Euler's totient function

This is also caled as phi function.

Euler's totient function denoted as $\phi(n)$.

$\phi(n)$ = number of positive integers less than 'n' that are relatively prime to n.

Ex: Find $\phi(n) = \phi(5)$

n=5 here.

Numbers that are less than 5 are 4, 3, 2 & 1.

GCD	Relatively prime.
GCD (1,5)=1	yes
GCD(2,5)=1	
GCD(3,5)=1	
GCD(4,5)=1	

$\boxed{\text{if GCD is 1 then two numbers are relatively prime}}$

There are 4 no. less than '5' are relatively prime.

$\therefore \phi(5) = 4$

Ex: find $\phi(8)$:

$n = 8$, No less than 8 = 7, 6, 5, 4, 3, 2, 1.

GCD	Relatively prime
$\gcd(1,8) = 1$	✓
$\gcd(2,8) = 2$	✗
$\gcd(3,8) = 1$	✓
$\gcd(4,8) = 4$	✗
$\gcd(5,8) = 1$	✓
$\gcd(6,8) = 2$	✗
$\gcd(7,8) = 1$.	✓

$\therefore \phi(8) = 4$

This is easy for only smaller numbers.

Formula for larger numbers.

$\phi(n) =$ i) if n is prime then $\phi(n) = n-1$

ii) $n = p \times q$ $\qquad \phi(n) = (p-1)(q-1)$.

p & q are prime

iii) $n = a \times b$

a = either composite & b is composite

$\phi(n) = n \times \left(1 - \frac{1}{p_1}\right)\left(1 - \frac{1}{p_2}\right) \ldots$

Ex:- Find $\phi(5)$ = n = prime

$$\phi(n) = n-1 \Rightarrow \phi(n) = 5-1$$

$$\therefore \phi(5) = 4$$

ii) find $\phi(31)$ n = 31 = prime.

$$\phi(31) = 31-1 \Rightarrow 30 \quad \phi(31) = 30$$

iii) $\phi(35) = 5 \times 7$ p = 5 q = 7. 5 & 7 are prime

$$\phi(n) = (p-1)(q-1)$$

$$= 4 \times 6 = 24$$

$$\phi(35) = 24$$

iii) $\phi(1000)$ $n = 1000 \Rightarrow 2^3 \times 5^3$

2 & 5 are prime.

$$\phi(n) = n \times \left(1 - \frac{1}{p_1}\right)\left(1 - \frac{1}{p_2}\right)$$

$$= 1000\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{5}\right)$$

$$= 1000\left(\frac{1}{2}\right)\left(\frac{4}{5}\right)$$

$$= \frac{4000}{10} = 400$$

find $\phi(7000)$

$$\Rightarrow 2^3 \times 5^3 \times 7^1$$

$$\therefore \phi(n) = n \times \left(1 - \frac{1}{p_1}\right)\left(1 - \frac{1}{p_2}\right)\left(1 - \frac{1}{p_3}\right)$$

$$= 7000 \times \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{5}\right)\left(1 - \frac{1}{7}\right).$$

$$\phi(n) \Rightarrow 2400$$

$\phi(369)$ $\phi(372)$