TOPIC : 2 – HOMOMORPHISM

Homomorphism :

Def : Group Homomorphism :

If $(G, \circ)$ and $(H, *)$ are groups and $f: G \to H$, then f is called a group homomorphism if for all $a, b \in G$, $f(a \cdot b) = f(a) * f(b)$

Def : Isomorphism :

If $f: (G, \circ) \to (H, *)$ is a homomorphism then f is an isomorphism If it is one-to-one and onto. in this case G, H are said to be isomorphic group.

Def : cyclic :

Group G is called cyclic if there is an element $x \in G$ such that for each $a \in G$, $a = x^n$ for some $n \in Z$

Find all sub groups of $(z_{12}, +)$ group

Solution

To determine all subgroups of the group $(z_{12}, +)$

Since $\{e\}$ and G are the trivial subgroups of Group G.

$\{0\}$ and z_{12} are the trivial subgroups of $(z_{12}, +)$

If G is group and $\phi \neq H \subseteq G$, with H is finite, then H is subgroup of G if and only if H is closed under the binary operation of G.

Clearly $\{0, 6\}$, $\{0, 4, 8\}$, $\{0, 3, 6, 9\}$, $\{0, 2, 4, 6, 8, 10\}$ are properties of subsets of group $(z_{12}, +)$

+	0	6
0	0	6
6	6	0

+	0	4	8
0	0	4	8
4	4	8	0
8	8	0	4

+	0	3	6	9
0	0	3	6	9
3	3	6	9	0
6	6	9	0	3
9	9	0	3	6

+	0	2	4	6	8	10
0	0	2	4	6	8	10
2	2	4	6	8	10	0
4	4	6	8	10	0	2
6	6	8	10	0	2	4
8	8	10	0	2	4	6
10	10	0	2	4	6	8

Find all the subgroups of $(Z_{11}^{*}, \cdot)$ group.

Solution :

To determine all subgroups of the group $(Z_{11}^{*}, \cdot)$

Since $\{e\}$ and G are the trivial subgroups of group G.

$\{1\}$ and Z_{11}^{*} are the trivial subgroup of $\{Z_{11}^{*}\}$

If G is a group and $\phi \neq H \subseteq G$, with H is finite, then H is a subgroup of G if and only if H is closed under the binary operation of G.

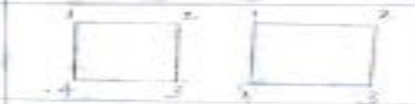
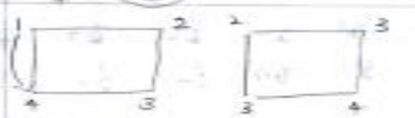
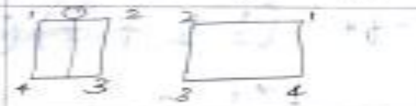
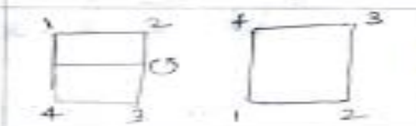
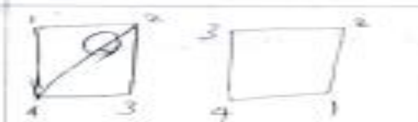
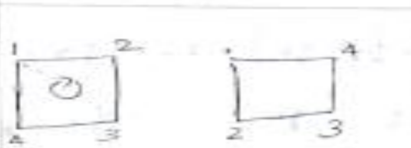
clearly $\{1, 10\}$, $\{1, 3, 4, 5, 9\}$ are proper subsets of group $(Z_{11}^{*}, \cdot)$.

$\cdot$	1	10
1	1	10
10	10	1

$\cdot$	1	3	4	5	9
1	1	3	4	5	9
3	3	9	1	4	5
4	4	1	5	9	3
5	5	4	9	3	1
9	9	5	3	1	4

from the table we observe that $\{1, 10\}$, $\{1, 3, 4, 5, 9\}$ are closed under the binary operation.

Hence, all sub group of $(Z_{11}^{*}, \cdot)$ are $\{1\}$, $\{1, 10\}$, $\{1, 3, 4, 5, 9\}$ Z_{11}^{*}.

SI No	Name	Rigid Motion (Before After)	Permutation of vertices
01	f_1 identity		$\begin{pmatrix}1&2&3&4\\1&2&3&4\end{pmatrix}$
02	f_2 rotate 90° clockwise		$\begin{pmatrix}1&2&3&4\\4&1&2&3\end{pmatrix}$
03	f_3 rotate 180° clockwise		$\begin{pmatrix}1&2&3&4\\3&4&1&2\end{pmatrix}$
04	f_4 rotate 90° counter clockwise		$\begin{pmatrix}1&2&3&4\\2&3&4&1\end{pmatrix}$
05	f_5 reflect		$\begin{pmatrix}1&2&3&4\\2&1&4&3\end{pmatrix}$
06	f_6 reflect		$\begin{pmatrix}1&2&3&4\\4&3&2&1\end{pmatrix}$
07	f_7 reflect		$\begin{pmatrix}1&2&3&4\\3&2&1&4\end{pmatrix}$
08	f_8 reflect		$\begin{pmatrix}1&2&3&4\\1&4&3&2\end{pmatrix}$

To composition table is :

	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8
f_1	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8
f_2	f_2	f_3	f_4	f_1	f_7	f_8	f_6	f_5
f_3	f_3	f_4	f_1	f_2	f_6	f_5	f_8	f_7
f_4	f_4	f_1	f_2	f_3	f_8	f_7	f_5	f_6
f_5	f_5	f_8	f_6	f_7	f_1	f_3	f_4	f_2
f_6	f_6	f_7	f_5	f_8	f_3	f_1	f_2	f_4
f_7	f_7	f_5	f_8	f_6	f_2	f_4	f_1	f_3
f_8	f_8	f_6	f_7	f_5	f_4	f_2	f_3	f_1

Example $f_3 \circ f_4 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$

$$= \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix} = f_2$$

Hence the composition o binary is a closure operation and is associative.