SIGNALS AND SYSTEMS

Trigonometric Fourier Series of Periodic Signals

Trigonometric Fourier series

- Represent periodic signals as a sum of sines and cosines.
- Analyze and understand the behavior of these signals in a more straightforward manner.
- The series decomposes a periodic signal into a fundamental frequency and its harmonics.
- The trigonometric form of the Fourier series is particularly useful in engineering and physics applications.

Derivation of the Fourier Series Coefficients

- Calculate the integrals for the coefficients a_0, an, and bn.
- a_0 - average value of the signal over a period.
- an and bn - the amplitudes of the cosine and sine terms, respectively, at the corresponding harmonic frequencies.

Derivation of the Fourier Series Coefficients

a_0 Coefficient

- The a_0 coefficient represents the DC component of the signal, which is the average value of the signal over one period.
- It is calculated as

$$a_0 = \frac{1}{T}\int_0^T f(t)dt$$

Derivation of the Fourier Series Coefficients

a_n Coefficient

- The an coefficients represent the amplitudes of the cosine terms in the Fourier series.
- They are calculated as

$$a_n = \frac{2}{T} \int_0^T x(t)\ \cos(nw_0 t)\, dt$$

Derivation of the Fourier Series Coefficients

b_n Coefficient

- The b_n coefficients represent the amplitudes of the sine terms in the Fourier series.
- They are calculated as

$$b_n = \frac{2}{T} \int_0^T x(t)\ \sin(nw_0 t)\, dt$$

- Consider a square wave with an amplitude of 1 and a period of T.
- The Fourier series representation of this square wave is:

$$f(t) = (4/\pi) * [\sin(\omega 0t) + (1/3)\sin(3\omega 0t) + (1/5)\sin(5\omega 0t) + ...]$$

- Square wave can be represented by a sum of odd harmonics.
- The amplitude of each harmonic is inversely proportional to the harmonic number

Fundamental Frequency

$\Omega 0$ - is the frequency of the first harmonic, $2\pi/T$

Harmonics

Multiples of the fundamental frequency, such as $3\omega_0$, $5\omega_0$,

Amplitude Decay

The amplitudes of the harmonics decrease as the harmonic number increases.

This is because higher harmonics have higher frequencies and contribute less to the overall shape of the signal.

Applications of Fourier Series

- Signal Analysis
- Signal Processing
- System Modeling

Thank you