Redesigning Common Mind & Business Towards Excellence

Build an Entrepreneurial Mindset Through Our Design Thinking FrameWork

SNS COLLEGE OF ENGINEERING

Kurumbapalayam (Po), Coimbatore – 641 107

An Autonomous Institution

Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A' Grade
Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

COURSE NAME : 23EET206 CONTROL SYSTEMS AND INSTRUMENTATION

II YEAR ECE /III SEMESTER

Unit 2- Time Domain and Frequency Domain Analysis

Topic 4 : Time Domain Analysis – Time Domain Specification

TIME DOMAIN SPECIFICATION

1. **Delay time** (t_d) is the time required to reach at 50% of its final value by a time response signal during its first cycle of oscillation.

Consider the step response of the second order system for t ≥ 0, when 'δ' lies between zero and one.

$$c(t) = 1 - \left(\frac{e^{-\delta\omega_n t}}{\sqrt{1-\delta^2}}\right)\sin(\omega_d t + \theta)$$

The final value of the step response is one.

Therefore, at $t = t_d$, the value of the step response will be 0.5. Substitute, these values in the above equation.

$$c(t_d) = 0.5 = 1 - \left(\frac{e^{-\delta\omega_n t_d}}{\sqrt{1-\delta^2}}\right)\sin(\omega_d t_d + \theta)$$

$$\Rightarrow \left(\frac{e^{-\delta\omega_n t_d}}{\sqrt{1-\delta^2}}\right)\sin(\omega_d t_d + \theta) = 0.5$$

$$t_d = \frac{1 + 0.7\delta}{\omega_n}$$

TIME DOMAIN SPECIFICATION

2. **Rise time** (t_r) is the time required to reach at final value by a under damped time response signal during its first cycle of oscillation. If the signal is over damped, then rise time is counted as the time required by the response to rise from 10% to 90% of its final value.

At $t = t_1 = 0$, $c(t) = 0$.

We know that the final value of the step response is one.

Therefore, at $t = t_2$, the value of step response is one. Substitute, these values in the following equation.

$$c(t) = 1 - \left(\frac{e^{-\delta\omega_n t}}{\sqrt{1-\delta^2}}\right)\sin(\omega_d t + \theta)$$

$$c(t_2) = 1 = 1 - \left(\frac{e^{-\delta\omega_n t_2}}{\sqrt{1-\delta^2}}\right)\sin(\omega_d t_2 + \theta)$$

$$\Rightarrow \left(\frac{e^{-\delta\omega_n t_2}}{\sqrt{1-\delta^2}}\right)\sin(\omega_d t_2 + \theta) = 0$$

$$\Rightarrow \sin(\omega_d t_2 + \theta) = 0$$

$$\Rightarrow \omega_d t_2 + \theta = \pi$$

$$\Rightarrow t_2 = \frac{\pi - \theta}{\omega_d}$$

Substitute t_1 and t_2 values in the following equation of **rise time**,

$$t_r = t_2 - t_1$$

$$\therefore\ t_r = \frac{\pi - \theta}{\omega_d}$$

TIME DOMAIN SPECIFICATION

3. **Peak time** (t_p) is simply the time required by response to reach its first peak i.e. the peak of first cycle of oscillation, or first overshoot.

We know the step response of second order system for under-damped case is

$$c(t) = 1 - \left(\frac{e^{-\delta\omega_n t}}{\sqrt{1-\delta^2}}\right)\sin(\omega_d t + \theta)$$

Differentiate $c(t)$ with respect to 't'.

$$\frac{\mathrm{dc}(t)}{\mathrm{dt}} = -\left(\frac{e^{-\delta\omega_n t}}{\sqrt{1-\delta^2}}\right)\omega_d\cos(\omega_d t + \theta) - \left(\frac{-\delta\omega_n e^{-\delta\omega_n t}}{\sqrt{1-\delta^2}}\right)\sin(\omega_d t + \theta)$$

Substitute, $t = t_p$ and $\frac{\mathrm{dc}(t)}{\mathrm{dt}} = 0$ in the above equation.

$$0 = -\left(\frac{e^{-\delta\omega_n t_p}}{\sqrt{1-\delta^2}}\right)\left[\omega_d\cos(\omega_d t_p + \theta) - \delta\omega_n\sin(\omega_d t_p + \theta)\right]$$

$$\Rightarrow \omega_n\sqrt{1-\delta^2}\cos(\omega_d t_p + \theta) - \delta\omega_n\sin(\omega_d t_p + \theta) = 0$$

$$\Rightarrow \sqrt{1-\delta^2}\cos(\omega_d t_p + \theta) - \delta\sin(\omega_d t_p + \theta) = 0$$

$$\Rightarrow \sin(\theta)\cos(\omega_d t_p + \theta) - \cos(\theta)\sin(\omega_d t_p + \theta) = 0$$

$$\Rightarrow \sin(\theta - \omega_d t_p - \theta) = 0$$

$$\Rightarrow sin(-\omega_d t_p) = 0 \Rightarrow -\sin(\omega_d t_p) = 0 \Rightarrow sin(\omega_d t_p) = 0$$

$$\Rightarrow \omega_d t_p = \pi$$

$$\Rightarrow t_p = \frac{\pi}{\omega_d}$$

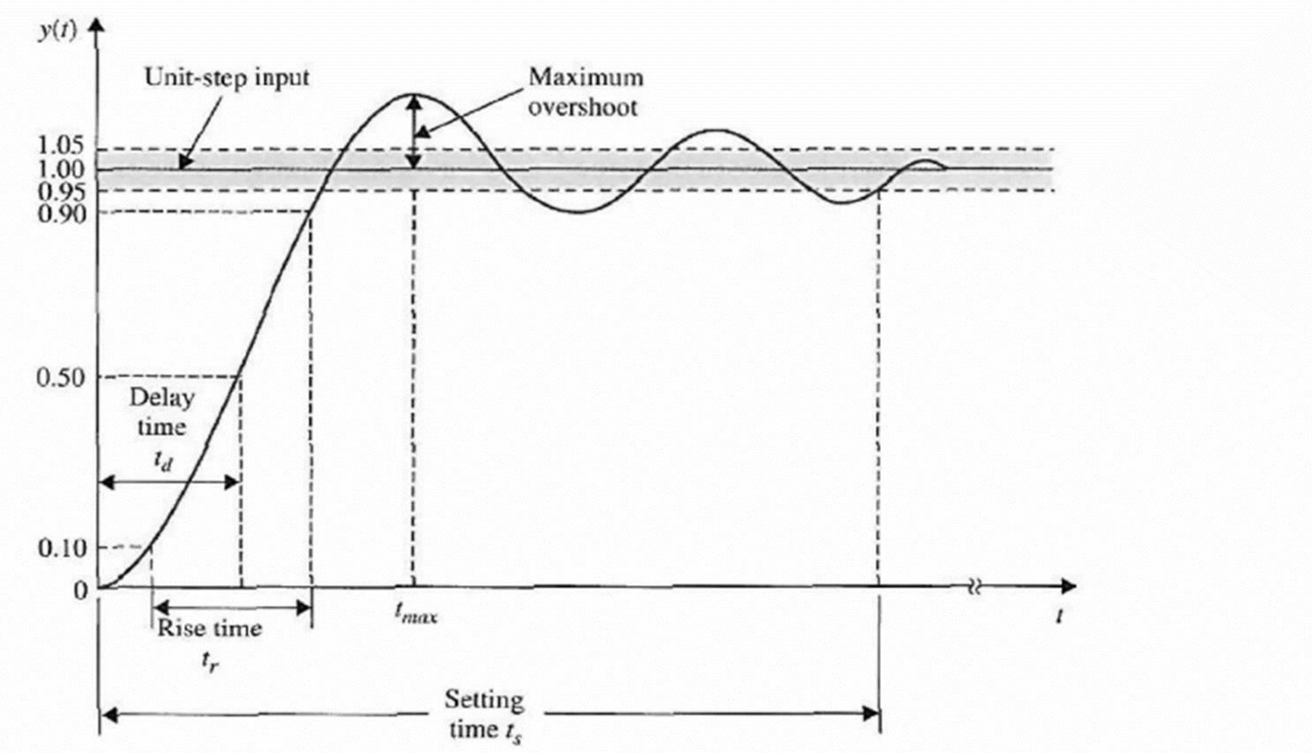

Figure 5-2 Typical unit-step response of a control system illustrating the time-domain specifications.

TIME DOMAIN SPECIFICATION

4. **Maximum overshoot** (M_p) is straight way difference between the magnitude of the highest peak of time response and magnitude of its steady state. Maximum overshoot is expressed in term of percentage of steady-state value of the response. As the first peak of response is normally maximum in magnitude, maximum overshoot is simply normalized difference between first peak and steady-state value of a response.

$$Maximum\ \%\ Overshoot = \frac{c(t_p) - c(\infty)}{c(\infty)} \times 100\%$$

At $t = t_p$, the response c(t) is -

$$c(t_p) = 1 - \left(\frac{e^{-\delta\omega_n t_p}}{\sqrt{1-\delta^2}}\right)\sin(\omega_d t_p + \theta)$$

Substitute, $t_p = \frac{\pi}{\omega_d}$ in the right hand side of the above equation.

$$c(t_P) = 1 - \left(\frac{e^{-\delta\omega_n\left(\frac{\pi}{\omega_d}\right)}}{\sqrt{1-\delta^2}}\right)\sin\left(\omega_d\left(\frac{\pi}{\omega_d}\right) + \theta\right)$$

TIME DOMAIN SPECIFICATION

Redesigning Common Mind & Business Towards Excellence

Build an Entrepreneurial Mindset Through Our Design Thinking FrameWork

$$\Rightarrow c(t_p) = 1 - \left(\frac{e^{-\left(\frac{\delta\pi}{\sqrt{1-\delta^2}}\right)}}{\sqrt{1-\delta^2}} \right) (-\sin(\theta))$$

We know that

$$\sin(\theta) = \sqrt{1-\delta^2}$$

So, we will get $c(t_p)$ as

$$c(t_p) = 1 + e^{-\left(\frac{\delta\pi}{\sqrt{1-\delta^2}}\right)}$$

Substitute the values of $c(t_p)$ and $c(\infty)$ in the peak overshoot equation.

$$M_p = 1 + e^{-\left(\frac{\delta\pi}{\sqrt{1-\delta^2}}\right)} - 1$$

$$\Rightarrow M_p = e^{-\left(\frac{\delta\pi}{\sqrt{1-\delta^2}}\right)}$$

TIME DOMAIN SPECIFICATION

Percentage of peak overshoot % M_p can be calculated by using this formula.

$$\%M_p = \frac{M_p}{c(\infty)} \times 100\%$$

By substituting the values of M_p and $c(\infty)$ in above formula, we will get the Percentage of the peak overshoot $\%M_p$ as

$$\%M_p = \left(e^{-\left(\frac{\delta\pi}{\sqrt{1-\delta^2}}\right)} \right) \times 100\%$$

TIME DOMAIN SPECIFICATION

Redesigning Common Mind & Business Towards Excellence

n Entrepreneurial Mindset Through Our Design Thinking FrameWork

5. **Settling time** (t_S) is the time required for a response to become steady. It is defined as the time required by the response to reach and steady within specified range of 2% to 5% of its final value.

The settling time for 5% tolerance band is -

$$t_s = \frac{3}{\delta\omega_n} = 3\tau$$

The settling time for 2% tolerance band is -

$$t_s = \frac{4}{\delta\omega_n} = 4\tau$$

Where, τ is the time constant and is equal to $\frac{1}{\delta\omega_n}$.

TIME DOMAIN SPECIFICATION

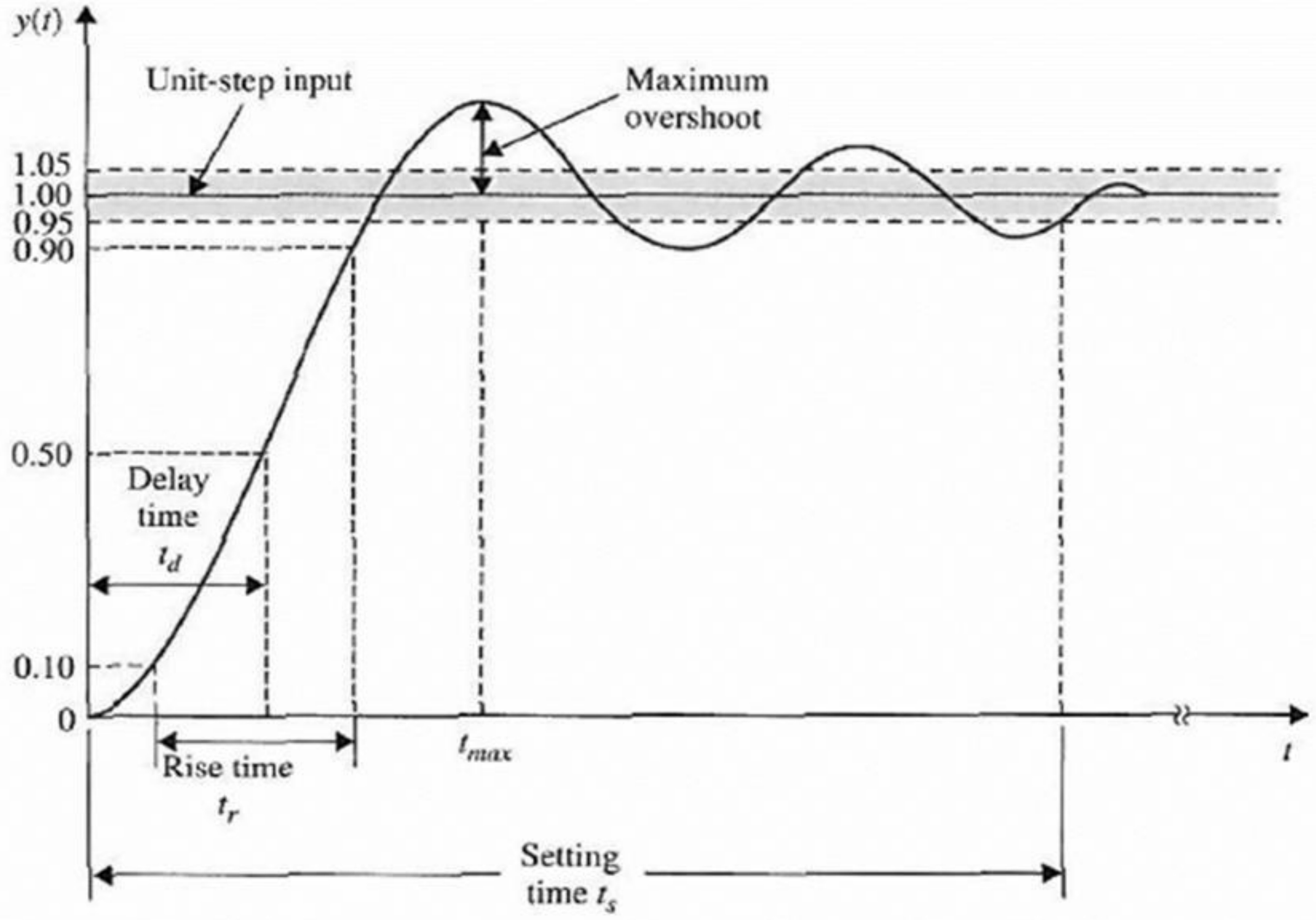

Figure 5-2 Typical unit-step response of a control system illustrating the time-domain specifications.

References

1. Nagrath, J., Gopal, M., “Control System Engineering”, New Age International Publishers, 7th Edition, 2021 (Unit I-III).
2. Benjamin.C.Kuo.,“Automatic Control Systems”, Prentice Hall of India, New Delhi, 9th Edition,2007 (Unit I-III).
3. Richard C. Dorf and Robert H. Bishop, “Modern Control Systems”, Addison, 12th Edition, 2010. (Unit I-III).
4. Katsuhiko Ogata, “Modern Control Engineering”, Prentice Hall of India, New Delhi, 5th Edition, 2009(Unit I-III).

Thank You