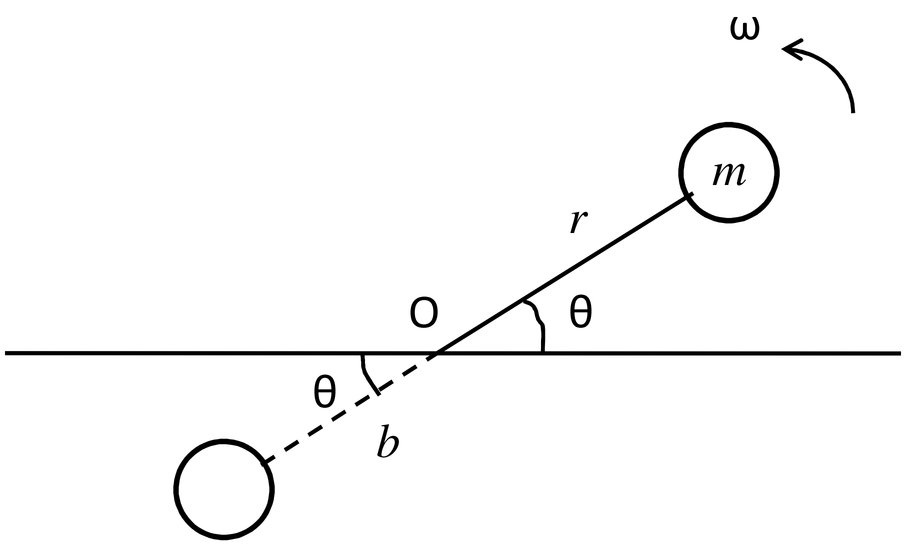
Balancing of rotating masses

- Balancing
  - Rotating masses
    - Single plane
    - Different planes
  - Reciprocating masses
    - Primary force and couple
    - Secondary force and couple

Balancing of single rotating weight by a weight rotating in the same plane

A weight W carried on a weight less arm of length r rotates with angular velocity ω as shown. For completely balancing the mass m, a weight B at distance b is attached to the same axle in a diametrically opposite direction.



$$m\omega^2 r = B\omega^2 b$$

$$\therefore mr = Bb$$

lancing a number of masses rotating in one plane by another weight rotating in the same plane

- Consider the masses m_1, m_2,m_3 revolving the at radii r_1,r_2,r_3 respectively in the same plane. Then each mass produces a centrifugal force acting radially outwards from the axis of rotation.
- If F is the vector sum,

$F= m_1r_1\omega^2+ m_2r_2\omega^2+ m_3r_3\omega^2$

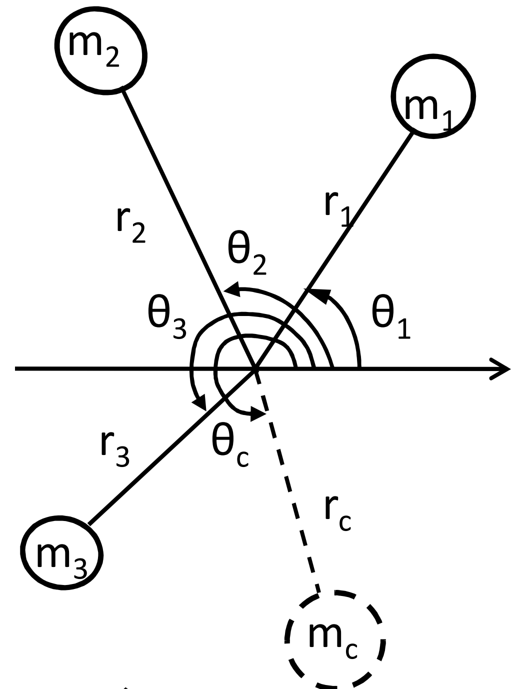
All the masses will be in balance if F=0.

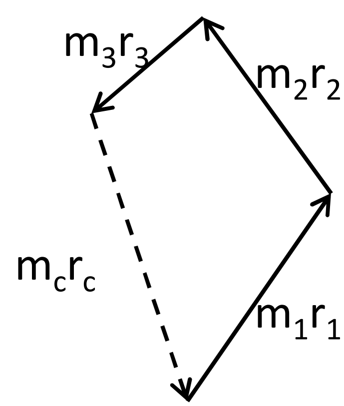
If F is not zero, a counter weight of mass m_c at radius r_c is introduced to balance the masses

$F= m_1r_1\omega^2+ m_2r_2\omega^2+ m_3r_3\omega^2 + m_cr_c\omega^2=0$

$m_1r_1+ m_2r_2+ m_3r_3 + m_cr_c=0$

Analytically, it can be solved by resolving the forces into components along and perpendicular to x axis.





Analytical method

$$\sum m_i r_i \cos\theta_i + m_c r_c \cos\theta_c = 0$$

$$\sum m_i r_i \sin\theta_i + m_c r_c \sin\theta_c = 0$$

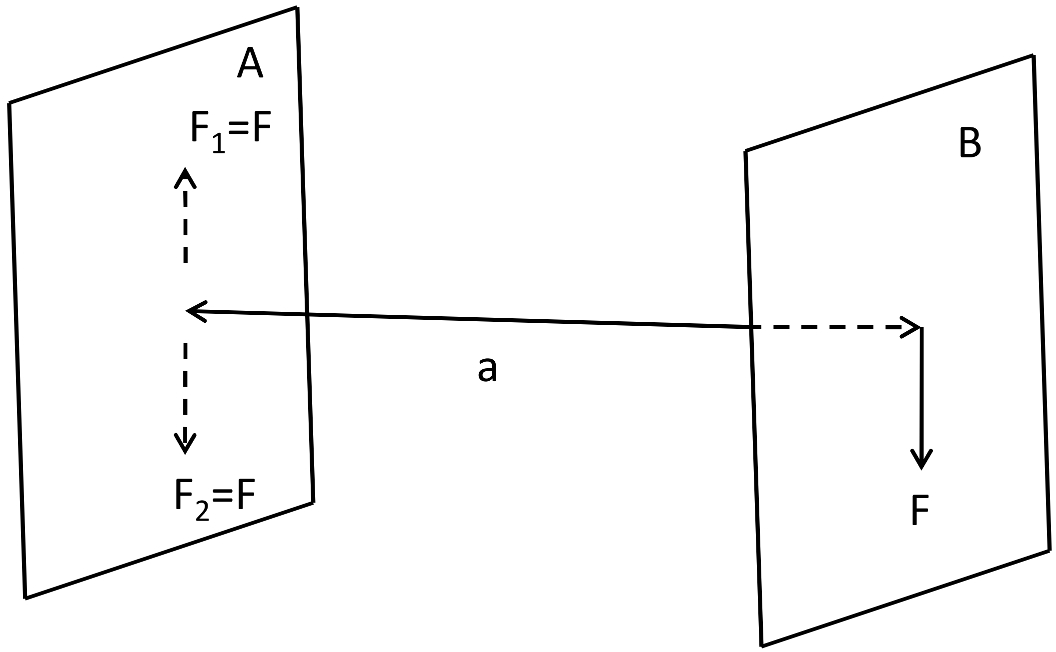
$$m_c r_c = \sqrt{\left(\sum m_i r_i \sin\theta_i\right)^2 + \left(\sum m_i r_i \cos\theta_i\right)^2}$$

$$\tan\theta_c = \frac{-\sum m_i r_i \sin\theta_i}{-\sum m_i r_i \cos\theta_i}$$

Balancing a number of masses rotating in different planes

- It is proposed to reduce the case to the case of several masses rotating in the same plane.
- In order to do so, the different masses rotating in different planes will have to be transferred to one plane called as a reference plane.

Transfer of force from one plane to another



Consider two planes A and B. Let F be the centrifugal force due to a rotating mass m acting in the plane B. The equilibrium in the planes will not be disturbed if two equal and opposite forces $F_1$and F_2each equal to and parallel to F. then F and F1 constitute a couple of magnitude F.a and F2=F will be also present. Thus the effect of transferring a rotating mass from one plane A to another plane, B is to introduce in the plane B
an unbalanced equal force in the same direction and
an unbalanced couple whose magnitude is equal to the product of the force and the distance between the two planes.

- Therefore, transfer of various forces in different planes to a reference plane will result in a number of unbalanced forces and unbalanced couples in the reference plane. Hence, to balance the system, the unbalanced forces and couples are to be balanced. The conditions to be satisfied are

$$\sum m_i r_i = 0$$

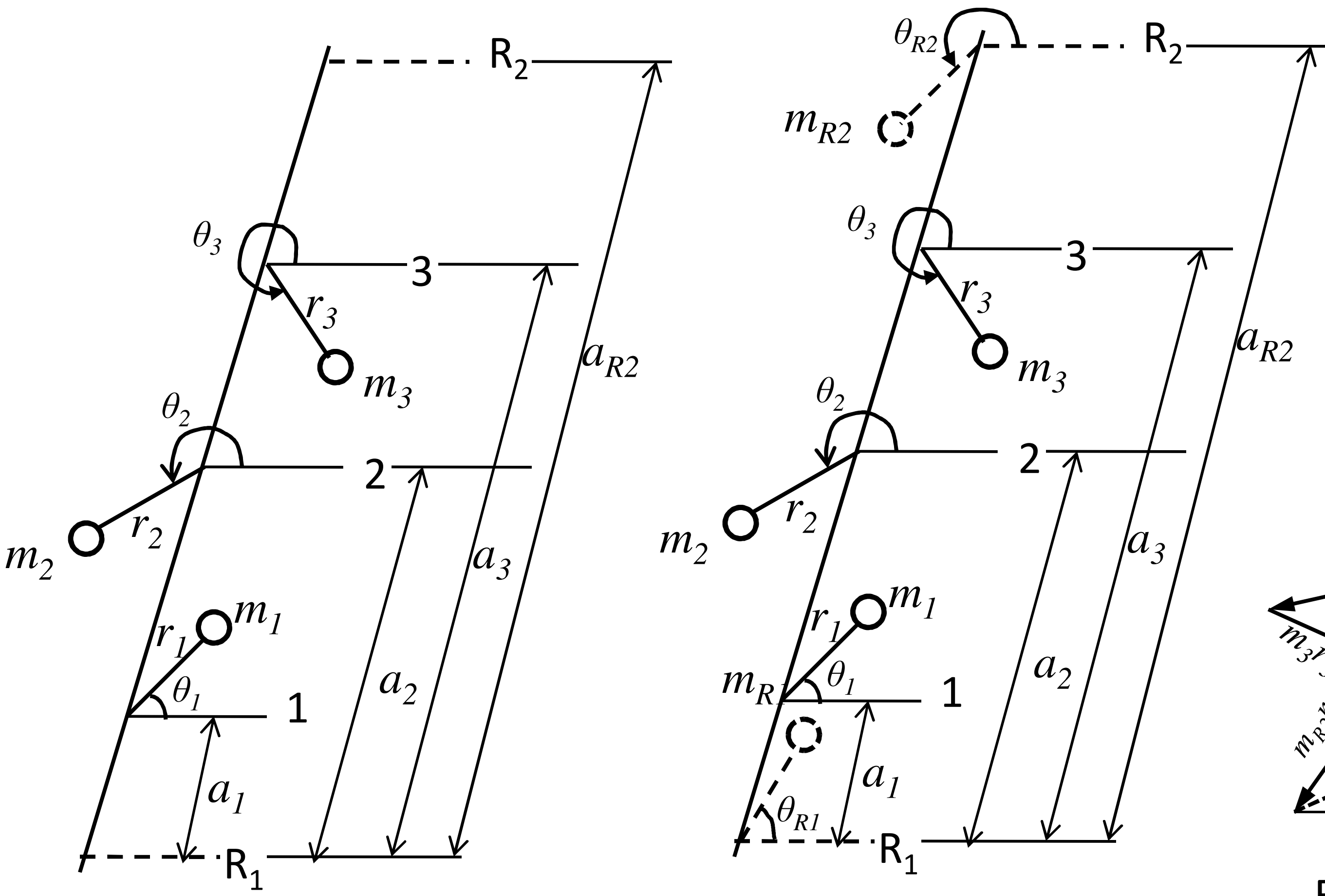
$$where\ i = 1\ to\ n\ (n = \ no.\ of\ masses)$$

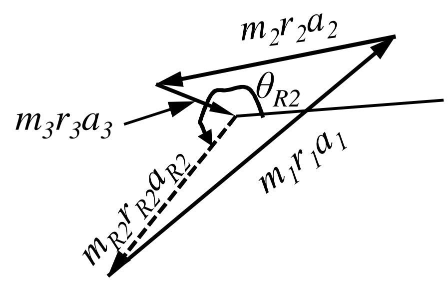
$$\sum m_i r_i a_i = 0$$

$$a_i = distance\ of\ mass\ i\ from\ the\ reference\ plane$$

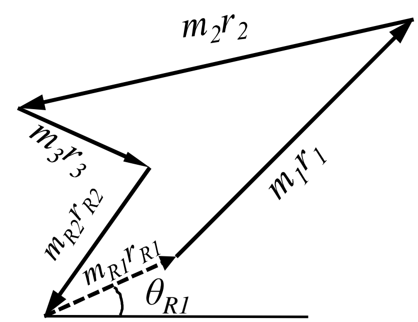
- couple acts in plane perpendicular to the plane in which the forces act. Its dire ction can be obtained by application of right hand screw rule. Thus the balancing couple will be at right angle to the forces producing it. However, in actual practice, the phase of couple diagram is changed through 90 degrees, thus enabling us to draw couples parallel to the forces.
- As a couple is produced by two forces, two balancing masses acting in two different planes will be required.

Masses in different planes





Couple polygon



Force polygon

In order to balance the masses completely, introduce two masses mR1 and mR2 at radii rR1 and rR2 in the reference planes R1 and R2 respectively. Then the conditions for complete balance are

$$m_1 r_1 \omega^2 + m_2 r_2 \omega^2 + m_3 r_3 \omega^2 + m_{R1} r_{R1} \omega^2 + m_{R2} r_{R2} \omega^2 = 0......(1)$$

$$\sum m_i r_i + m_{R1} r_{R1} + m_{R2} r_{R2} = 0.....(2)$$

$$m_1 r_1 a_1 \omega^2 + m_2 r_2 a_2 \omega^2 + m_3 r_3 a_3 \omega^2 + m_{R2} r_{R2} a_{R2} \omega^2 = 0......(3)$$

$$\sum m_i r_i a_i + m_{R2} r_{R2} a_{R2} = 0........(4)$$

The above equations can be solved analytical ly or graphicall y

Resolving the forces into components ,

$$\sum m_i r_i a_i \cos \theta_i + m_{R2} r_{R2} a_{R2} \cos \theta_{R2} = 0.......(5)$$

$$\sum m_i r_i a_i \sin \theta_i + m_{R2} r_{R2} a_{R2} \sin \theta_{R2} = 0........(6)$$

from Eq. (5) and (6)

$$m_{R2} r_{R2} a_{R2} = \sqrt{\left(\sum m_i r_i a_i \sin \theta_i\right)^2 + \left(\sum m_i r_i a_i \cos \theta_i\right)^2}........(7)$$

$$\tan \theta_{R2} = \frac{-\sum m_i r_i a_i \sin \theta_i}{-\sum m_i r_i a_i \cos \theta_i}.......... ....(8)$$

Subsitutin g the values from Eq. (7) and (8) in Eq.(2),

Eq.(2) can be solved by taking its components

$$m_{R1}r_{R1} = \sqrt{\left(\sum m_i r_i \sin\theta_i + m_{R2}r_{R2}\sin\theta_{R2}\right)^2 + \left(\sum m_i r_i \cos\theta_i + m_{R2}r_{R2}\cos\theta_{R2}\right)^2} \quad ...(9)$$

$$\tan\theta_{R1} = \frac{-\left(\sum m_i r_i \sin\theta_i + m_{R2}r_{R2}\sin\theta_{R2}\right)}{-\left(\sum m_i r_i \cos\theta_i + m_{R2}r_{R2}\cos\theta_{R2}\right)} \quad ....\ ....\ ..(10)$$

The Eq. (2) and (4) can be solved graphically by solving first Eq.(4) by drawing couple polygon to determine $m_{R2}r_{R2}a_{R2}$ and θ_{R2} and then Eq.(2) through force polygon to determine $m_{R1}r_{R1}$ and θ_{R1} .

A rotating disc of 1 m diameter has two eccentric masses of 0.5 kg each at radii of 50 mm and 60 mm at angular positions of 0°and 150°, respectively. a balancing mass of 0.1 kg is to be used to balance the rotor. What is the radial position of the balancing mass? [GATE-2005]

(a) 50 mm (b) 120 mm (c) 150 mm (d) 280 mm

$$0.1 \text{ x } r = \sqrt{(0.5 \text{ x } 50\cos 0 + 0.5 \text{ x } 60\cos 150)^2 + (0.5 \text{ x } 50\sin 0 + 0.5 \text{ x } 60\sin 150)^2}$$

$$r = 15.03/0.1 = 150.3mm$$

Balancing of reciprocating parts

- Acceleration of reciprocating parts in an engine is given by

$$f = \omega^2 r\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

If m is the mass of the reciprocating parts,

then force required to accelerate the mass m is

$$F = m\omega^2 r\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

$mr\omega^2 \cos\theta$ is called the primary accelerating force and

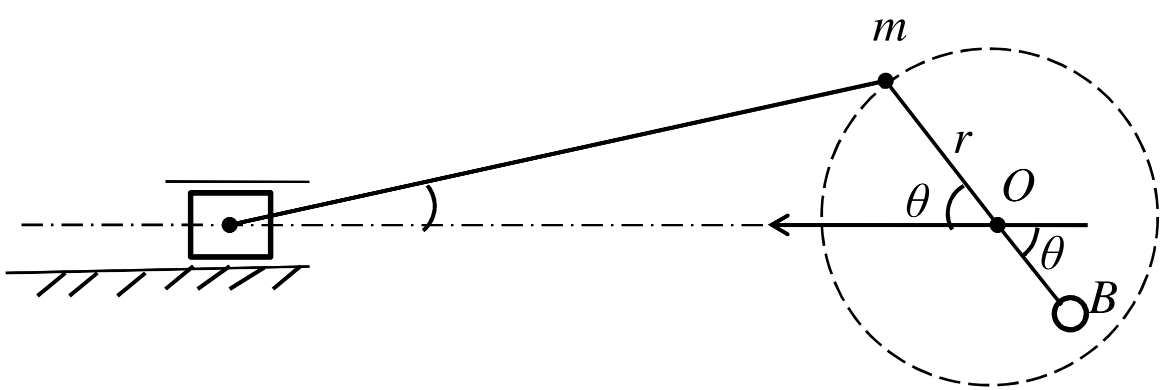
$mr\omega^2 \dfrac{\cos 2\theta}{n}$ is called the secondary accelerating force.

$mr\omega^2 \cos\theta$ is called the primary force and

$mr\omega^2 \dfrac{\cos 2\theta}{n}$ is called the secondary force.

since n is greater than 1, *for all engines except high speed engines, balancing of* sec*ondary force is nelgected.*

Partial balancing of Primary force



- The primary force acts along the line of stroke as shown in the figure. The primary force can be considered as a component of a centrifugal force due to a mass m at the crank radius r. Thus, balancing of primary force is equivalent to balancing of mass m.
- This can be done by attaching a mass B at a radius b in diametrically opposite direction as shown such that $Bb = mr$
- By the above method, the primary force is completely balanced, but the vertical component of the centrifugal force due to $B = Bb\omega^2 \sin\theta$ remains and its maximum value, $Bb\omega^2$ is again equal to the maximum magnitude of the primary force, $mr\omega^2$.
- In other words, the effect of the above method of balancing is toun change the direction of maximum unbalanced force from that along the line of stroke to that perpendicular to it.

As a compromise, a fraction, *c* of the reciprocating parts is balanced such that

$Bb = cmr$

As a result unbalanced force along the line of stroke = $(1-c)mr\omega^2\cos\theta$

Similarly, unbalanced force perpendicular to the line of stroke = $cmr\omega^2\sin\theta$

$\therefore$ Resultant unbalanced primary force at any instant

$$= \sqrt{\left[(1-c)mr\omega^2\cos\theta\right]^2 + \left[cmr\omega^2\sin\theta\right]^2}$$

$$= mr\omega^2\sqrt{\left[(1-c)\cos\theta\right]^2 + \left[c\sin\theta\right]^2}$$

The resultant unbalanced force will be minimum when c=1/2. However, the unbalanced force along the line stroke is more harmful than in a direction perpendicular to it. So, the common practice is to balance two-third of the reciprocating parts.

…tial balancing of reciprocating parts of two cylinder locomotives

- The two cylinders of locomotive have their cranks at 90 degrees.
- On account of partial balancing of reciprocating parts, there is an
- 1. unbalanced force along the line of stroke and 2. unbalanced force perpendicular to the line of stroke.
- The effect of unbalanced force along the line of stroke is to produce
  - a)the variation of tractive force along the line of stroke
  - b) the unbalanced couple

- The effect of unbalanced force perpendicular to the line of stroke is to produce
  - c) the variation of pressure on rails which results in hammering action on the rails. The maximum magnitude of the unbalanced force perpendicular to the line of stroke is termed as hammer blow.

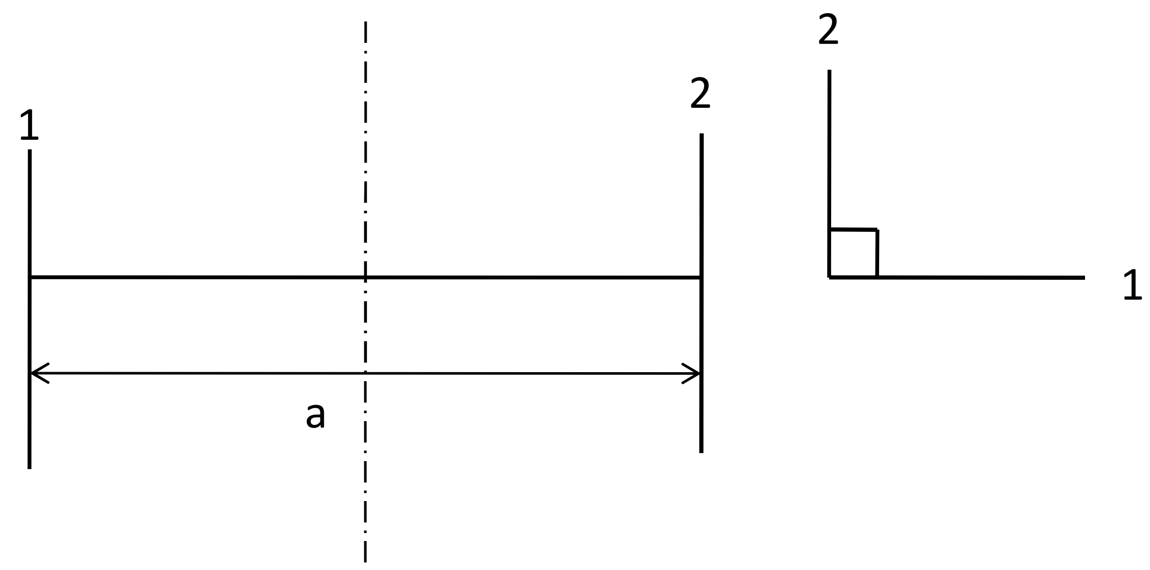
- Consider a two cylinder engine with cranks at 90 degrees as shown.

m - mass of reciprocating parts per cylinder

c - fraction of reciprocating parts balanced

r - radius of crank

a - the distance between cylinder centre lines



a) Variation of tractive force :

unbalanced force along the line of stroke of

cylinder 1

$$= (1-c) m r \omega^2 \cos\theta$$

cylinder 2

$$= (1-c) m r \omega^2 \cos(90+\theta)$$

Resultant unbalanced tractive force along

line of stroke $F_t = (1-c) m r \omega^2 \cos\theta + (1-c) m r \omega^2 \cos(90+\theta)$

$$F_t = (1-c) m r \omega^2 (\cos\theta - \sin\theta)$$

for F_t to be maximum or minimum

$$\frac{dF_t}{d\theta} = 0$$

$$-\sin\theta - \cos\theta = 0$$

$$\tan\theta = -1$$

$$\theta = 135^\circ \text{ or } 315^\circ$$

$$F_t \text{ max. and min.} = (1\text{-}c)mr\omega^2\left(\mp\frac{1}{\sqrt{2}}\mp\frac{1}{\sqrt{2}}\right)$$

$$= \mp\sqrt{2}(1-c)mr\omega^2$$

Maximum and minimum values of F_t are referred to as variation in tractive force

b) Swaying couple

- Unbalanced forces along line of stroke for cylinders 1 and 2 constitute a couple. This couple is measured about line YY in figure and its maximum magnitude is known as swaying couple and occurs at θ=45° and 225°.

$$Couple\ ,C = (1-c)mr\omega^2 \cos\theta\left(\frac{-a}{2}\right) + (1-c)mr\omega^2\cos(90+\theta)\frac{a}{2}$$

$$= -(1-c)mr\omega^2\frac{a}{2}(\cos\theta + \sin\theta)$$

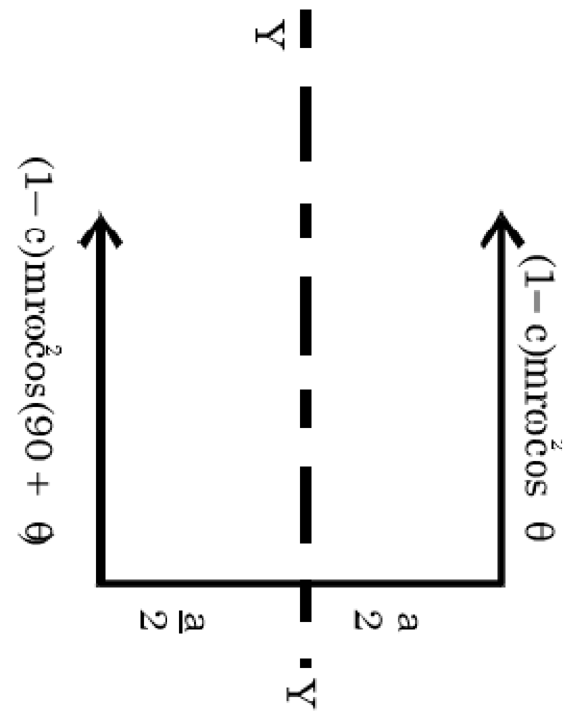
$$\text{the couple is maximum or minimum when } \frac{dC}{d\theta} = 0$$

$$\frac{dC}{d\theta} = -(1-c)mr\omega^2\frac{a}{2}(-\sin\theta + \cos\theta) = 0$$

$$\tan\theta = 1$$

$$\theta = 45^o\ and\ 225^o$$

$$C\max\ or\ \min = \mp(1-c)mr\omega^2\frac{a}{\sqrt{2}}$$



c) Hammer blow

- The unbalanced force perpendicular to the line of stroke due to balance mass B_1 at radius b to balance reciprocating parts only= $B_1 b\omega^2 \sin\theta$
- The maximum magnitude of this force is known is hammer blow and occurs at $\theta=90^o$ and 270^o
- Hammer blow =$B_1 b\omega^2$
- If P is the downwards pressure on rail due to dead load and other loads, then net pressure =P± $B_1 b\omega^2$.
- If P- $B_1 b\omega^2$ is negative, then the wheels will be lifted from the rails. Therefore the limiting condition in order that the wheel does not lift from rails is P- $B_1 b\omega^2 =0$.
- The permissible value of angular speed obtained from the condition is $\omega=(P/B_1 b)^{1/2}$

Balancing of secondary force

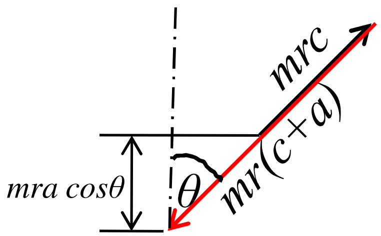
$$F_S = mr\omega^2 \frac{\cos 2\theta}{n}$$

$$= mr\omega^2 \frac{r}{l} \cos 2\omega t$$

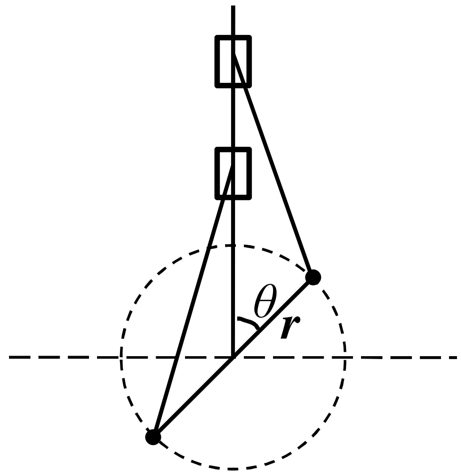
$$= m(2\omega)^2 \cdot \frac{r^2}{4l} \cdot \cos 2\omega t$$

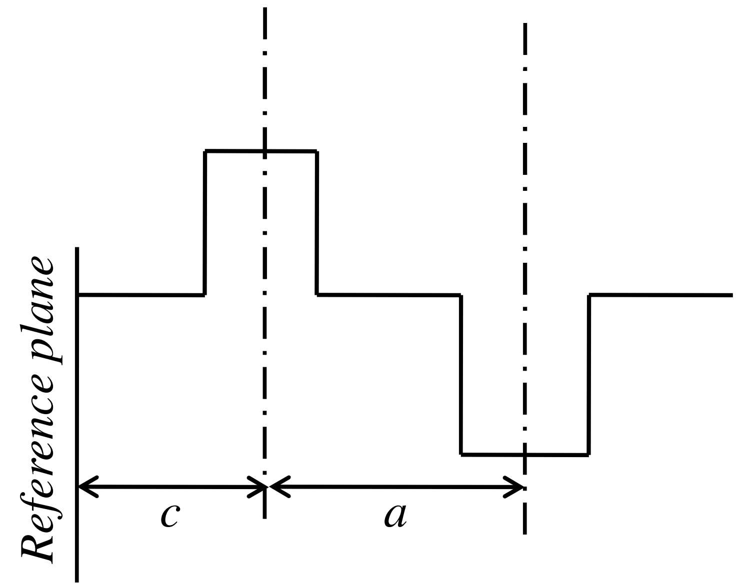
Balancing of In-line engines

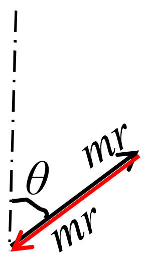
Two cylinder in-line engines:



Primary couple polygon







Primary force polygon

Consider a two cylinder vertical engine having equal reciprocating masses and cranks 180^o apart as shown. The crank angles are θ and $180+\theta$. Length of each crank is r and the distance of cylinder center lines from the reference plane are c and a. the reference plane is at the centre of left hand main bearing.

Two cylinder in-line engines….

- Primary forces: From the primary force polygon diagram, it is completely balanced.
- Primary couples: From the primary couple polygon diagram, resultant primary couple is proportional to the vertical component of the resultant mra.
- Resultant primary couple= $mr\omega^2 a\cos\theta$
- It is maximum at $\theta=0$ and 180^o and the maximum value=$\pm mr\omega^2 a$

- Secondary forces:

$$F_S = m(2\omega)^2 \cdot \frac{r^2}{4l} \cdot \cos 2\omega t$$

$$taking\ r_1 = \frac{r^2}{4l}$$

The speed of this crank is twice the speed of the actual cranks. Therefore, crank pins lie in line since when the actual crank angles are θ and $180 + \theta$, the secondary crank angles are 2θ and $360 + 2\theta$ respectively.

Two cylinder in-line engines….

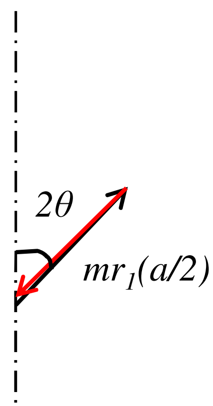
- Resultant centrifugal force is proportional to $2mr_1$.

$$\text{Resultant secondary force} = 2m(2\omega)^2 \frac{r^2}{4l} \cos 2\theta$$

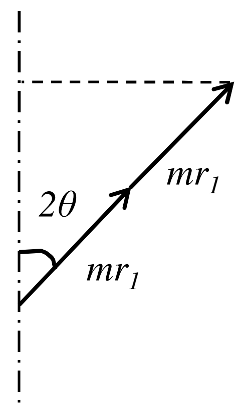
The maximum values occur when $\theta = 0^\circ, 90^\circ, 180^\circ$ and 270°

i.e. four times per revolution of the actual crank and the

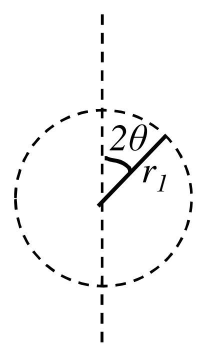
$$\text{maximum value} = 2m\frac{\omega^2 r^2}{l}$$

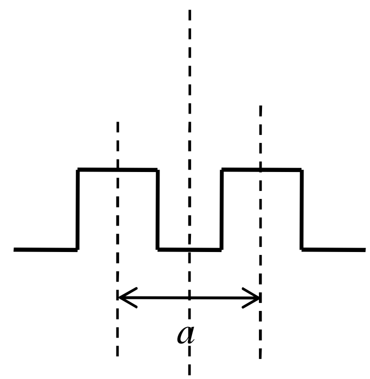


Secondary couple polygon



Secondary force polygon

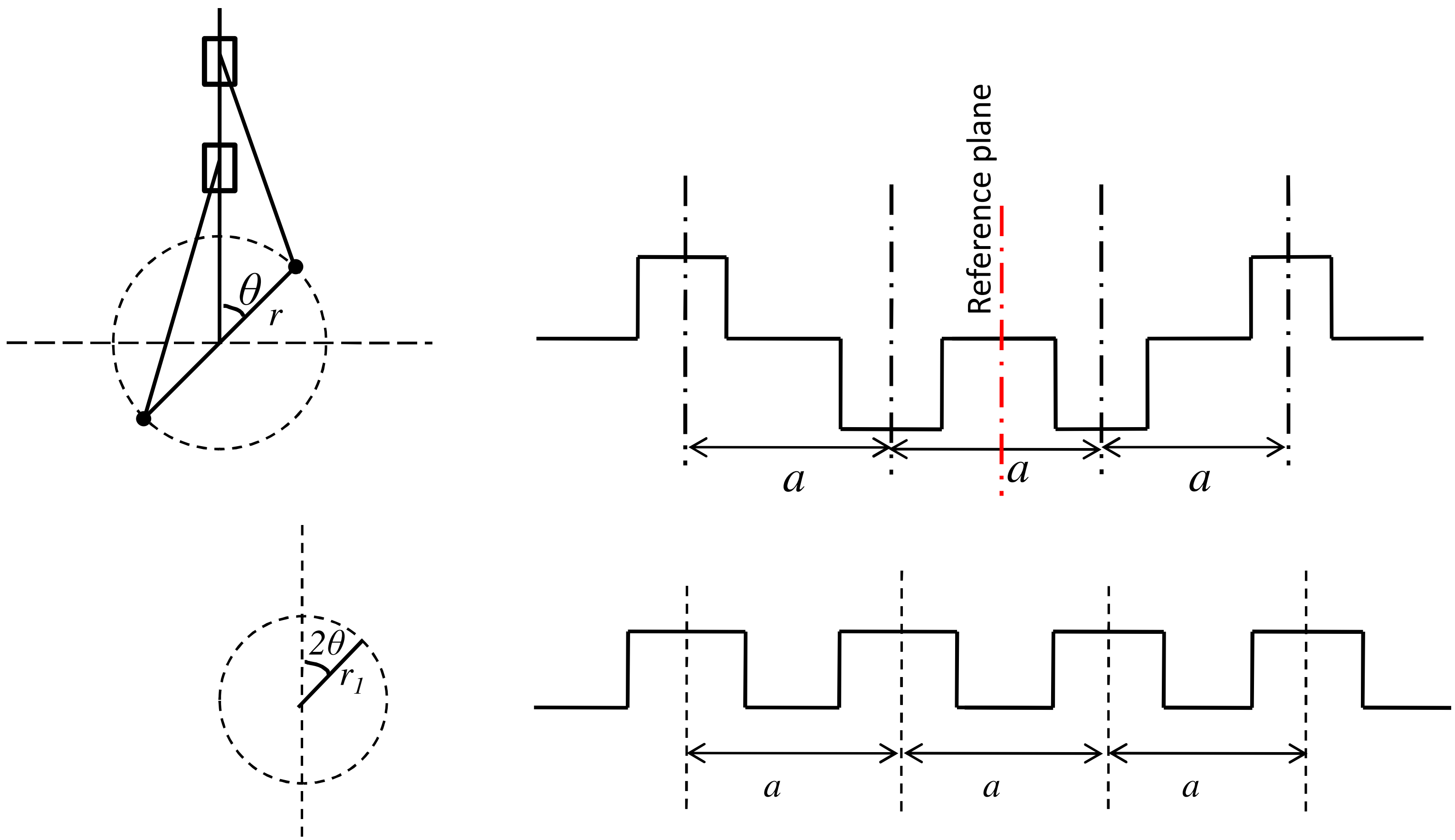




Two cylinder in-line engines….

- Secondary couples: From the secondary couple polygon, secondary couples are balanced.
- Resultant secondary couple=0.

Four cylinder in-line engine



Primary forces are balanced
Primary couples are balanced

$$\text{Resultant secondary force} = 4m(2\omega)^2 \frac{r^2}{4l} \cos 2\theta$$

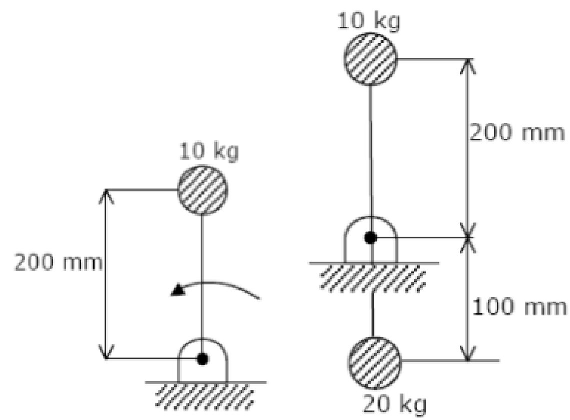
Secondary couples are balanced

Questions

A rigid body shown in the Fig. (a) has a mass of 10 kg. It rotates with a uniform angular velocity 'ω'. A balancing mass of 20 kg is attached as shown in Fig. (b). The percentage increase in mass moment of inertia as a result of this addition is

(a) 25% (b) 50%

(c) 100% (d) 200%



[GATE-2004]

$$I_1 = 10 \times (0.2)^2 = 0.4 kgm^2$$

$$I_2 = 10 \times (0.2)^2 + 20 \times 0.1^2 = 0.6 kg - m^2$$

$$\% Increase = \frac{I_2 - I_1}{I_1} \times 100 = 50\%$$

Which one of the following can completely balance several masses revolving in different planes on a shaft?

(a)A single mass in one of the planes of the revolving masses

(b) A single mass in any one plane

(c)Two masses in any two planes

(d)Two equal masses in any two planes.

Ans. (c)

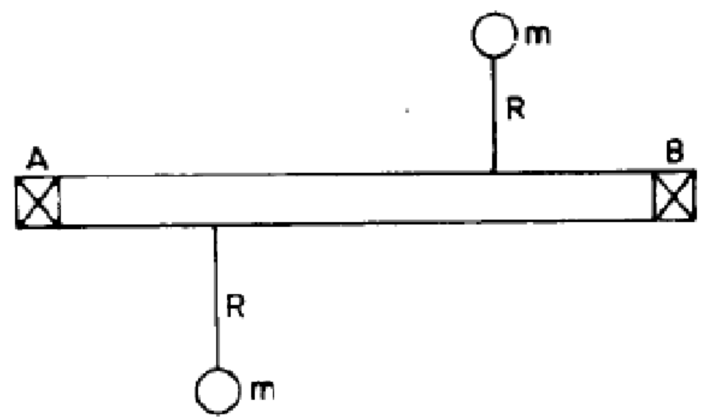
A rotor supported at A and B, carries two masses as shown in the given figure. The rotor is

(a) dynamically balanced

(b) statically balanced

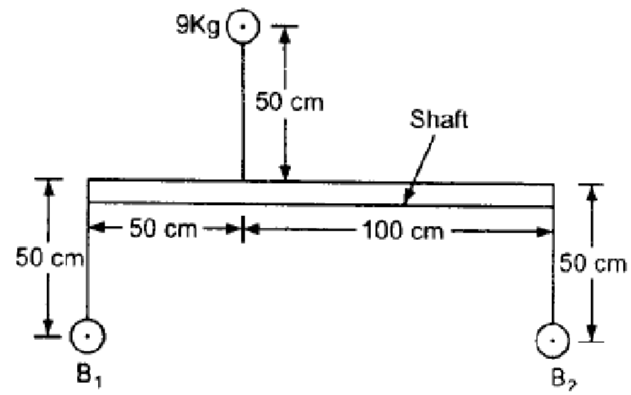
(c) statically and dynamically balanced

(d) not balanced.



Ans. (b)

- Masses B1, B2 and 9 kg are attached to a shaft in parallel planes as shown in the figure. If the shaft is rotating at 100 rpm, the mass B2 is
  (a) 3 kg (b) 6 kg (c) 9 kg
  (d) 27 kg



Solution:

9 x0.5=B2 x 1.5

B2=3 kg

Static balancing is satisfactory for low speed rotors but with increasing speeds, dynamic balancing becomes necessary. This is because, the

(a) Unbalanced couples are caused only at higher speeds

(b) Unbalanced forces are not dangerous at higher speeds

(c) Effects of unbalances are proportional to the square of the speed

(d) Effects of unbalances are directly proportional to the speed

Ans. (C)

- The balancing weights are introduced in planes parallel to the plane of rotation of the disturbing mass. To obtain complete dynamic balance, the minimum number of balancing weights to be introduced in different planes is
  (a) 1 (b) 2 (c) 3 (d) 4
- Ans. b

A rigid rotor consists of a system of two masses located as shown in the given figure. The system is

(a) statically balanced

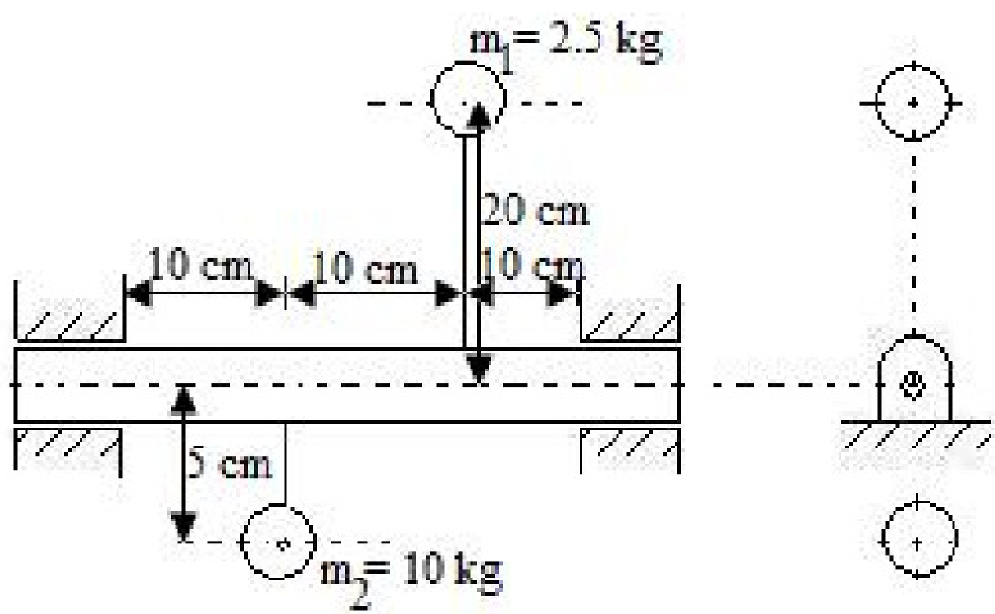
(b) dynamically balanced

(c) statically unbalanced

(d) both statically and dynamically unbalanced

Ans. a

centre of masses lie on the axis of rotation.



If the ratio of the length of connecting rod to the crank radius increases, then

(a) Primary unbalanced forces will increase
(b) Primary unbalanced forces will decrease
(c) Secondary unbalanced forces will increase
(d) Secondary unbalanced forces will decrease

Ans. (d) Secondary force only involves ratio of length of connecting rod and crank radius.

If n increases, value of secondary force will decrease.

- A single cylinder, four-stroke I.C. engine rotating at 900 rpm has a crank length of 50 mm and a connecting rod length of 200 mm. If the effective reciprocating mass of the engine is 1.2 kg, what is the approximate magnitude of the maximum 'secondary force' created by the engine?

(a) 533 N (b) 666 N (c) 133 N

(d) None of the above

Sol: Maximum secondary force= $mr\omega^2/n$

Ans. (b)

- Consider the following statements:

An in-line four-cylinder four-stroke engine is completely balanced for

1. primary forces 2. secondary forces

3. primary couples 4. secondary couples

Of these statements:

(a) 1, 3 and 4 are correct (b) 1, 2 and 4 are correct

(b) (c) 1 and 3 are correct (d) 2 and 4 are correct