TOPIC:9- SOLUTION OF LINEAR RECURRENCE RELATIONS

Linear recurrence relation

A recurrence relation of the form $c_0 a_n + c_1 a_{n-1} + c_2 a_{n-2} + \dots c_k a_{n-k} = f(n)$ is called a linear recurrence relation of degree k with constant coefficients, where $c_0, c_1, \dots c_k$ are real numbers & $c_k \neq 0$. The recurrence relation is called linear, because each a_r is raised to the power 1 and there are no products such as $a_r \cdot a_s$.

Note:

If $f(n) = 0$, the recurrence relation is said to be homogeneous; otherwise it is said to be non-homogeneous.

Methods of solving recurrence relations are

1. Iteration
2. Characteristic roots &
3. Generating fun.

Solution of recurrence relation:

Consider the recurrence relation $c_0 y_{n+2} + c_1 y_{n+1} + c_2 y_n = f(n)$. The solution of the above recurrence relation is $y_n = H_{(h)}S + P_{(p)}S$ (ie) ~~$y_n = a_n^{(h)} + a_n^{(p)}$~~ $y_n = y_n^{(h)} + y_n^{(p)}$

where H.S = Homogeneous Solution

P.S = Particular Solution.

Rules to find H.S:

1. First write the characteristic equation
$$c_0 r^2 + c_1 r + c_2 = 0.$$
2. Solve the characteristic equation & get the roots.

Problem.

1. Solve the recurrence relation $a_{n+1} - a_n = 3n^2 - n$; $n \geq 0$. $a_0 = 3$.

Soln:

The characteristic eqn. is $r - 1 = 0$ (ie) $r = 1$

The H.S is $a_n^{(H)} = C \cdot 1^n = C$

Since the R.S. of the R.R is $3n^2 - n = (3n^2 - n) \cdot 1^n$, let ~~Assume~~ the particular soln of the R.R be assumed as $a_n^{(P)} = (A_0 n^2 + A_1 n + A_2) n$, since 1 is a characteristic root of the R.R.

(ie) $a_n = (A_0 n^3 + A_1 n^2 + A_2 n)$

Using this in the recurrence relation we have

$$\{A_0 (n+1)^3 + A_1 (n+1)^2 + A_2 (n+1)\} - \{A_0 n^3 + A_1 n^2 + A_2 n\} = 3n^2 - n$$

$$\text{(ie)} \{A_0 [\cancel{n^3} + 3n^2 + 3n + 1 - \cancel{n^3}] + A_1 [\cancel{n^2} + 2n + 1 - \cancel{n^2}] + A_2 [\cancel{n} + 1 - \cancel{n}]\} = 3n^2 - n$$

$$A_0 (3n^2 + 3n + 1) + A_1 (2n + 1) + A_2 = 3n^2 - n$$

Comparing like terms, we have

$A_0 = 1$, $3A_0 + 2A_1 = -1$ & $A_0 + A_1 + A_2 = 0$.

Solving these equations, we get

$A_0 = 1$, $A_1 = -2$ & $A_2 = 1$

$$\therefore a_n^{(P)} = n^3 - 2n^2 + n$$
$$= n(n-1)^2$$

$\therefore$ The general soln. of the recurrence relation is $a_n = a_n^{(h)} + a_n^{(P)} = C + n(n-1)^2$. Given that $a_0 = 3$

$a_n = C + n(n-1)^2 \Rightarrow C = 3$

$\therefore$ The required soln. of the R.R is

$$a_n = 3 + n(n-1)^2.$$

2. Solve the recurrence relation
$a_{n+2} - 6a_{n+1} + 9a_n = 3(2^n) + 7(3^n), \quad n \geq 0$
Given that $a_0 = 1$ and $a_1 = 4$.

Soln: The characteristic equation is
$r^2 - 6r + 9 = 0 \Rightarrow (r-3)^2 = 0$
$r = 3, 3$
$\therefore$ The homogeneous soln. is $a_n^{(h)} = (C_1 + C_2 n) 3^n$.

Nothing that 3 is a double root of the characteristic equation, we assume the particular soln. of the R.R. as $a_n^{(p)} = A_0 2^n + A_1 n^2 \cdot 3^n$

Usin this in given eqn.
$A_0 \cdot 2^{n+2} + A_1 (n+2)^2 \cdot 3^{n+2} - 6\{A_0 \cdot 2^{n+1} + A_1 (n+1)^2 \cdot 3^{n+1}\}$
$+ 9\{A_0 \cdot 2^n + A_1 n^2 \cdot 3^n\} = 3(2^n) + 7(3^n)$

$A_0 2^n [4 - 12 + 9] + A_1 3^n \{9(n+2)^2 - 18(n+1)^2 + 9n^2\}$
$= 3(2^n) + 7(3^n)$

(ie) $A_0 \cdot 2^n + A_1 3^n [9\cancel{n^2} + \cancel{36n} + 36 - \cancel{18n^2} - \cancel{36n} - 18 + \cancel{9n^2}]$
$= 3(2^n) + 7(3^n)$

(ie) $A_0 2^n + A_1 3^n \times 18 = 3 \cdot (2^n) + 7 \cdot (3^n)$

Comparing like terms, we get $A_0 = 3$ & $A_1 = 7/18$
$\therefore a_n^{(p)} = 2^n + \frac{7}{18} n^2 \cdot 3^n$

Hence the general soln. of the R.R is
$a_n = a_n^{(h)} + a_n^{(p)}$

(ie) $a_n = (C_1 + C_2 \cdot n) 3^n + 3 \cdot 2^n + \frac{7}{18} n^2 \cdot 3^n$

Given that $a_0 = 1 \quad \therefore C_1 + 3 = \cancel{0}1 \Rightarrow C_1 = \cancel{0}\ -2$
$a_1 = 4 \quad 3C_1 + 3C_2 + \cancel{6} + 7/6 = 4 \Rightarrow C_2 = \cancel{17}/18$

$\therefore$ The required soln. is
$a_n = \frac{5}{18} n \cdot 3^n + 2^n + \frac{7}{18} n^2 \cdot 3^n$ //.

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