TOPIC:2.-Problems on Strong Induction

Strong Induction

In this form, we use the same basic step as before, but we use a different inductive step. We assume that $P(j)$ is true for $j = 1, 2, \ldots k$ and show that $P(k+1)$ must also be true based on this assumption. This is called Strong induction (second principle of Mathematical induction).

Basic step : The proposition $P(1)$ is shown to be true.

Inductive step: It is shown that

$$[P(1) \wedge P(2) \wedge \cdots \wedge P(k)] \rightarrow P(k+1).$$

is true for every positive integer k.

1) Show that if n is an integer greater than 1, then 'n' can be written as the product of primes.

Let P(n) be the proposition that n can be written as the product of primes.

Basic step : P(2) is true, since 2 can be written as the product of one prime.

Inductive step : Assume that P(j) is true for all positive integer j with $j \le K$. To complete the inductive step, it must be shown that P(K+1) is true under this assumption.

There are 2 cases to consider, namely

Case : 1 If (K+1) is prime, we immediately see that P(K+1) is true.

Case : 2 If (K+1) is composite

Then it can be written as product of two positive integer a and b with $2 \le a < b \le K+1$. By the induction hypothesis, both a and b can be written as the product of primes. Thus, if (K+1) is composite it can be written as the product of primes.