SIGNALS AND SYSTEMS

Introduction to Fourier Series

Fourier series

- Fourier series is a powerful mathematical tool
- Represent periodic functions as a sum of sines and cosines.
- It has vast applications in various fields, including signal processing, physics, and engineering.

Periodic Functions

- Functions that repeat at regular intervals.
- The period is the length of one repetition cycle.
- A function x(t) is periodic if there exists a positive number T such that $x(t + T) = x(t)$ for all t.
- The smallest positive value of T for which

  $x(t + T) = x(t)$ is called the period of x(t).
- Ex. Sine waves, square waves, saw tooth waves, triangular waves

Trigonometric Fourier Series

- Trigonometric Fourier series represents a periodic function as an infinite sum of sine and cosine terms, each with a specific frequency and amplitude.

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n w_0 t + b_n \sin n w_0 t$$

- The coefficients are determined by the function and can be calculated using integrals.
- The frequency ω is determined by the period of the function It represents the number of cycles per unit time.

Complex Fourier Series

- ➢ Provides a more concise and elegant way to represent periodic functions using complex exponentials.
- ➢ It uses a single coefficient a_n to represent both amplitude and phase.

Trigonometric

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos nw_0 t + b_n \sin nw_0 t$$

Coefficients: $a_n,\ b_n$

Complex

$$x(t) = \sum_{n=-\infty}^{\infty} a_n e^{jkwt}$$

Coefficient: a_n

Convergence of Fourier Series

- The convergence of a Fourier series refers to whether the series converges to the original function as the number of terms in the series increases.
- The conditions for convergence depend on the properties of the function.

Dirichlet's Condition

- The function must be piecewise smooth and have a finite number of discontinuities in one period.

Convergence of Fourier Series

Convergence at Discontinuities

- At points of discontinuity, the Fourier series converges to the average of the left and right limits of the function.

Convergence at Continuity

- At points of continuity, the Fourier series converges to the value of the function itself.

Applications of Fourier Series

- Signal Processing
- Heat Transfer
- Image Analysis
- Electrical Engineering


Thank you