7–3 Consider the table of Prob. 7–2 where the primary dimensions of several variables are listed in the mass–length–time system. Some engineers prefer the force–length–time system (force replaces mass as one of the primary dimensions). Write the primary dimensions of three of these (density, surface tension, and viscosity) in the force–length–time system.

7–4 Write the primary dimensions of the *universal ideal gas constant* R_u. (*Hint*: Use the *ideal gas law*, $PV = nR_uT$ where P is pressure, V is volume, T is absolute temperature, and n is the number of moles of the gas.) *Answer:* $\{m^1L^2t^{-2}T^{-1}N^{-1}\}$

7–5 On a periodic chart of the elements, molar mass (M), also called *atomic weight,* is often listed as though it were a dimensionless quantity (Fig. P7–5). In reality, atomic weight is the mass of 1 mol of the element. For example, the atomic weight of nitrogen $M_{\text{nitrogen}} = 14.0067$. We interpret this as 14.0067 g/mol of elemental nitrogen. What are the primary dimensions of atomic weight?

6 C 12.011	7 N 14.0067	8 O 15.9994
14 Si 28.086	15 P 30.9738	16 S 32.060

FIGURE P7–5

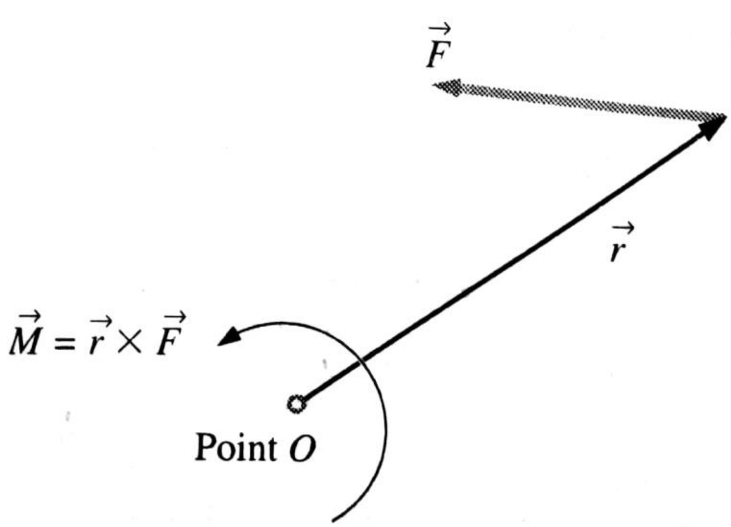
Prob. 7–4, rewrite the primary dimensions of the universal gas constant in this alternate system of primary dimensions.

7–7 We define the *specific ideal gas constant* R_{gas} for a particular gas as the ratio of the universal gas constant and the molar mass (also called *molecular weight*) of the gas, $R_{gas} = R_u/M$. For a particular gas, then, the ideal gas law is written as follows:

$$PV = mR_{gas}T \qquad \text{or} \qquad P = \rho R_{gas}T$$

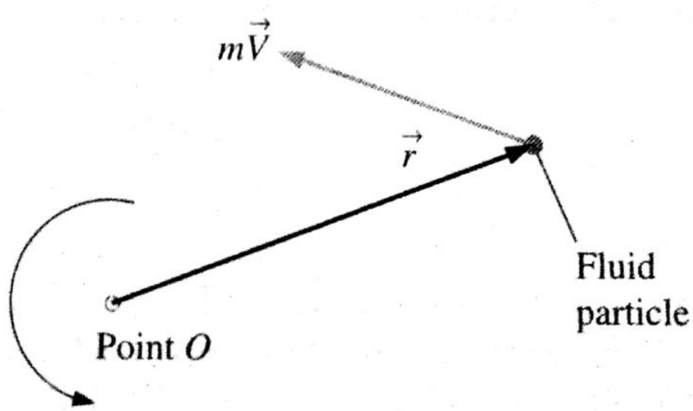
where P is pressure, V is volume, m is mass, T is absolute temperature, and ρ is the density of the particular gas. What are the primary dimensions of R_{gas}? For air, $R_{air} = 287.0$ J/kg · K in standard SI units. Verify that these units agree with your result.

7–8 The *moment of force* ($\vec{M}$) is formed by the cross product of a moment arm ($\vec{r}$) and an applied force ($\vec{F}$), as sketched in Fig. P7–8. What are the primary dimensions of moment of force? List its units in primary SI units.



7–12 Write the primary dimensions of each of the following variables, showing all your work: (*a*) acceleration a; (*b*) angular velocity ω; (*c*) angular acceleration α.

7–13 *Angular momentum*, also called *moment of momentum* ($\vec{H}$), is formed by the cross product of a moment arm ($\vec{r}$) and the linear momentum ($m\vec{V}$) of a fluid particle, as sketched in Fig. P7–13. What are the primary dimensions of angular momentum? List the units of angular momentum in primary SI units. *Answers:* $\{m^1L^2t^{-1}\}$, kg·m²/s



$$\vec{H} = \vec{r} \times m\vec{V}$$

FIGURE P7–13

7–14 Write the primary dimensions of each of the following variables, showing all your work: (*a*) specific heat at constant pressure c_p; (*b*) specific weight ρg; (*c*) specific enthalpy h.

7–15 Th

(b) heat flux $\dot{q}$ (*Hint*: rate of heat transfer per unit area); (c) heat transfer coefficient h (*Hint*: heat flux per unit temperature difference).

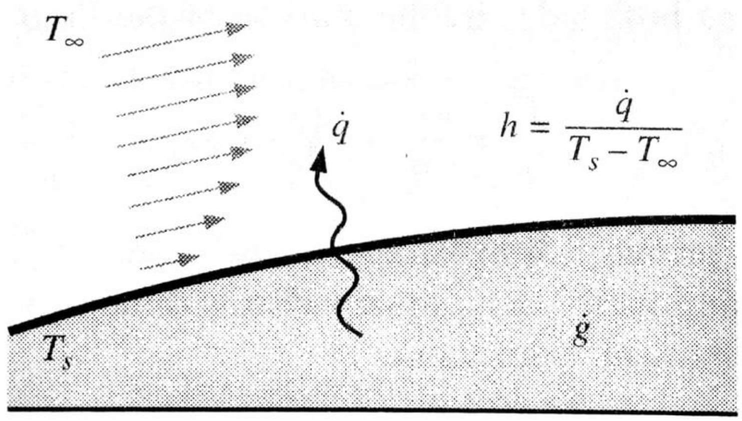


FIGURE P7–16

7–17 Thumb through the appendices of your thermodynamics book, and find three properties or constants not mentioned in Probs. 7–1 to 7–16. List the name of each property or constant and its SI units. Then write out the primary dimensions of each property or constant.

Dimensional Homogeneity

7–18C Explain the *law of dimensional homogeneity* in simple terms.

7–19 In Chap. 4, we defined the *material acceleration*, which is the acceleration following a fluid particle (Fig. P7–19),

$$\vec{a}(x, y, z, t) = \frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \vec{\nabla})\vec{V}$$

(*a*) What are the primary dimensions of the gradient operator $\vec{\nabla}$? (*b*) Verify that each additive term in the equation has the same dimensions. *Answers:* (*a*) $\{L^{-1}\}$; (*b*) $\{L^1t^{-2}\}$

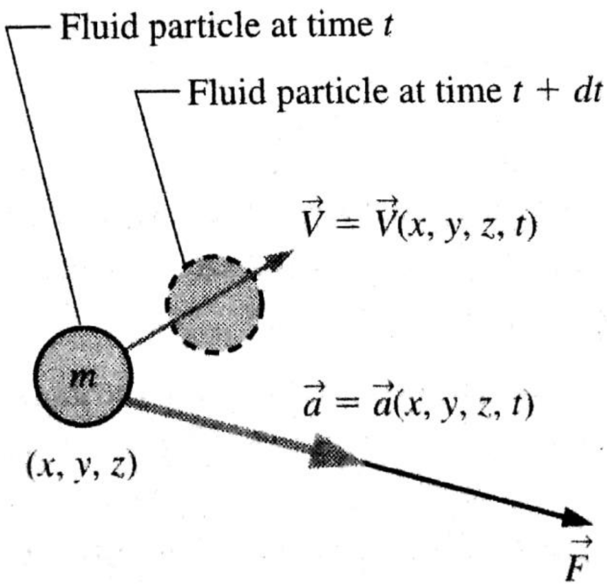


FIGURE P7–19

7–20 Newton's second law is the foundation for the differential equation of conservation of linear momentum (to be discussed in Chap. 9). In terms of the material acceleration following a fluid particle (Fig. P7–19), we write Newton's second law as follows:

$$\vec{F} = m\vec{a} = m\left[\frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \vec{\nabla})\vec{V}\right]$$

Or, dividing both sides by the mass m of the fluid particle,

$$\frac{\vec{F}}{m} = \frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \vec{\nabla})\vec{V}$$

Write the primary dimensions of each additive term in the equation, and verify that the equation is dimensionally homogeneous. Show all your work.

7–21 In Chap. 4 we defined *volumetric strain rate* as the rate of increase of volume of a fluid element per unit volume (Fig. P7–21). In Cartesian coordinates we write the volumetric strain rate as

$$\frac{1}{\mathcal{V}}\frac{D\mathcal{V}}{Dt} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

Write the primary dimensions of each additive term, and verify that the equation is dimensionally homogeneous. Show all your work.

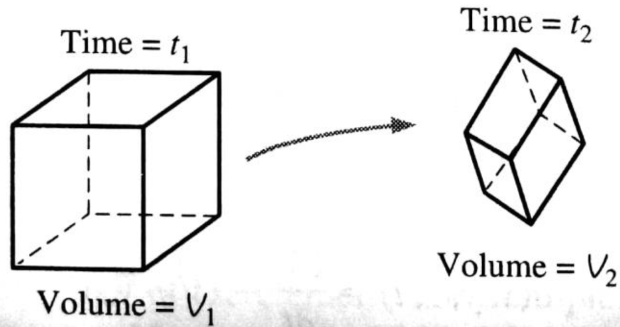


FIGURE P7–21

Dimensionless Parameters and the Method of Repeating Variables

7–45 Using primary dimensions, verify that the Archimedes number (Table 7–5) is indeed dimensionless.

7–46 Using primary dimensions, verify that the Grashof number (Table 7–5) is indeed dimensionless.

7–47 Using primary dimensions, verify that the Rayleigh number (Table 7–5) is indeed dimensionless. What other established nondimensional parameter is formed by the ratio of Ra and Gr? *Answer:* the Prandtl number

7–48 Consider a liquid in a cylindrical container in which both the container and the liquid are rotating as a rigid body (solid-body rotation). The elevation difference h between the center of the liquid surface and the rim of the liquid surface is a function of angular velocity ω, fluid density ρ, gravitational acceleration g, and radius R (Fig. P7–48). Use the method of repeating variables to find a dimensionless relationship between the parameters. Show all your work. *Answer:* $h/R = f(\text{Fr})$

7–49 Consider the case in which the container and liquid of Prob. 7–48 are initially at rest. At $t = 0$ the container begins to rotate. It takes some time for the liquid to rotate as a rigid body, and we expect that the liquid's viscosity is an additional relevant parameter in the unsteady problem. Repeat Prob. 7–48, but with two additional independent parameters included, namely, fluid viscosity μ and time t. (We are interested in the development of height h as a function of time and the other parameters.)

7–51 Repeat Prob. 7–50, but with an additional independent parameter included, namely, the speed of sound c in the fluid. Use the method of repeating variables to generate a dimensionless relationship for Kármán vortex shedding frequency f_k as a function of free-stream speed V, fluid density ρ, fluid viscosity μ, cylinder diameter D, and speed of sound c. Show all your work.

7–52 A stirrer is used to mix chemicals in a large tank (Fig. P7–52). The shaft power $\dot{W}$ supplied to the stirrer blades is a function of stirrer diameter D, liquid density ρ, liquid viscosity μ, and the angular velocity ω of the spinning blades. Use the method of repeating variables to generate a dimensionless relationship between these parameters. Show all your work and be sure to identify your Π groups, modifying them as necessary. *Answer:* $N_p = f(\text{Re})$

7–55 The *Richardson number* is defined as

$$\text{Ri} = \frac{L^5 g\,\Delta\rho}{\rho \dot{V}^2}$$

Miguel is working on a problem that has a characteristic length scale L, a characteristic velocity V, a characteristic density difference $\Delta\rho$, a characteristic (average) density ρ, and of course the gravitational constant g, which is always available. He wants to define a Richardson number, but does not have a characteristic volume flow rate. Help Miguel define a characteristic volume flow rate based on the parameters available to him, and then define an appropriate Richardson number in terms of the given parameters.

7–56 Consider fully developed **Couette flow**—flow between two infinite parallel plates separated by distance h, with the top plate moving and the bottom plate stationary as illustrated in Fig. P7–56. The flow is steady, incompressible, and two-dimensional in the xy-plane. Use the method of repeating variables to generate a dimensionless relationship for the x-component of fluid velocity u as a function of fluid viscosity μ, top plate speed V, distance h, fluid density ρ, and distance y. Show all your work. *Answer:* $u/V = f(\text{Re}, y/h)$

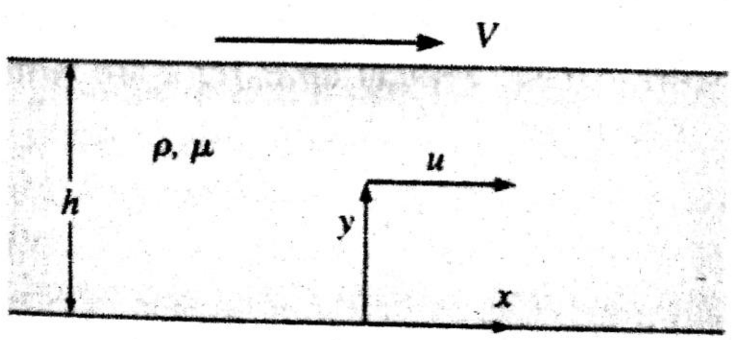


FIGURE P7–56

7–57

7–58 The speed of sound c in an ideal gas is known to be a function of the ratio of specific heats k, absolute temperature T, and specific ideal gas constant R_{gas} (Fig. P7–58). Showing all your work, use dimensional analysis to find the functional relationship between these parameters.

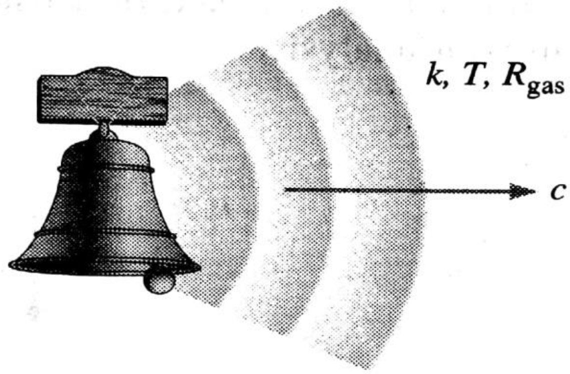


FIGURE P7–58

7–59 Repeat Prob. 7–58, except let the speed of sound c in an ideal gas be a function of absolute temperature T, universal ideal gas constant R_u, molar mass (molecular weight) M of the gas, and ratio of specific heats k. Showing all your work, use dimensional analysis to find the functional relationship between these parameters.

7–60 Repeat Prob. 7–58, except let the speed of sound c in an ideal gas be a function only of absolute temperature T and specific ideal gas constant R_{gas}. Showing all your work, use dimensional analysis to find the functional relationship between these parameters. *Answer:* $c/\sqrt{R_{gas}T}$ = constant

7–61 Repeat Prob. 7–58, except let speed of sound c in an ideal gas be a function only of pressure P and gas density ρ. Showing all your work, use dimensional analysis to find the functional relationship between these parameters. Verify that your results are consistent with the equation for speed of sound in an ideal gas, $c = \sqrt{kR_{gas}T}$.

7–74 Repeat Prob. 7–73 for the case in which the propeller operates in a compressible gas instead of a liquid.

7–75 In the study of turbulent flow, turbulent viscous dissipation rate ε (rate of energy loss per unit mass) is known to be a function of length scale l and velocity scale u' of the large-scale turbulent eddies. Using dimensional analysis (Buckingham pi and the method of repeating variables) and showing all of your work, generate an expression for ε as a function of l and u'.

7–76 The rate of heat transfer to water flowing in a pipe was analyzed in Prob. 7–23. Let us approach that same problem, but now with dimensional analysis. Cold water enters a pipe, where it is heated by an external heat source (Fig. P7–76). The inlet and outlet water temperatures are T_{in} and T_{out}, respectively. The total rate of heat transfer $\dot{Q}$ from the surroundings into the water in the pipe is known to be a function of mass flow rate $\dot{m}$, the specific heat c_p of the water, and the temperature difference between the incoming and outgoing water. Showing all your work, use dimensional analysis to find the functional relationship between these parameters, and compare to the analytical equation given in Prob. 7–23. (*Note*: We are pretending that we do not know the analytical equation.)

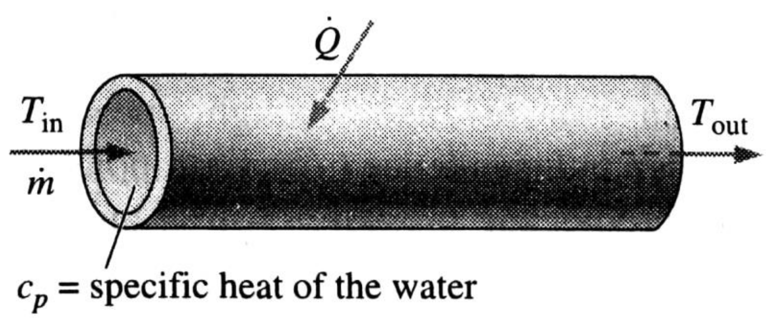


FIGURE P7–76