

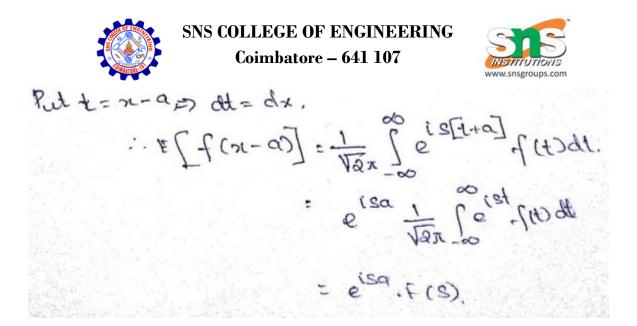
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TOPIC : 8 – Properties of Fourier Transform

Properties of Fourier Transform: 1. sinear Property: F{afin>+bqin>} = afis)+bGies). Read: F (afini) + bg(n)) = 1/ ST Seisz po finit by postdy. = afige st forda] + bfiss for gooth = a F(S) + b G(S) ushere F(S) and GISS are the Fornier transforms of find and good

2. Shifting Property: -5f F(f(a)) = F(s), then F[-f(a-a)]= e F(s) roof: $F(f(x-\alpha)) = \frac{1}{\sqrt{a\pi}} \int_{-\infty}^{\infty} e^{isn} f(x-\alpha) dx,$



where gite Tooln: F & f (ax) = - 1 [eisx, f (ax) dx. Pull t=ax $dt = adx \Rightarrow dn = \frac{dt}{a}$: E {f(ax)}= i [i Sect(a) fit)dt],

By definition, so ist of Libolt

SNS COLLEGE OF ENGINEERING Coimbatore – 641 107 · Ff f (ax) = - + (2), arb and = - 1 F (2), a co. $F\left(\frac{1}{2}\right) = \frac{1}{101}F\left(\frac{1}{2}\right)$ where $a \neq 0$. A. JJ F ff (2)} = F(S), then F feiox f(2) = F(S+a) Proof: ffeiax f(x) = the feiax eisz f(m) dx. = ton icerain finida. ... F{ eiax frong= F (s+a). Respectings: Modubation thosean. -If Effcold = E (is); then Eff (20). Corsand = - F (S-a)+F (Sta) Proof: Fficond = F(s) = 1 fin e is dr. Ff. frow. corsand: Jan Je S. Corsan frow dr. But Coxan: e + e



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 $\int_{0}^{\infty} (Sx) \left(e^{i\alpha x} - i\alpha x \right)$ $F[f(n)(\alpha)(\alpha)] = \frac{1}{2} \int \frac{1}{2\pi} \int \frac{1}{2} \int \frac{1}{2\pi} \int \frac{1}{2} \int \frac{1}{2\pi} \int \frac{1}{2\pi$ = $\frac{1}{2} \left[F(rs + \alpha) + F(r - \alpha) \right]$

Theorem b:

$$Jf \in \{f(x)\} = F(x), \text{theo} \in \{x\} \{(x)\} = (-i) \frac{d^n}{dx^n} \{f(x)\}.$$

Property 7:

$$\Im F \{f(x)\} = F(x), \text{ theo } F \{f'(x)\} = (is)^{n} F(x) if$$

 $f(x), f'(x), \dots f''(x) \text{ all lead to zero } as x \to \pm\infty$.
where $f''(x) = \frac{dh}{dx^{n}} (f(x)).$

Proof:

$$F \left\{ f'(x) \right\} = \frac{1}{\sqrt{2\pi}} \int_{\infty}^{\infty} f'(x) e^{iSx} dx.$$

$$= \frac{1}{\sqrt{2\pi}} \int_{0}^{\infty} e^{iSx} d(f(x))$$

$$= \frac{1}{\sqrt{2\pi}} \int_{0}^{\infty} e^{iSx} d(f(x)) e^{iSx} \int_{0}^{\infty} f(x) e^{iSx} dx.$$

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Since
$$f(x) \rightarrow 0$$
 as $x \rightarrow \infty$, we have.
 $F\{f'(x)\} = (-is) \int \frac{1}{\sqrt{2x}} \int f(x) e^{iSx} dx = (-is)F(i)$
 $F\{f'(x)\} = (-is) \int \frac{1}{\sqrt{2x}} \int f(x) e^{iSx} dx = (-is)F(i)$
 $F\{f''(x)\} = \frac{1}{\sqrt{2x}} \int f'(s) e^{iSx} \int \frac{1}{-\infty} - \frac{1}{\sqrt{2x}} \int e^{iSn} (is)f(s)$
 $F\{f''(x)\} = (-is)F\{f'(x)\}$
 $F\{f''(x)\} = (-is)^2F(s)$ Using D.
In general,
 $F\{f''(x)\} = (-is)^2F(s)$ Using D.
 $F\{f''(x)\} = (-is)^2F(s)$ Using D.

Property 8:

$$Jf \in \{f(x)\} = E(x) \text{ then } \in \{\int_{a}^{x} f(x) dx\}; \frac{E(x)}{(-is)}$$
Proof > Let $\phi(x) = \int_{a}^{x} f(x) dx$.
 $\phi'(x) = f(x).$



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Alloo
$$F(p'(n)) = (-is)F(p(n))$$

 $= (-is)F(f(n))dn$
 $F(f(n)) = F(n)$
 $F(f(n))dn = F(n)$
 $= F(n)$
 $= F(n)$
 $= F(n)$