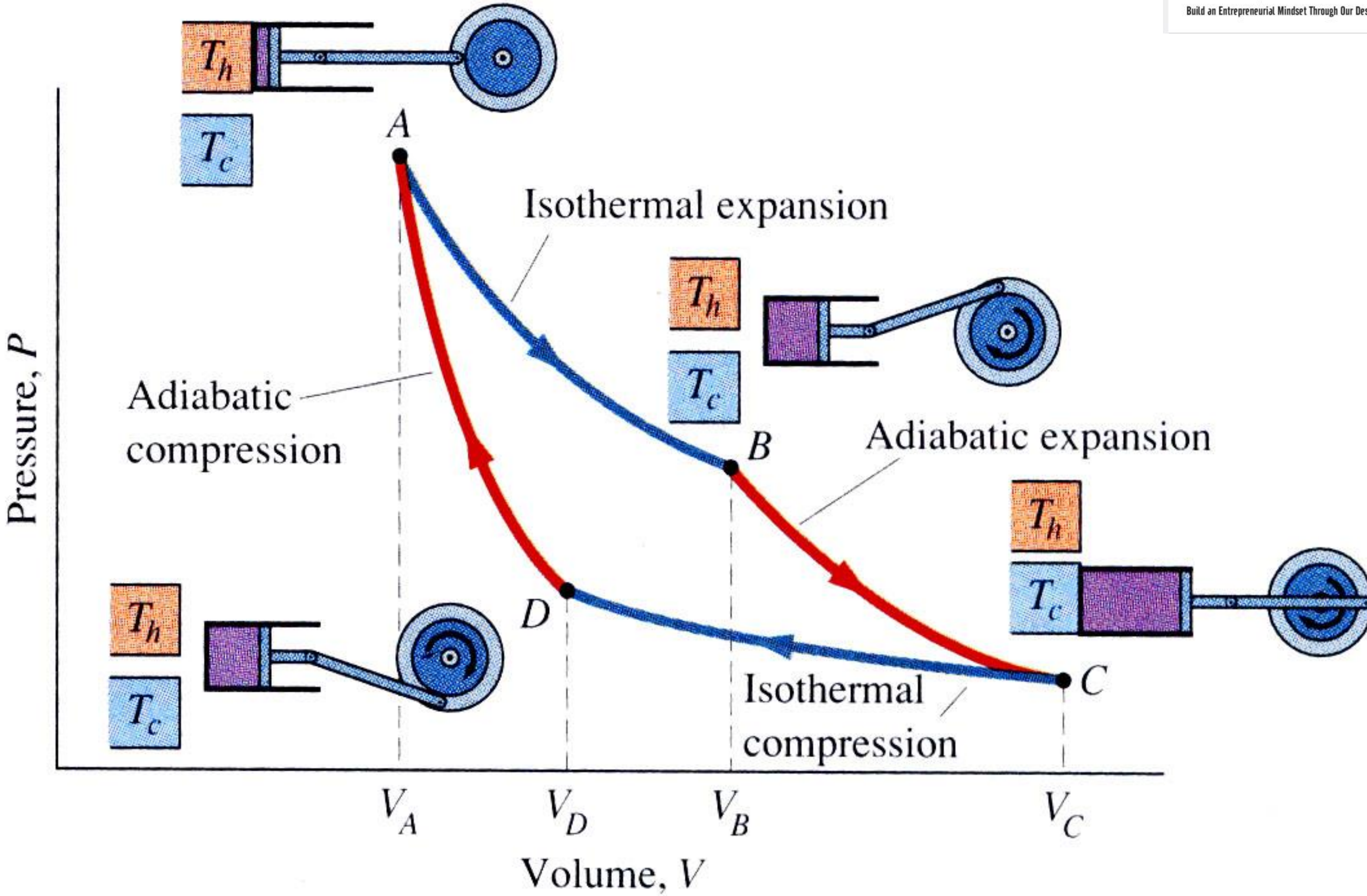
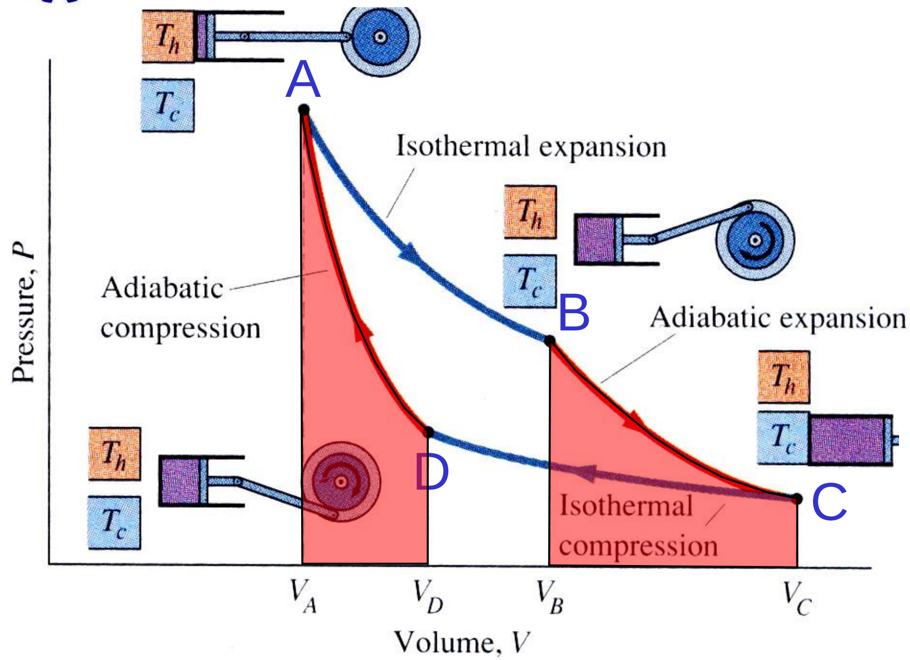
Carnot cycle



Carnot cycle



A - B

$$\Delta U_{AB} = 0 \quad W_{AB} = Q_h > 0$$

B - C

$$Q = 0 \quad W_{BC} = -\Delta U_{BC} > 0$$

C - D

$$\Delta U_{CD} = 0 \quad W_{CD} = -Q_c < 0$$

D - A

$$Q = 0 \quad W_{DA} = -\Delta U_{DA} < 0$$

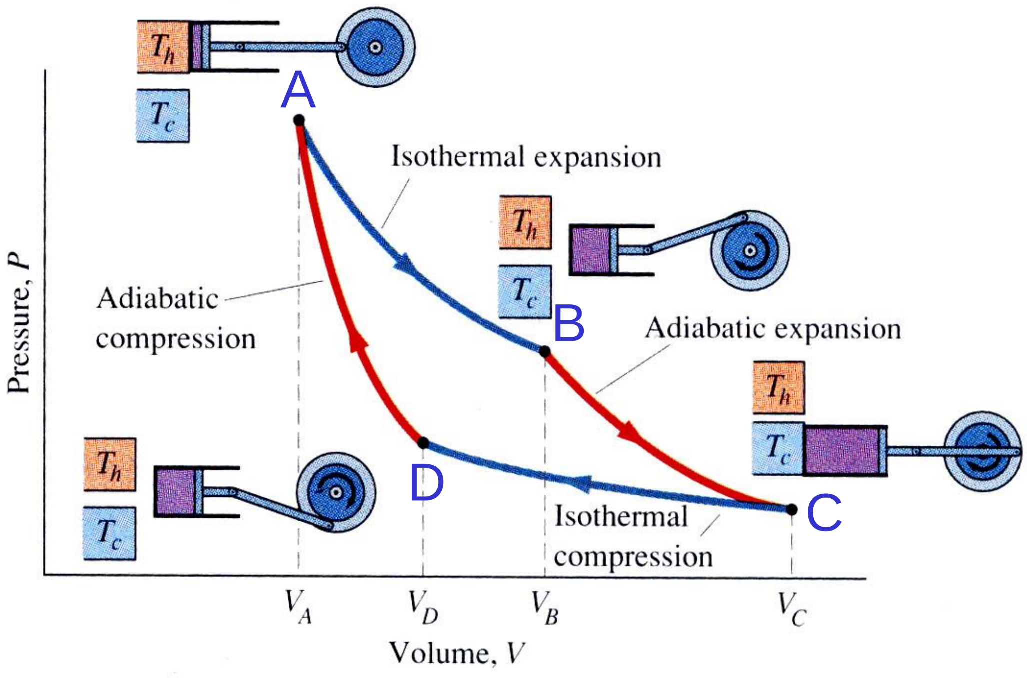
What is the total work by the gas?

$$\Delta U_{AB} + \Delta U_{BC} + \Delta U_{CD} + \Delta U_{DA} = 0 \;\Rightarrow\; \Delta U_{BC} + \Delta U_{DA} = 0$$

$$W_{BC} + W_{DA} = -(\Delta U_{BC} + \Delta U_{DA}) = 0$$

$$W_{total} = W_{AB} + W_{CD} = Q_h - Q_c$$

Carnot cycle



What is the total work done by the gas?

$$W_{total} = W_{AB} + W_{CD}$$

$$W_{AB} = nRT_h \ln\left(\frac{V_B}{V_A}\right)$$

$$W_{CD} = -nRT_c \ln\left(\frac{V_C}{V_D}\right)$$

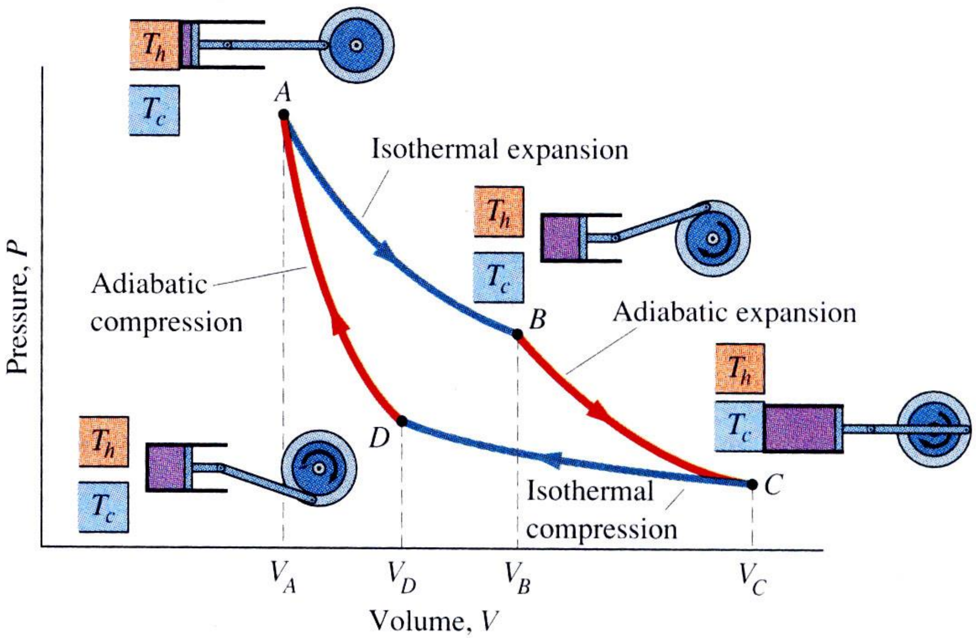
After some calculations we get

$$\frac{V_B}{V_A} = \frac{V_C}{V_D} \qquad \left|\frac{W_{CD}}{W_{AB}}\right| = \frac{Q_c}{Q_h} = \frac{T_c}{T_h}$$

Efficiency

$$e = \frac{W}{Q_h} = \frac{Q_h - Q_c}{Q_h} = \frac{T_h - T_c}{T_h}$$

Carnot cycle

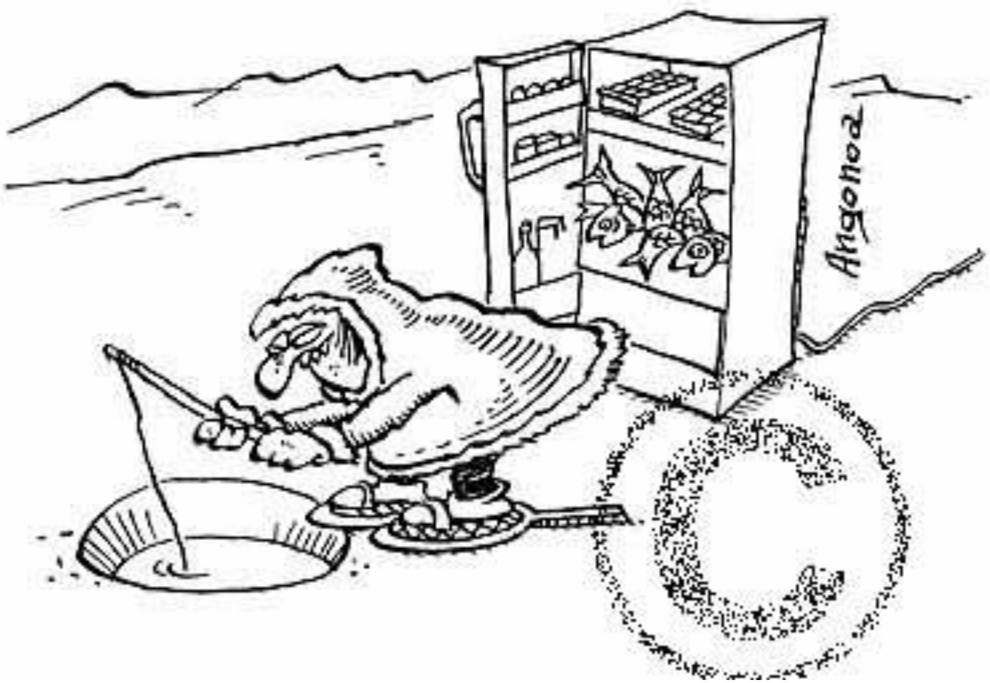


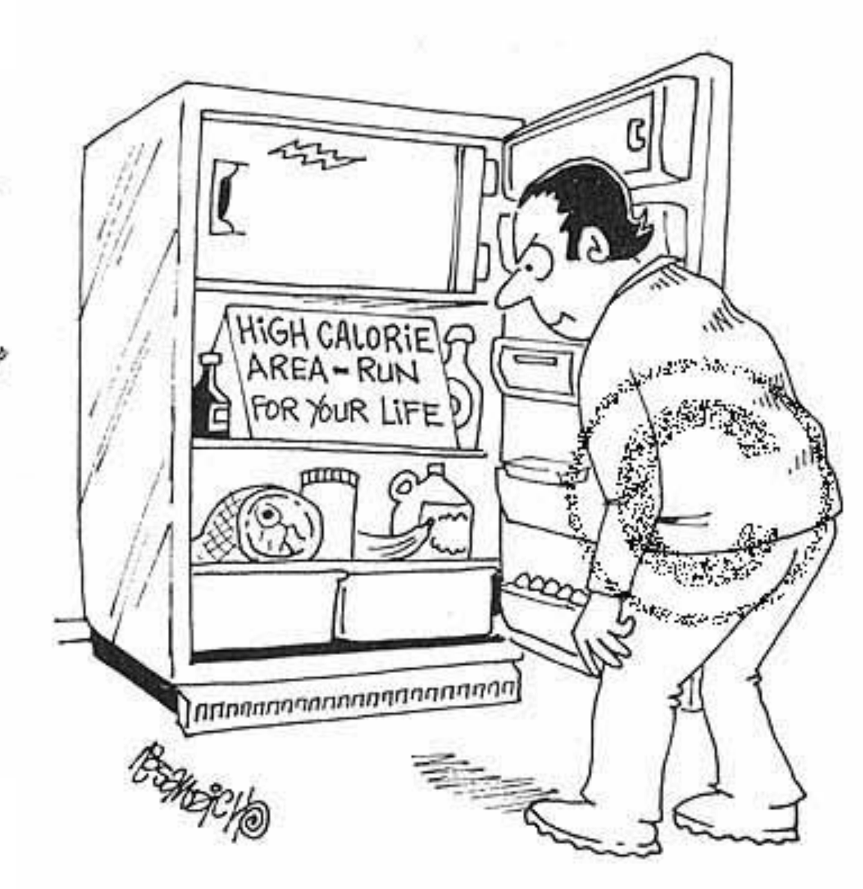
Efficiency

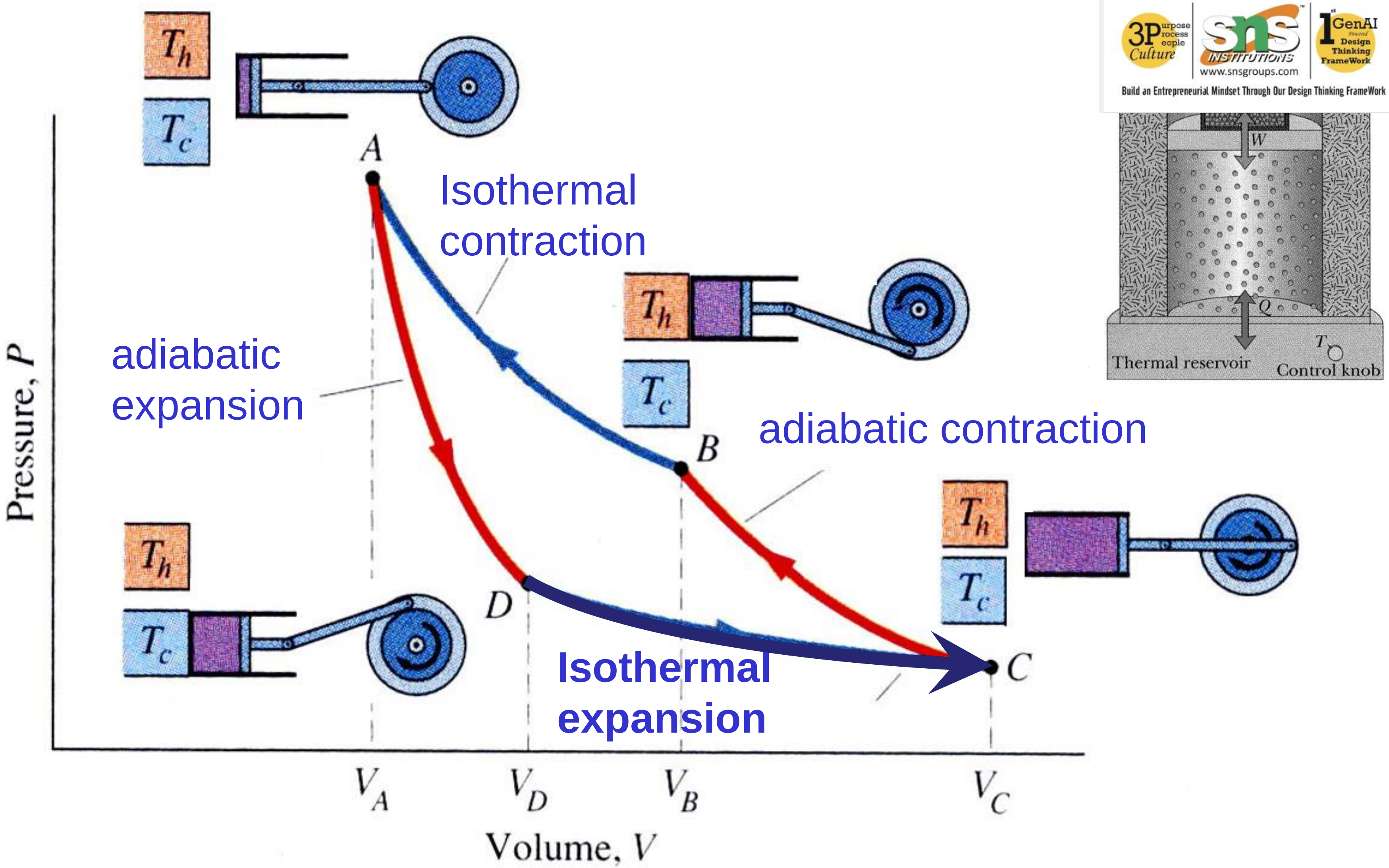
$$e = \frac{W}{Q_h} = \frac{Q_h - Q_c}{Q_h} = \frac{T_h - T_c}{T_h}$$

- For highest efficiency we would want to run our engine between a very hot and a very cold reservoirs.
- Large temperature difference, $T_h - T_c$, and low temperature of the cold reservoir, T_c, are very helpful.
- Efficiency can in principle reach 100% for $T_c = 0$, but we normally do not have such reservoirs available…

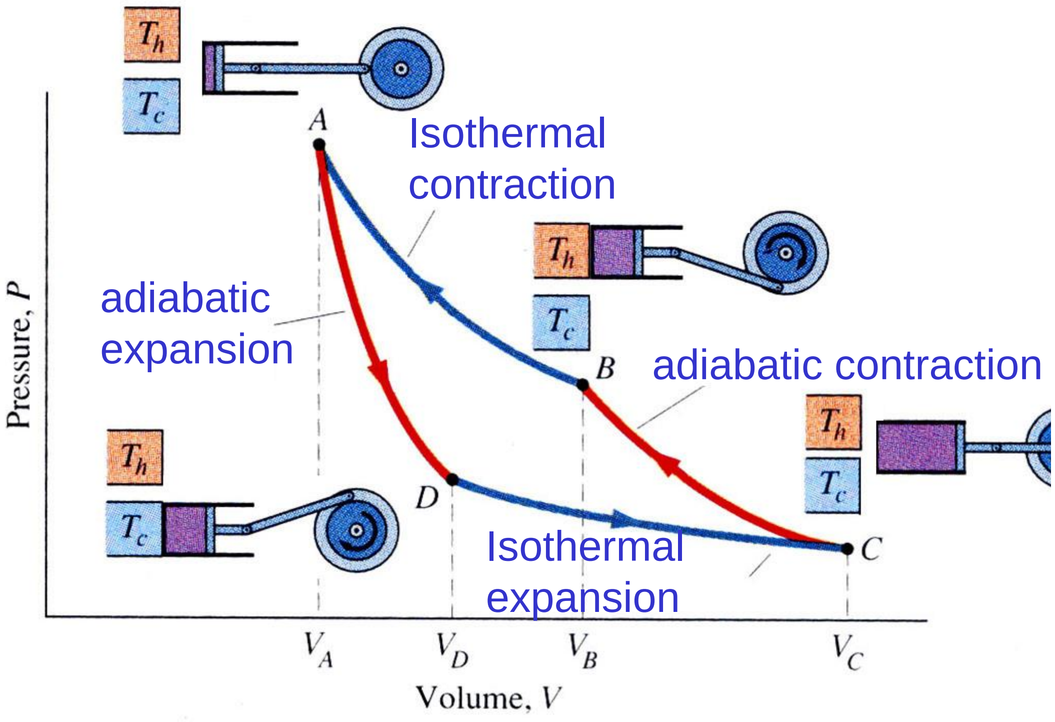
Refrigerators and Air conditioners







The key part is the isothermal process DC – the only time the system is in contact with the cold reservoir. The gas is made to expand taking heat Q_c from the cold reservoir and cooling it down even more.



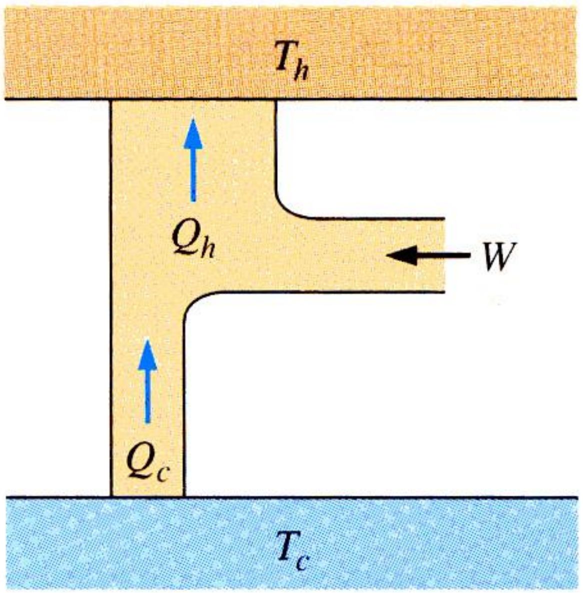


FIGURE 22-7 Energy flow diagram for a real refrigerator. The device takes in mechanical work and transfers heat from the cool to the hot reservoir.

Inverting the cycle - building a refrigerator

Mind the conventions: Q_h is positive when obtained from the Hot Reservoir; Q_c is positive, when rejected to the Cold Reservoir; W is positive, when done by the gas… Now we have: $Q_h < 0$, $Q_c < 0$, $W < 0$.

$$W = Q_h - Q_c \qquad |Q_c| = |Q_h| - |W| \qquad \frac{Q_c}{Q_h} = \frac{T_c}{T_h}$$

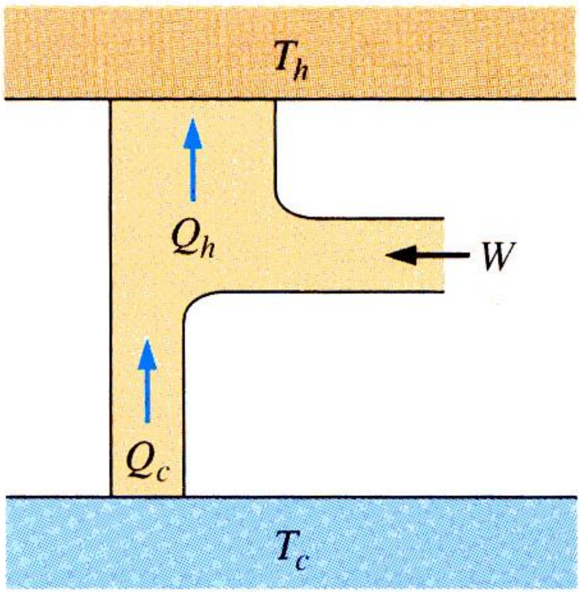


FIGURE 22-7 Energy flow diagram for a real refrigerator. The device takes in mechanical work and transfers heat from the cool to the hot reservoir.

Coefficient of Performance, COP

$$COP = \frac{Q_c}{W} = \frac{Q_c}{Q_h - Q_c}$$

For a Carnot cycle

$$COP = \frac{T_c}{T_h - T_c}$$

COP goes to 0, when T_h goes to infinity;

COP is equal to 1, when $T_h = 2 \cdot T_c$;

COP becomes infinitely large if T_h is close to T_c !

A hell of a theorem….

Suggestion #1. Hell is a place, where people are supposed to suffer.

Suggestion #2. Some physicist and engineers are sinners and go to hell… and they also bring their tools with them.

Suggestion #3. The basic laws of thermodynamics work in hell as well.

Theorem: Hell is isothermal.

Prove:
If hell were not isothermal, one would be able to build a heat engine running between two different temperatures.

The work provided by the engine could then be used to run a refrigerator (air conditioner) to create a comfortable environment.

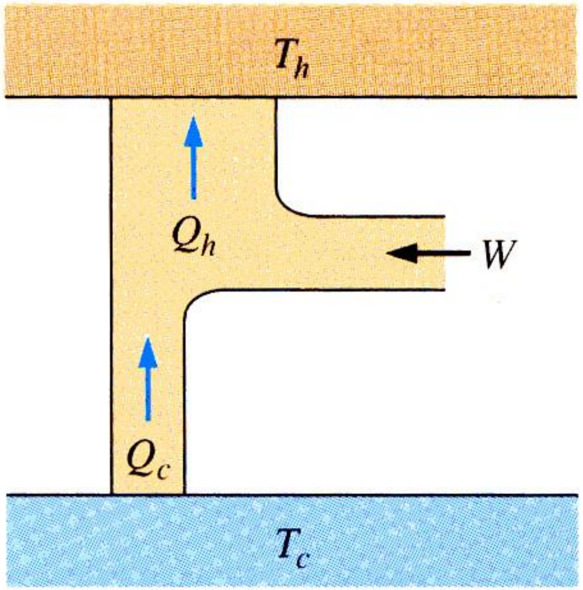


FIGURE 22-7 Energy flow diagram for a real refrigerator. The device takes in mechanical work and transfers heat from the cool to the hot reservoir.

For a Carnot cycle

$$COP = \frac{T_c}{T_h - T_c}$$

A home freezer, working between -18 °C inside and 30 °C outside.

What is the maximal possible COP?

How much electrical energy will be required to cool 0.53 kg of water by 1°C?

How much heat will be rejected to the air in the kitchen?

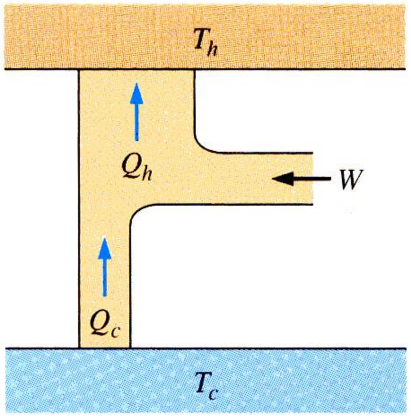
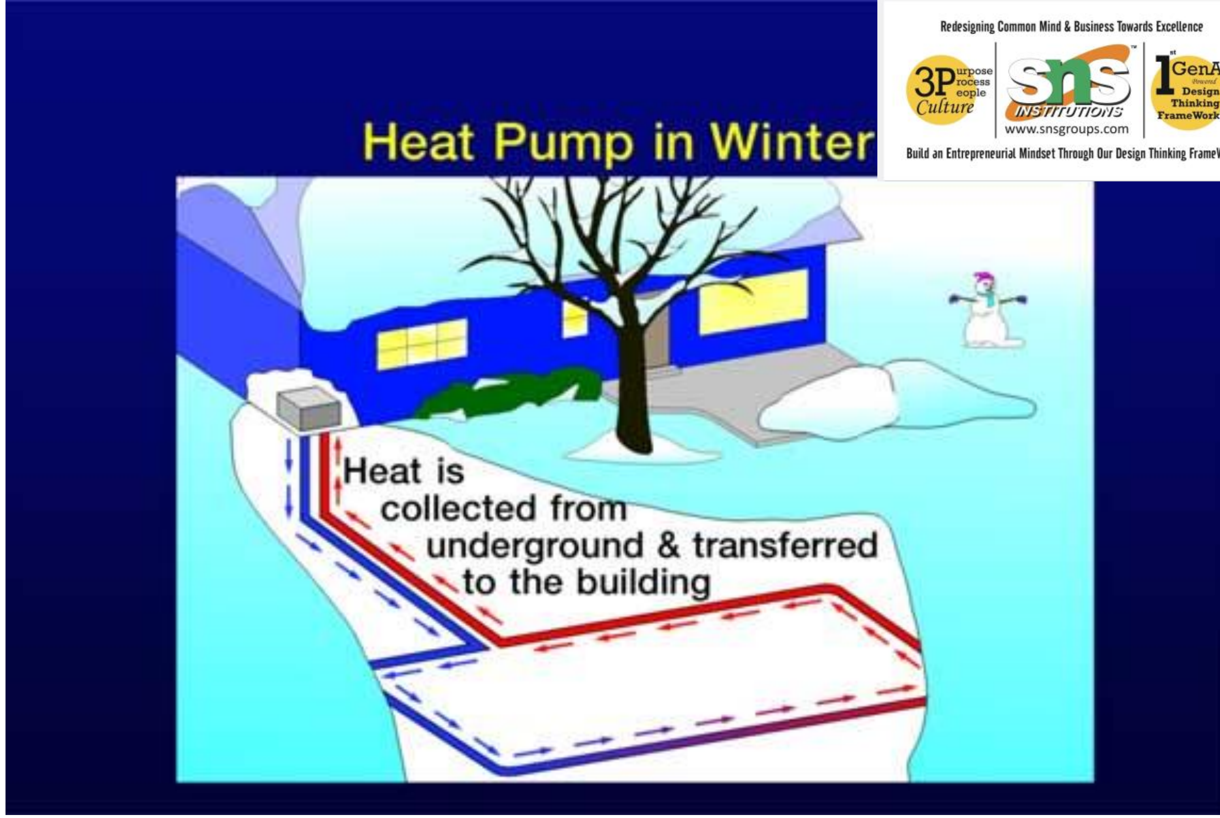


FIGURE 22-7 Energy flow diagram for a real refrigerator. The device takes in mechanical work and transfers heat from the cool to the hot reservoir.



For a Carnot cycle

$$COP = \frac{T_c}{T_h - T_c}$$

What is the ratio between the work done by the motor in the fridge and heat transferred to the house?

$$\frac{Q_h}{W} = \frac{Q_c + W}{W} = COP + 1$$

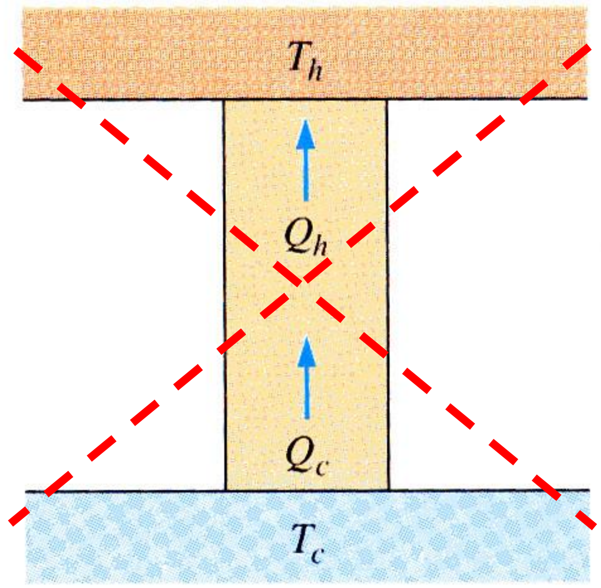
For 25 °C inside the house and 10 °C found 2 m under the ground COP = 19

$$Q_h = (COP + 1) \cdot W = 20 \cdot W$$

An alternative formulation of the second law

It is impossible to construct a **refrigerator** operating **in a cycle** whose **sole effect** is transferring heat from a cooler object to a hotter one.

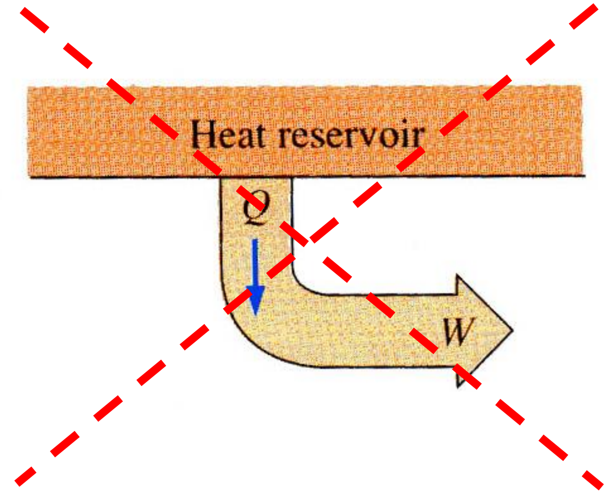
Perfect refrigerator (impossible thing.)



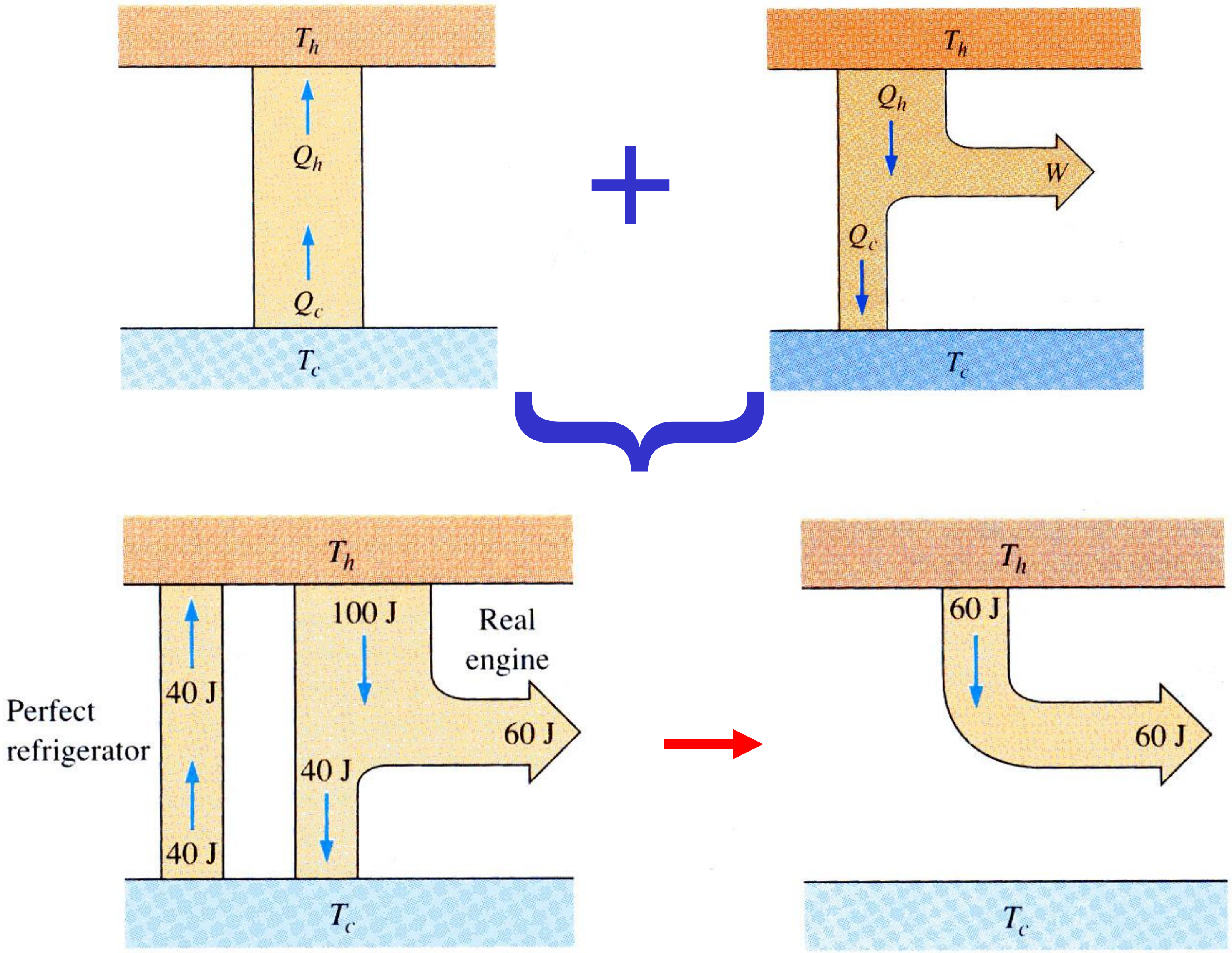
How does it relate to

It is **impossible** to construct a heat engine operating in a **cycle** that extracts **heat from a reservoir** and delivers and **equal amount of work**

?

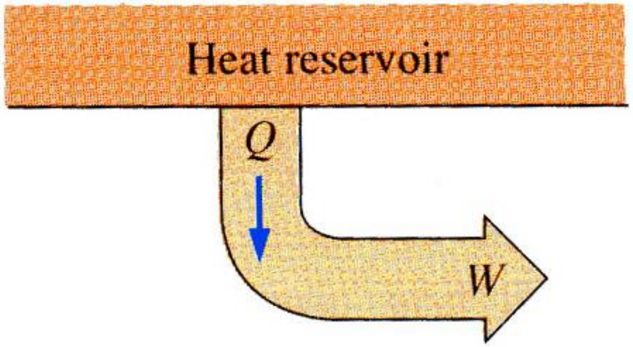


If a perfect refrigerator were possible we could build a perf engine, which is prohibited by the 2nd law



(b)

Let's see:



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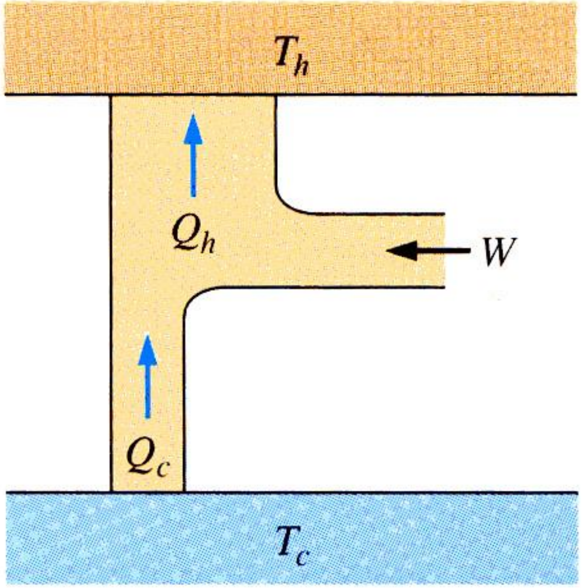

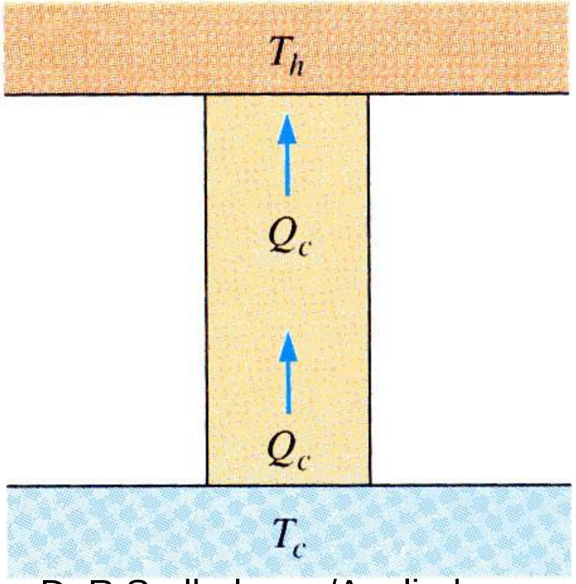

A **perfect** heat engine would allow a **perfect** refrigerator.