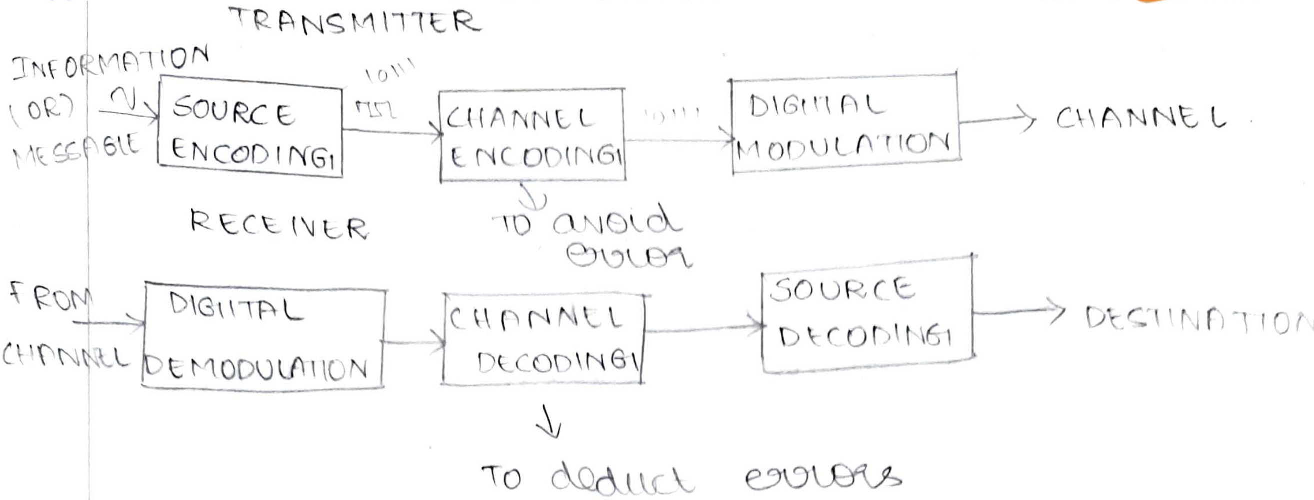
BLOCK DIAGRAM OF DIGITAL COMMUNICATION :



UNIT - I

SAMPLING AND QUANTIZATION

Low Pass Sampling :

(Sampling theorem :

A continuous time signal can be completely represented in its channel & recovered back if the sampling frequency is twice of the highest frequency content of the signal.

i.e., $\boxed{\begin{array}{c} f_s \geq 2W \\ \text{(or)} \\ f_s \geq 2f_m \end{array}}$

where, f_s is the sampling frequency
W is the highest frequency.

The sampling theorem is represented by two statements)

Statement 1 : (Process of sampling)

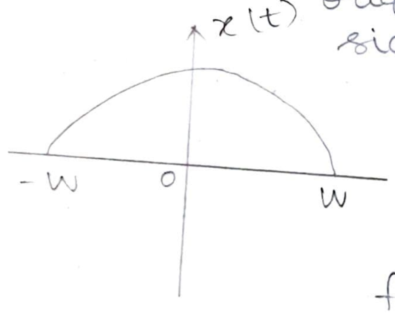
A band limited signal of finite energy which has no frequency components higher than W hertz can be completely described by specifying the values of signal at instance of time separated by 1/2 W seconds.

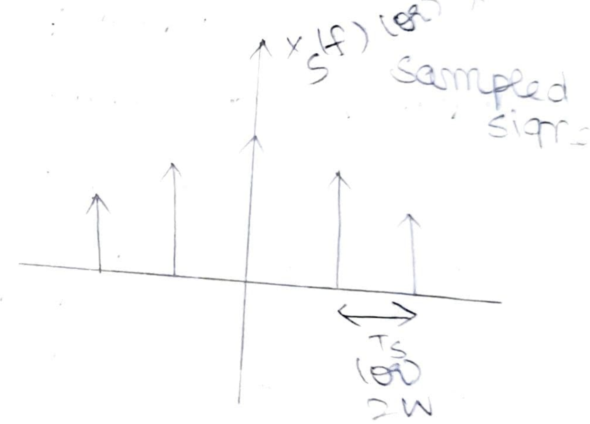
Statement 2 : (Reconstruction from sampling)

A band limited signal of finite energy which has no frequency components higher than W hertz may be completely recovered from knowledge of its samples taken at the rate of 2W samples / second.

2M (or) RAQ

Low pass sampling :





$f_s \geq 2W$

$f_s = 2W$ (Nyquist criteria)

$T_s = \frac{1}{f_s}$ where $T_s = 2W$

Proof for Sampling theorem :

Let x(t) be the original signal which is sampled at uniform rate T_s second and denoted by $x(nT_s)$ where,

n is an integer and

T_s is the sampling period and

$1/T_s = f_s$ where,

f_s is sampling frequency

In sampling theorem there are parts,

Part i) Representation of x(t) in terms of sample x(nTs).

Part ii) Reconstruction of x(t) from its sample.

Part i) Representation of x(t) in terms of sample x(nTs):

There are four steps available in part - i.

Step i) Define $x\delta(t)$.

Step ii) Fourier transform of $x\delta(t)$ i.e., $X\delta(f)$.

Step iii) Relationship between $X(f)$ and $X\delta(f)$

Step iv) Relationship between $x(t)$ and $x(nTs)$

Step i) Define $x\delta(t)$:-

$x\delta(t)$ can be defined as the product of $x(t)$ and $\delta(t)$ where, $\delta(t)$ is impulse train function (or) Dirac function. which will be maximum only at origin and it is given as,

$$\delta(t) = \sum_{n=-\infty}^{\infty} \delta(t - nTs) \quad \text{--- (1)}$$

where,

$(t - nTs)$ is a delta function at time $t = nTs$

and nT_s is instantaneous am...

So,

$$x_\delta(t) = x(t) \times \delta(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\, \delta(t-n...$$

Sub $x(t) = x(nT_s)$

Step. ii) Fourier transform of $x_\delta(t)$ i.e., ...

Taking Fourier transform on equat... we get,

$$X_\delta(f) = FT\left[\sum_{n=-\infty}^{\infty} x(nT_s)\, \delta(t-nT_s)\right] \quad \text{--- (5)}$$

$$X_\delta(f) = FT\,[\text{Product of } x(t)\ \&\ \delta(t)]$$

$$X_\delta(f) = FT\,[x(t)] * FT\,[\delta(t)] \quad \text{---}$$

$\because$ FT $\quad x(t) = X(f)$.

$$\delta(t) = f_s \sum_{n=-\infty}^{\infty} \delta(f-nf_s).$$

$$X_\delta(f) = X(f) * f_s \sum_{n=-\infty}^{\infty} \delta(f-nf_s) \quad \text{---}$$

Step. iii) By using shifting property,

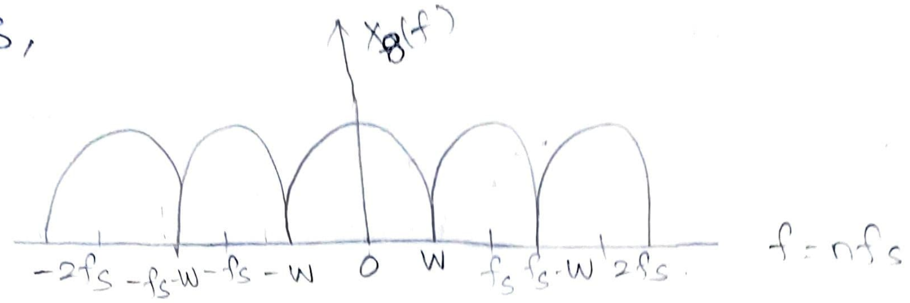
The above eqn can be written as,

$$\boxed{X_\delta(f) = f_s \sum_{n=-\infty}^{\infty} X(f-nf_s)} \quad \text{---}$$

$$X_\delta(f) = f_s X(f+2f_s) + f_s X(f+f_s) + f_s X(f) + f_s X(f-f_s) + f_s X(f-2f_s) \cdots \quad \text{--- (7)}$$

The RHS part of eqn (7) indicates that $X(f)$ is placed at regular intervals $\pm f_s, \pm 2f_s \ldots$

which means $X(f)$ is periodic function. It can be diagramatically represented, expressed as follows,



Step. iii) Relationship between $X(f)$ and $X\delta(f)$

$$\left.\begin{aligned} X\delta(f) &= f_s X(f) \\ X(f) &= X\delta(f) \cdot 1/f_s \end{aligned}\right\} \quad \text{—— (8)}$$

Step. iv) Relationship between $x(t)$ & $x(nT_s)$

To find out the relation between $x(t)$ & $x(nT_s)$ let us consider DTFT. i.e., Discrete Time Fourier Transform and it is defined as $X(\Omega)$, DTFT

$$\boxed{X(\Omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\Omega n}} \quad \text{—— (9)}$$

From eqn (9) we may obtain $X(f)$ as follows, $\Omega = 2\pi f$

$$\boxed{X(f) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j2\pi f n}} \quad \text{—— (10)}$$

|||ly

$$\boxed{X\delta(f) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j2\pi f/f_s n}} \quad \text{—— (11)}$$

Sub $x(n) = x(nT_s)$.

$1/f_s = T_s$ in eqn (11).

$$X\delta(f) = \sum_{n=-\infty}^{\infty} x(nT_s)\, e^{-j2\pi f n T_s} \quad \text{—— (12)}$$

$$X(f) = 1/f_s \sum_{n=-\infty}^{\infty} x(nT_s)\, e^{-j2\pi f n T_s}$$

(from eqn (8)).

$$x(t) = IFT\left\{ 1/f_s \sum_{n=-\infty}^{\infty} x(nT_s)\, e^{-j2\pi f n T_s} \right\}$$

Part ii) Reconstruction of x(t) from its sample :

This part consists of 3 steps :

i) step : Applying IFT on X(f)

ii) step : use of interpolation fn expansion

iii) step : Passing the signal via low pass filter.

Step. i) : Inverse Fourier Transformation (IFT) on X(f).

considering eqn (14) the IFT can be written as,

$$x(t) = \int_{-\infty}^{\infty} \left\{ 1/f_s \sum_{n=-\infty}^{\infty} x(nT_s)\, e^{-j2\pi f n T_s} \right\} e^{j2\pi f t}\, df$$

Since we are using the term W as frequency the integration limits can be changed to $-W \le f \le W$ so that,

$$x(t) = \int_{-W}^{W} 1/f_s \sum_{n=-\infty}^{\infty} x(nT_s)\, e^{-j2\pi f n T_s}\, e^{j2\pi f t}\, df$$

Interchange the order of summation and integration we get,

$$x(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\ \frac{1}{f_s} \int_{-W}^{W} e^{j2\pi f(t-nT_s)}\,df \quad \text{(17)}$$

$$x(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\ \frac{1}{f_s} \left[\frac{e^{j2\pi f(t-nT_s)}}{j2\pi(t-nT_s)}\right]_{-W}^{W}$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s)\ \frac{1}{f_s} \left[\frac{e^{j2\pi W(t-nT_s)} - e^{-j2\pi W(t-nT_s)}}{j2\pi(t-nT_s)}\right]$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s)\ \frac{1}{f_s} \left[\frac{\cancel{2}j\sin 2\pi W(t-nT_s)}{j2\pi(t-nT_s)}\right]$$

as $\sin\theta = \dfrac{e^{j\theta} - e^{-j\theta}}{2j}$

$$= \sum_{n=-\infty}^{\infty} x(nT_s)\ \frac{1}{f_s} \left[\frac{\sin 2\pi W(t-nT_s)}{\pi(t-nT_s)}\right]$$

sub $T_s = \frac{1}{f_s} = \frac{1}{2W}$

$$x(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\ \frac{\sin \pi(2Wt-n)}{\pi(2Wt-n)} \quad \text{(18)}$$

Eqn (18) can be further reduced as,

$$x(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\ \operatorname{sinc}(2Wt-n) \quad \text{(19)}$$

$$\left[\because \frac{\sin\pi\theta}{\theta} = \operatorname{sinc}\theta\right]$$

Step ii) use of Interpolation fn

Interpolation is a process of expanding the given function (or) equation.

Expanding eqn (19) by substituting values we get,

$$x(t) = x(0)\,\text{sinc}(2wt) + x(\pm T_s)\,\text{sinc}(\ldots) + x(\pm 2T_s)\,\text{sinc}(2wt \pm 2) + \ldots \quad (20)$$

spectrum like structure

where,

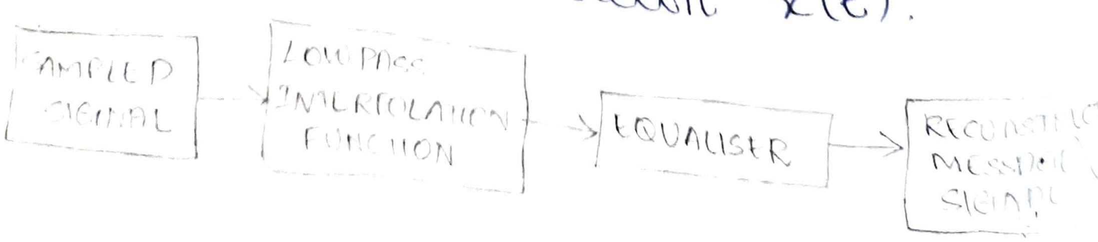
the sinc is the interpolating function and the samples $x(nT_s)$ are weighed by sinc fn.

iii) Step : Reconstruction filter by using Low pass filter :

When an interpolated signal (eqn (20)) passed through a LPF of band width $-w \le f \le w$.

Then the original signal can be reconstructed.

The sinc function are passed through equalizer to obtain $x(t)$.



ALIASING :

When a high frequency interferes with low frequency and appears as low frequency is called as Aliasing.

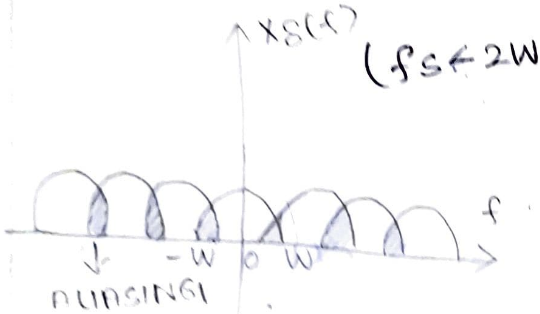
EFFECTS OF ALIASING :

i) Because of the interference distortion is generated.

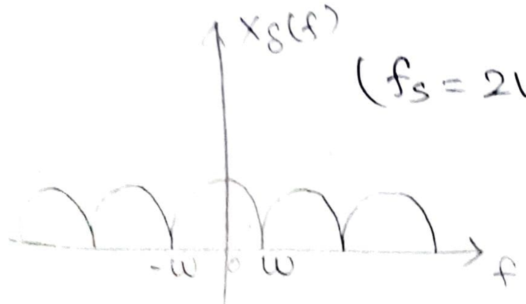
ii) The data is lost & it cannot be recovered.

iii) Aliasing mainly occurs when sampling theorem condition is not satisfied properly.

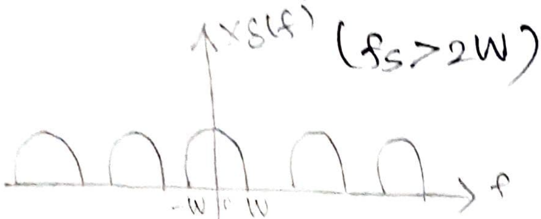
i) Under sampling :



ii) Critical sampling :



iii) Over sampling :



$\boxed{f_s = 2W \text{ (Nyquist rate)}}$

$T_s = 1/f_s = \frac{1}{2W}$ (Nyquist interval)

METHODS TO ELIMINATE ALIASING (OR) IMPLEMENTATION OF ANTI-ALIASING FILTER:

Aliasing can be avoided in two ways, i) choosing $f_s > 2W$

which is oversampling.

This method is also called as pre-filtering anti-aliasing filter

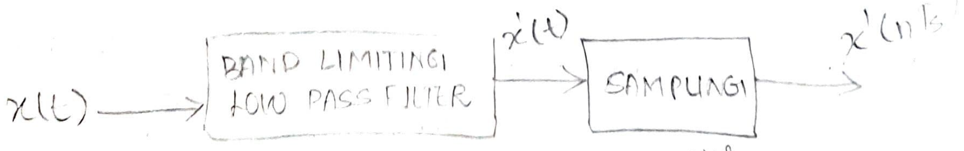
The only drawback of this method is it increases transmission bandwidth

ii) Band limiting the signal.

Generally the sampling rate will be $f_s = 2W$ (i.e, Critical sampling)

But some signals will have higher frequency components which may cause aliasing.

To avoid that before sampling the signal must be passed to a band limited low pass filter. So that, higher frequency components may be eliminated and the aliasing is avoided.



QUANTIZATION :-

It is defined as the process of transforming the sampled amplitude of message signal into discrete amplitude taken from a finite set of possible amplitudes.

The sampled value obtained by the sampling process is quantized to the nearest standard level.

[illegible] → | Quantizer | → [illegible]

TYPES OF QUANTIZATION :

Quantization
- Uniform (step size constant)
  - Midtread
  - Midrise
  - Biased
- Non-uniform (step size [illegible])
  - A law
  - μ law

In quantization the total amplitude range is divided into a number of standard levels. [illegible]

[illegible]