① Demonstrate that R is a valid inference from the premises $P \rightarrow Q$, $Q \rightarrow R$ and P.

Here given premises are

(1) $P \rightarrow Q$

(2) $Q \rightarrow R$

(3) P

{1}	1) $P \rightarrow Q$	Rule P
{2}	2) P	Rule P
{1,2}	3) Q	Rule T $(P, P \rightarrow Q \Rightarrow Q)$
{4}	4) $Q \rightarrow R$	Rule P
{1,2,4}	5) R	Rule T $(P, P \rightarrow Q \Rightarrow Q$

show that $(P \to Q) \wedge (R \to S)$, $(Q \to M) \wedge (S \to N)$,

2.

$\neg(M \wedge N)$ and $(P \to R) \implies \neg P$.

Given premises are $(P \to Q) \wedge (R \to S)$,

$(Q \to M) \wedge (S \to N)$, $\neg(M \wedge N)$ and $(P \to R)$

Conclusion is $\neg P$.

{1}	1) $(P \to Q) \wedge (R \to S)$	Rule P
{1}	2) $P \to Q$	Rule T $(P \wedge Q \implies P)$
{1}	3) $R \to S$	Rule T $(P \wedge Q \implies Q)$
{4}	4) $(Q \to M) \wedge (S \to N)$	Rule P
{4}	5) $Q \to M$	Rule T $(P \wedge Q \implies P)$
{4}	6) $S \to N$	Rule T $(P \wedge Q \implies Q)$
{1,4}	7) $P \to M$	Rule T $(P \to Q, Q \to R \implies P$
{1,4}	8) $R \to N$	Rule T $(P \to Q, Q \to R \implies P$
{9}	9) $P \to R$	Rule P

$\{1,4,9\}$	10) $P \rightarrow N$	Rule T $(P \rightarrow a, a \rightarrow R \Rightarrow P$
$\{1,4,9\}$	11) $\neg N \rightarrow \neg P$	Rule T (Taking $\neg$)
$\{1,4\}$	12) $\neg M \rightarrow \neg P$	Rule T (Taking $\neg$)
$\{1,4,9\}$	13) $(\neg M \vee \neg N) \rightarrow \neg P$	Rule T $(a \rightarrow b, c \rightarrow b \Rightarrow$ $(a \vee c) \rightarrow b)$
$\{1,4,9\}$	14) $\neg (M \wedge N) \rightarrow \neg P$	Rule T (Demorgan's law
$\{15\}$	15) $\neg (M \wedge N)$	Rule P
$\{1,4,9,15\}$	16) $\neg P$	Rule T $(P, P \rightarrow Q \Rightarrow Q)$

3.

Prove that the following argument is valid :

$$p \rightarrow \neg q, \quad r \rightarrow q, \quad r \Rightarrow \neg P$$

Given premises are $p \rightarrow \neg q, \quad r \rightarrow q, \quad r$

Conclusion is $\neg P$.

$\{1\}$	1) r	Rule P
$\{2\}$	2) $r \rightarrow q$	Rule P
$\{1,2\}$	3) q	Rule T $(P, P \rightarrow Q \Rightarrow Q)$
$\{4\}$	4) $p \rightarrow \neg q$	Rule P
$\{1,2,4\}$	5) $\neg P$	Rule T $(P \rightarrow \neg Q, Q \Rightarrow \neg P)$