

Next extended Euclidean algorithm

$$\text{gcd}(a, b) = ax + by$$

$$3 = 93x + 219y$$

$$3 = \text{as } 27 - (6 \times 4)$$

$$\text{hence } 3 = 27 - 4 \times 6$$

$$\text{sub } 6 \text{ as } 33 - (1 \times 27)$$

$$\Rightarrow 3 = 27 - 4 \times (33 - (1 \times 27))$$

$$3 = 27 - 4 \times (33 - 1 \times 27) \quad \text{sub } 27 \text{ as } 93 - (2 \times 33)$$

$$= 93 - (2 \times 33) - 4 \times 33 + (5 \times 27)$$

$$= -4 \times 33 + 5 \times (93 - 2 \times 33)$$

$$= -4 \times 33 + 5 \times 93$$

$$= -4 \times (219 - 2 \times 93) + 5 \times 93$$

$$= 33(93) + (-14)(219)$$

$$\text{hence } x = 33 \quad y = -14$$

$$\begin{aligned} 12x + 4y &= 2 \\ 3 \times 4y &= 2 \\ y &= \end{aligned}$$

$$\begin{aligned} &= 3 \text{ as } 27 - (6 \times 4) \\ &= \end{aligned}$$

$$1 = 0$$

$$3 = 219y$$

$$3 / 219 = 0.013$$

$$2 = 93x + 219(0.013)$$

$$2 = 93x + 2.847$$

$$93x = 2 - 2.847$$

$$x = 0.153 / 93$$

Matrices:-

In cryptography, we need to handle matrices. Here we use linear algebra that is used in cryptography.

$[2 \ 1 \ 5 \ 11]$	$\begin{bmatrix} 2 \\ 4 \\ 5 \end{bmatrix}$	$\begin{bmatrix} 2 & 3 & 5 \\ 14 & 10 & 11 \\ 30 & 90 & 6 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
Row matrix	column matrix	Square matrix	Zero matrix	Inverse matrix

Product of row matrix (1×3) by (3×1) will be (1×1)

$$\begin{aligned} \text{Ex: } [5 \ 2 \ 1] \begin{bmatrix} 7 \\ 8 \\ 2 \end{bmatrix} &\Rightarrow (5 \times 7) + (2 \times 8) + (1 \times 2) \\ &= 35 + 16 + 2 \\ &= \underline{\underline{53}} \end{aligned}$$

Product of 2×3 by 3×4 will be 2×4 .

$$\text{Scalar} = 3 \begin{bmatrix} 5 & 2 & 1 \\ 3 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 15 & 6 & 3 \\ 9 & 6 & 3 \end{bmatrix}$$

Determinant is for square matrix.

$$\begin{bmatrix} 5 & 2 \\ 3 & 9 \end{bmatrix} = (5 \times 9) - (3 \times 2) \\ = 20 - 6 \\ = \underline{14} \rightarrow \text{determinant.}$$

$$\begin{bmatrix} 5 & 2 & 1 \\ 3 & 0 & -4 \\ 2 & 1 & 6 \end{bmatrix} = 5 \begin{bmatrix} 0 & -4 \\ 1 & 6 \end{bmatrix} + 2 \begin{bmatrix} 3 & -4 \\ 2 & 6 \end{bmatrix} + 1 \begin{bmatrix} 3 & 0 \\ 2 & 1 \end{bmatrix}$$

$$= 5[0 + 4] + 2[18 + 8] + 1[3 - 0]$$

$$= 5(+4) + 2(26) + 3$$

$$= +20 + 46 + 3$$

$$= \underline{3 \ 69}$$

Galois Fields

$\mathbb{GF}(p)$ is set of integers $\{0, 1, \dots, p-1\}$ with arithmetic operations modulo prime p .

This is finite field.

modular polynomial arithmetic:-

Finite field $\mathbb{GF}(p)$

For given prime p , we have define $\mathbb{GF}(p)$ as the set \mathbb{Z}_p of integers $\{0, 1, \dots, p-1\}$ together with the arithmetic operations modulo p .

Addition

$$f(x) = a_2x^2 + a_1x + a_0$$

$$g(x) = b_1x + b_0$$

$$\begin{aligned} f(x) + g(x) &= (a_2x^2 + a_1x + a_0) + (b_1x + b_0) \\ &= a_2x^2 + (a_1 + b_1)x + a_0 + b_0. \end{aligned}$$

Finite fields are represented by coefficients z_p such as given as GF(p) notations.

GF(p) where p is prime number.

Addition:

$$f(x) = 5x^2 + 4x + 6$$

$$g(x) = 5x + 6$$

$$\begin{aligned} f(x) + g(x) &= [(5x^2 + 4x + 6) + (5x + 6)] \pmod{7} \\ &= (5x^2 + 9x + 12) \pmod{7} \end{aligned}$$

$$f(x) + g(x) = 5x^2 + 2x + 5$$

Subtraction:

$$f(x) - g(x) = [(5x^2 + 4x + 6) - (5x + 6)] \pmod{7}$$

$$= (5x^2 + 4x + 6 - 5x - 6) \pmod{7}$$

$$= [5x^2 - x] \pmod{7}$$

$$f(x) - g(x) = 5x^2 + 6$$

Multiplication:

$$f(x) * g(x) = [(5x^2 + 4x + 6) * (5x + 6)] \pmod{7}$$

$$= (25x^3 + 30x^2 + 20x^2 + 24x + 30x + 36) \pmod{7}$$

$$= (25x^3 + 50x^2 + 54x + 36) \pmod{7}$$

$$f(x) * g(x) = 4x^3 + x^2 + 5x + 1$$

Division

$$f(x) = 5x^2 + 4x + 6$$

$$g(x) = 2x + 1$$

$$\begin{array}{r} 2x+1 \overline{) 5x^2+4x+6} \\ \underline{5x^2+4x+6} \end{array}$$

$$5/2 = 5 \times (2^{-1}) \pmod{7}$$

$$= 5 \times 4 \pmod{7}$$

$$= 20 \pmod{7}$$

$$= 6.$$

first quotient = 6x

To prove

$$(2x+1)(6x) \pmod{7}$$

$$= (12x^2 + 6x) \pmod{7}$$

$$= \underline{\underline{5x^2 + 6x}} \pmod{7}.$$

Step 2: Now subtract $5x^2 + 6x$ from $5x^2 + 4x + 6$

$$(5x^2 + 6x) - (5x^2 + 4x + 6) \pmod{7}$$

$$= (5x^2 + 4x + 6) - (5x^2 + 6x) \pmod{7}$$

$$= (5x^2 + 4x + 6 - 5x^2 - 6x) \pmod{7}$$

$$= (4x - 6x + 6) \pmod{7}$$

$$= (-2x + 6) \pmod{7}$$

$$= \underline{\underline{5x + 6}}$$

Step 3: divd $5x + 6$ with $g(x)$

$$\begin{array}{r} 2x+1 \overline{) 5x+6} \end{array}$$

$$5/2 = \underline{\underline{6x}}$$

$$\text{second quotient} = 6$$

$$\therefore 6x + 6$$

To prove:

$$\begin{aligned} & (6x + 6)(2x + 1) \pmod{7} = f(x) \\ &= (6x + 6)(2x + 1) \\ &= (12x^2 + 6x + 12x + 6) \pmod{7} \\ &= (12x^2 + 18x + 6) \pmod{7} \\ &= 5x^2 + 4x + 6 = f(x) \end{aligned}$$

Hence proved.

Block cipher Principles

A stream cipher - encrypts stream of data to convert to the ciphertext.

A Block cipher - encrypts a block of data.

It is based on three principles.

- Number of rounds ✓
- Design of function ✓
- Key schedule algorithm ✓

Number of rounds:

The number of rounds judges the strength of Block cipher algorithm. When there is more number of rounds, it is difficult for cryptanalysis to break the algorithm.

Even if the function is weak, number of rounds would make the algorithm tough to break.

Design of function F:

Function F of the block cipher, such that is impossible for cryptanalysis to unscramble the situation.

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