TOPIC : 1 – Rings

Ring

Def Ring

Let R be a non-empty set on which we have two closed binary operations denoted by + and .

Then $(R, +, \cdot)$ is a ring if for all $a, b, c \in R$, the following conditions are satisfied

(a) $a+b = b+a$ commutative law of +

(b) $a+(b+c) = (a+b)+c$ Associative law of +

(c) There exists $z \in R$ such that $a+z = z+a = a$ for every $a \in R$ Existence of an identity for +

(d) For each $a \in R$ there is an element $b \in R$ with $a+b = b+a = z$. Existence of inverse under +

(e) $a.(b.c) = (a.b).c$ Associative law of .

(f) $a \cdot (b+c) = a \cdot b + a \cdot c$
$(b+c) \cdot a = b \cdot a + c \cdot a$ Distributive Laws of . over +

Def: Ideal

A non-empty subset I of a ring R is called a subring of R if (S, +, .) - that is, S under the addition and multiplication of R, restricted to S - is a ring.