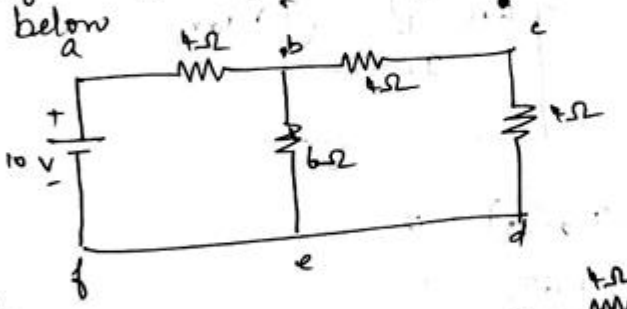
RECIPROCITY THEOREM:

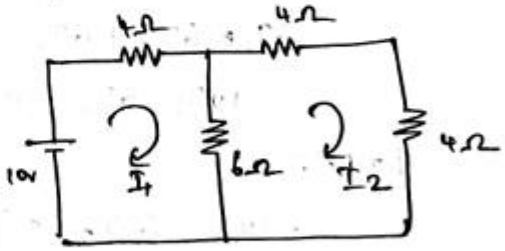
The reciprocity theorem states that in a linear, bilateral network a voltage source V volts in a branch gives rise to current I in another branch, the ratio V/I is constant when the positions of V and I are interchanged.

Problem:

Verify reciprocity theorem for the network shown in Fig below



Sol:



Loop 1:

$$10 = 4I_1 + 6(I_1 - I_2)$$

$$10 = 4I_1 + 6I_1 - 6I_2$$

$$10 = 10I_1 - 6I_2 \rightarrow (1)$$

Loop 2:

$$0 = 6(I_2 - I_1) + 4I_2 + 4I_2$$

$$0 = 6I_2 - 6I_1 + 8I_2$$

$$0 = -6I_1 + 14I_2 \rightarrow (2)$$

Solve (1) & (2)

$$\Delta \begin{bmatrix} 10 & -6 \\ -6 & 14 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 10 & -6 \\ -6 & 14 \end{vmatrix} = 140 - 36 = 104$$

$$\Delta_1 = \begin{vmatrix} 10 & -6 \\ 0 & 14 \end{vmatrix} = 140$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{140}{104} =$$

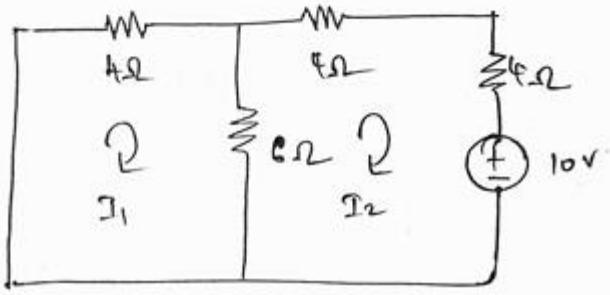
$$\Delta_2 = \begin{vmatrix} 10 & 10 \\ -6 & 0 \end{vmatrix} = 0 + 60$$

$$I_2 = \frac{\Delta_1}{\Delta} = \frac{60}{104} = 0.577\ A$$

Voltage - Current Ratio

$$\frac{V}{I} = \frac{10}{0.577} = 17.33\ \Omega$$

Change the position of voltage source



$$4I_1 + 6(I_1 - I_2) = 0$$

$$4I_1 - 6I_2 + 6I_1 = 0$$

$$10I_1 - 6I_2 = 0 \quad \rightarrow (1)$$

Loop 2:

$$-10 = 6(I_2 - I_1) + 4I_2 + 4I_2$$

$$-10 = 6I_2 - 6I_1 + 4I_2 + 4I_2$$

$$-10 = -6I_1 + 14I_2 \quad \rightarrow (2)$$

$$\begin{bmatrix} 10 & -6 \\ -6 & 14 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -10 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 10 & -6 \\ -6 & 14 \end{vmatrix} = 140 - 36$$

$$= 104$$

$$\Delta_2 = \begin{vmatrix} 10 & 0 \\ -6 & -10 \end{vmatrix} = -100$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{-100}{104} = 0.96$$

$$I_1 = \frac{\Delta_1}{\Delta}$$

$$\Delta_1 = \begin{vmatrix} 0 & -6 \\ -10 & 14 \end{vmatrix} = 0 - 60$$

$$= -60$$

$$I_1 = \frac{-60}{104} = -0.577\ A$$

Voltage - Current Ratio

$$\frac{V}{I} = \frac{10}{0.577} = 17.33$$