duration then it can be power signal. Hence calculate its power directly.

If the signal is periodic but of finite duration, then it can be energy signal. Hence calculate its energy directly.

If the signal is not periodic, then it can be energy signal. Hence calculate its energy directly.

Example Problems

Ex. 1.3.31 *Find whether the following signals are power or energy signals. Determine power and energy of these signals.*

$g(t) = 5\cos\left(17\pi t + \frac{\pi}{4}\right) + 2\sin\left(19\pi t + \frac{\pi}{3}\right)$

AU : May-16, Marks 4

Sol. :

$$g(t) = 5\cos\left(17\pi t + \frac{\pi}{4}\right) + 2\sin\left(19\pi t + \frac{\pi}{3}\right)$$

$$\text{Here } f_1 = \frac{17}{2}\,Hz \text{ and } f_2 = \frac{19}{2}\,Hz$$

$$\text{Hence } T_1 = \frac{2}{17}\text{ sec. and } T_2 = \frac{2}{19}\text{ sec.}$$

$$\text{Since } \frac{T_1}{T_2} = \frac{2/17}{2/19} = \frac{19}{17} \text{ is rational, the signal is periodic.}$$

The period is $T = 17\,T_1 = 19\,T_2 = 2$ *sec.* Since this is periodic signal, it must be a power signal. Let us calculate the power, i.e.,

$$P = \frac{1}{T}\int_0^T g^2(t)\,dt = \frac{1}{2}\int_0^2 \left[5\cos\left(17\pi t + \frac{\pi}{4}\right) + 2\sin\left(19\pi t + \frac{\pi}{3}\right)\right]^2_{dt}$$

$$= \frac{1}{2}\int_0^2 \left[25\cos^2\left(17\pi t + \frac{\pi}{4}\right) + 20\cos\left(17\pi t + \frac{\pi}{4}\right)\sin\left(19\pi t + \frac{\pi}{3}\right) + 4\sin^2\left(19\pi t + \frac{\pi}{3}\right)\right] dt$$

Example Problems

Here use $\cos^2 x = \frac{1+\cos 2x}{2}$, $\sin^2 x = \frac{1-\cos 2x}{2}$ and

2 cos x sin y = $\cos(x-y)+\cos(x+y)$. Then above equation will be,

$$P = \frac{1}{2}\int_0^2 \left\{\frac{25}{2}\left[1+\cos 2\left(17\pi t+\frac{\pi}{4}\right)\right]+10\left[\cos\left(2\pi t+\frac{\pi}{12}\right)+\cos\left(36\pi t+\frac{7\pi}{12}\right)\right] +\frac{4}{2}\left[1-\cos 2\left(19\pi t+\frac{\pi}{4}\right)\right]\right\} dt$$

$$= \frac{1}{2}\left\{\frac{25}{2}\int_0^2 dt+\frac{25}{2}\int_0^2 \cos 2\left(17\pi t+\frac{\pi}{4}\right)dt +10\int_0^2 \cos\left(2\pi t+\frac{\pi}{12}\right)dt+\right.$$

$$\left.10\int_0^2 \cos\left(36\pi t+\frac{7\pi}{12}\right)dt+2\int_0^2 dt-2\int_0^2 \cos 2\left(19\pi t+\frac{\pi}{4}\right)dt\right\}$$

Here first and fifth integration terms will be non zero. Rest of the terms are evaluation of cosine waves over a complete cycle. Hence they will be zero. Hence,

$$P = \frac{1}{2}\left\{\frac{25}{2}[t]_0^2 +2\,[t]_0^2\right\}$$

$$= \frac{1}{2}\left\{\frac{25}{2}\times(2-0)+2\times(2-0)\right\} = \frac{29}{2}W$$

Since the power is finite and non-zero, this signal is a **power signal.**


Thank you