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TOPIC: 11 - Convolution theorem

Convolection Theorem. Def: Convolution of 2 functions, The convolution of f(n) and gens is defined by (Ixg) (m) = is S f(+)g(x+)dt convolution Theorem :-If for for and Fly (a) = G(s), the € { (f×g) (n)} = F (s) G(s) where ~ (f × g) (n) = 1 \$ f(+).g(x-t)dt, the Conduction of findand give. EgIA(B). verify convolution theorem for $f(n) = g(n) = e^{-n^2}$. Elaho: Griven frad=grad=e=312 KI. B.T. ESTCNDS= Tan Stands F{e-x2} = 1 ST SE e-ESN dr.



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$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}x + \frac{1}{2}sx} dx,$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(x - \frac{1}{2}s)^{2} + \frac{sy}{4}} dx,$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}x + \frac{1}{2}s} dx,$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{$$

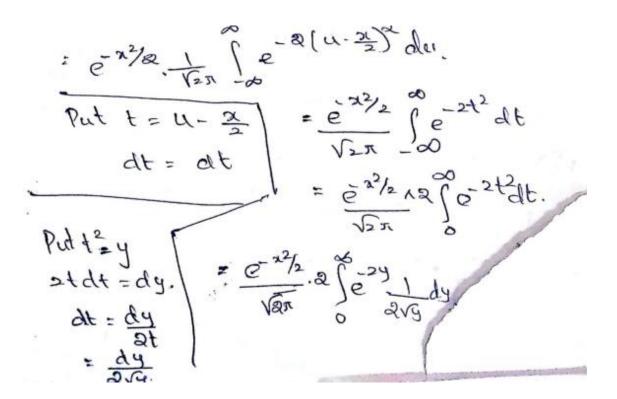
SNS COLLEGE OF ENGINEERING Coimbatore - 641 107 dt = ay = e 4 (Jar) je - " u 1/2 du = 24 1: Set 1- 4 = 百年(南).乐. F{f(m)} = ta e 12. $F \{ g(a) \} = F \{ f(a) \} = \sqrt{2} e^{-62/4}$ $-F\left\{f(n)\right\} \left\{-F\left\{g(n)\right\}\right\} = \frac{1}{2}e^{-\frac{r^2}{2}} \rightarrow 0$ -frad + grad = 1 frad gran - wider. e^{-x^2} , $e^{-x^2} = \frac{1}{\sqrt{2\pi}} \int e^{-x^2} e^{-(x-x)^2} dx$. $=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-(\chi-\eta)^{2}-\eta^{2}} d\eta = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-\chi^{2}} + 2\sqrt{2}\eta d\eta d\eta$ $= \frac{1}{\sqrt{6\pi}} \int_{0}^{-\chi^{2}} - \frac{2}{\sqrt{24\pi}} \frac{1}{\sqrt{24\pi}} \frac{1}{\sqrt{6\pi}} \int_{0}^{\infty} \frac{1}{\sqrt{6\pi}} \left(\frac{\pi^{2}}{\sqrt{6\pi}} - \frac{2}{\sqrt{6\pi}} \frac{1}{\sqrt{6\pi}} \frac{\pi^{2}}{\sqrt{6\pi}} - \frac{2}{\sqrt{6\pi}} \frac{1}{\sqrt{6\pi}} \frac{\pi^{2}}{\sqrt{6\pi}} - \frac{2}{\sqrt{6\pi}} \frac{1}{\sqrt{6\pi}} \frac{\pi^{2}}{\sqrt{6\pi}} \frac{1}{\sqrt{6\pi}} \frac{1}{\sqrt{6\pi}} \frac{\pi^{2}}{\sqrt{6\pi}} \frac{1}{\sqrt{6\pi}} \frac{1}{\sqrt{6\pi}} \frac{1}{\sqrt{6\pi}} \frac{\pi^{2}}{\sqrt{6\pi}} \frac{1}{\sqrt{6\pi}} \frac{1}$ $= \frac{1}{\sqrt{2\pi}} \int_{0}^{\infty} \frac{1}{2^2} \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \frac{1}$ = ton je - 2 [22 + (u - 2)2 - 22] du · to per 2[(4-2/3)] 22] dy =.

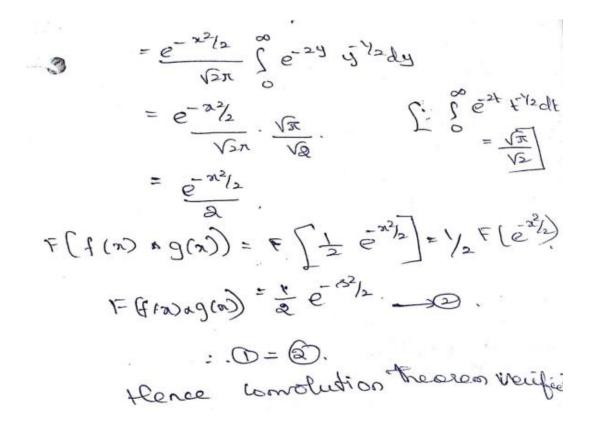


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Using Parsevals identity evaluate $\int_{0}^{\infty} \frac{dx}{(2^{2}+x^{2})^{2}} \xrightarrow{(1)} \int_{0}^{\infty} \frac{\partial x^{2}}{(\alpha^{2}+x^{2})^{2}} dn \quad i \neq \alpha > 0.$ Folm: $kl_{1} + 5 \cdot 7 \cdot i \neq f(\alpha) = e^{-\alpha x}$ $Hhen F_{S}(f(x_{2})) = \sqrt{\frac{2}{\pi}} \frac{S}{S^{2}+\alpha^{2}} = F(S) \longrightarrow 0.$ $F_{C}(f(\alpha_{2})) = \sqrt{\frac{2}{\pi}} \frac{\alpha}{S^{2}+\alpha^{2}} = F(S) \longrightarrow 0.$

$$H(c_{A_{0}} = f(-x_{1}): e^{-\alpha x},$$

$$\int_{0}^{\infty} [e^{-\alpha x}]^{2} dx : \int_{\infty}^{\infty} (\int_{\overline{x}}^{2} \frac{s}{s^{2} + \alpha^{2}})^{2} ds. \quad uniq 0.$$

$$\partial \int_{0}^{\infty} e^{-2\alpha x} dx : \frac{2}{\pi} \int_{-\infty}^{\infty} \frac{s^{2}}{(s^{2} + \alpha^{2})^{2}} ds.$$

$$\int_{0}^{\infty} \frac{e^{-2\alpha x}}{-2\alpha} \int_{0}^{\infty} \frac{s^{2}}{(s^{2} + \alpha^{2})^{2}} ds.$$

$$\int_{0}^{\infty} \frac{e^{-2\alpha x}}{-2\alpha} \int_{0}^{\infty} \frac{s^{2}}{(s^{2} + \alpha^{2})^{2}} ds.$$

$$\int_{0}^{\infty} \frac{x^{2}}{(x^{2} + \alpha^{2})^{2}} dx = \frac{\pi}{A}.$$

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(i).
$$\int_{-\infty}^{\infty} |f(x)|^2 dx = \int_{-\infty}^{\infty} |f(x)|^2 dx$$
.
Here $f(x) = e^{-\alpha x}$.
 $\int_{-\infty}^{\infty} |e^{-\alpha x}|^2 dx = \int_{-\infty}^{\infty} (\sqrt{\frac{2}{\pi}} \cdot \frac{\alpha}{\alpha^2 + s^2})^2 dx$ using (2)
 $\frac{1}{\alpha} = \frac{2}{\pi} \cdot 2 \int_{0}^{\infty} \frac{\alpha^2}{(s^2 + \alpha^2)^2} dx$.
 $\int_{0}^{\infty} \frac{\alpha x}{(s^2 + \alpha^2)^2} dx \ge \frac{\pi}{\alpha^3}$,