Topic: 1.10 –Problems On Reduction to Quadratic Form

$2x_1x_2 + 2x_1x_3 - 2x_2x_3$

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix}$$

$S_1 = 0$

$$S_2 = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$= (-1) + (-1) + (-1)$

$= -3$

$S_3 = 0 - 1(0+1) + (-1)$

$= -1 - 1$

$S_3 = -2$

$\lambda^3 - 3\lambda + 2 = 0$

$$\begin{array}{c|cccc} 1 & 1 & 0 & -3 & 2 \\ & 0 & 1 & 1 & -2 \\ \hline & 1 & 1 & -2 & 0 \end{array}$$

$\lambda^3 + \lambda^2 - 2 = 0$

$(\lambda - 1)(\lambda + 2)(\lambda + 1) = 0$

$\lambda = 1, 1, -2$

$(A - \lambda I)X = 0$

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} -\lambda & 1 & -1 \\ 1 & -\lambda & -1 \\ 1 & -1 & -\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$\lambda = 1$,

$$\begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$-x_1 + x_2 + x_3 = 0$

$x_1 - x_2 - x_3 = 0$

$x_1 - x_2 - x_3 = 0$

$x_1 = 0$

$x_2 = -x_3$

$\frac{x_2}{-1} = \frac{x_3}{1}$

$\lambda = -2$

$x_1 = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$

$x_2 = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$

Let $x_3 = \begin{bmatrix} l \\ m \\ n \end{bmatrix}$

$(l\ m\ n) \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} = 0$

$-l+m+n=0$

$X_3 X_2^T = 0$

$(l\ m\ n)\begin{pmatrix}0\\-1\\1\end{pmatrix}=0$

l	m	n
-1	1	-1
0	-1	1 0

$$\frac{l}{1+1} = \frac{m}{0+1} = \frac{n}{1}$$

$$\frac{1}{2} = \frac{m}{1} = \frac{n}{1}$$

$$X_3 = \begin{bmatrix}2\\1\\1\end{bmatrix}$$

$$N = \begin{bmatrix}-1 & 0 & 2\\1 & -1 & 1\\1 & 1 & 1\end{bmatrix}$$

$$N = \begin{bmatrix}-1/\sqrt{3} & 0 & 2/\sqrt{6}\\1/\sqrt{3} & -1/\sqrt{2} & 1/\sqrt{6}\\1/\sqrt{3} & 1/\sqrt{2} & 1/\sqrt{6}\end{bmatrix}$$

$$N^T = \begin{bmatrix}-1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3}\\0 & -1/\sqrt{2} & 1/\sqrt{2}\\2/\sqrt{6} & 1/\sqrt{6} & 1/\sqrt{6}\end{bmatrix}$$

$$AN = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} -1/\sqrt{3} & 0 & 2/\sqrt{6} \\ 1/\sqrt{3} & -1/\sqrt{2} & 1/\sqrt{6} \\ 1/\sqrt{3} & 1/\sqrt{2} & 1/\sqrt{6} \end{bmatrix}$$

$$= \begin{bmatrix} 2/\sqrt{3} & 0 & 2/\sqrt{6} \\ -2/\sqrt{3} & -1/\sqrt{2} & 1/\sqrt{6} \\ -2/\sqrt{3} & 1/\sqrt{2} & 1/\sqrt{6} \end{bmatrix}$$

$$ANN^T = \begin{bmatrix} -1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} \\ 2/\sqrt{6} & 1/\sqrt{6} & 1/\sqrt{6} \end{bmatrix} \begin{bmatrix} 2/\sqrt{3} & 0 & 2/\sqrt{3} \\ -2/\sqrt{3} & 1/\sqrt{2} & 1/6 \\ -2/\sqrt{6} & 1/\sqrt{2} & 1/6 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$C.F := -2y_1^2 + y_2^2 + y_3^2 = 0$$

(2) $2x_1^2 + x_2^2 + x_3^2 + 2x_1x_2 - 2x_1x_3 - 4x_2x_3$

$$A = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 1 & -2 \\ -1 & -2 & 1 \end{bmatrix}$$

Charact. equ is $\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$

$S_1 = 4$

$$S_2 = \begin{vmatrix} 1 & -2 \\ -2 & 1 \end{vmatrix} + \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} + \begin{vmatrix} 2 & -1 \\ -1 & 1 \end{vmatrix}$$

$$= (1-4) + (2-1) + 2(-1)$$

$$= -3 + 1 + 1 = -1$$

$S_3 = 2(1-4) - 1(1-2) - 1(-2+1)$

$= -6+1+1$

$= -4$

$\lambda^3 - 4\lambda^2 - \lambda + 4 = 0$

$\boxed{\lambda = 1}$

$1 - 4 - 1 + 4 = 0$

1	1	-4	-1	4
	0	1	-3	-4
	1	-3	-4	0

$(\lambda - 1)(\lambda^2 - 3\lambda - 4)$

$(\lambda - 1)(\lambda - 4)(\lambda + 1)$

$\lambda = 1, -1, 4$

$\boxed{\lambda = 1}$

$$\begin{pmatrix} 2-\lambda & 1 & -1 \\ 1 & 1-\lambda & -2 \\ -1 & -2 & 1-\lambda \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$(2-\lambda)x_1 + x_2 - x_3 = 0$

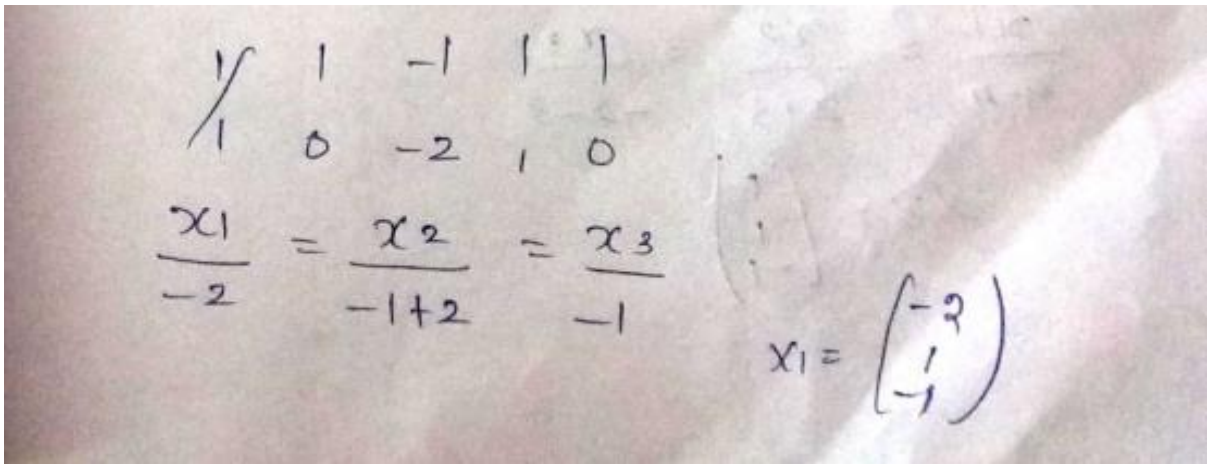
$x_1 + (1-\lambda)x_2 - 2x_3 = 0$

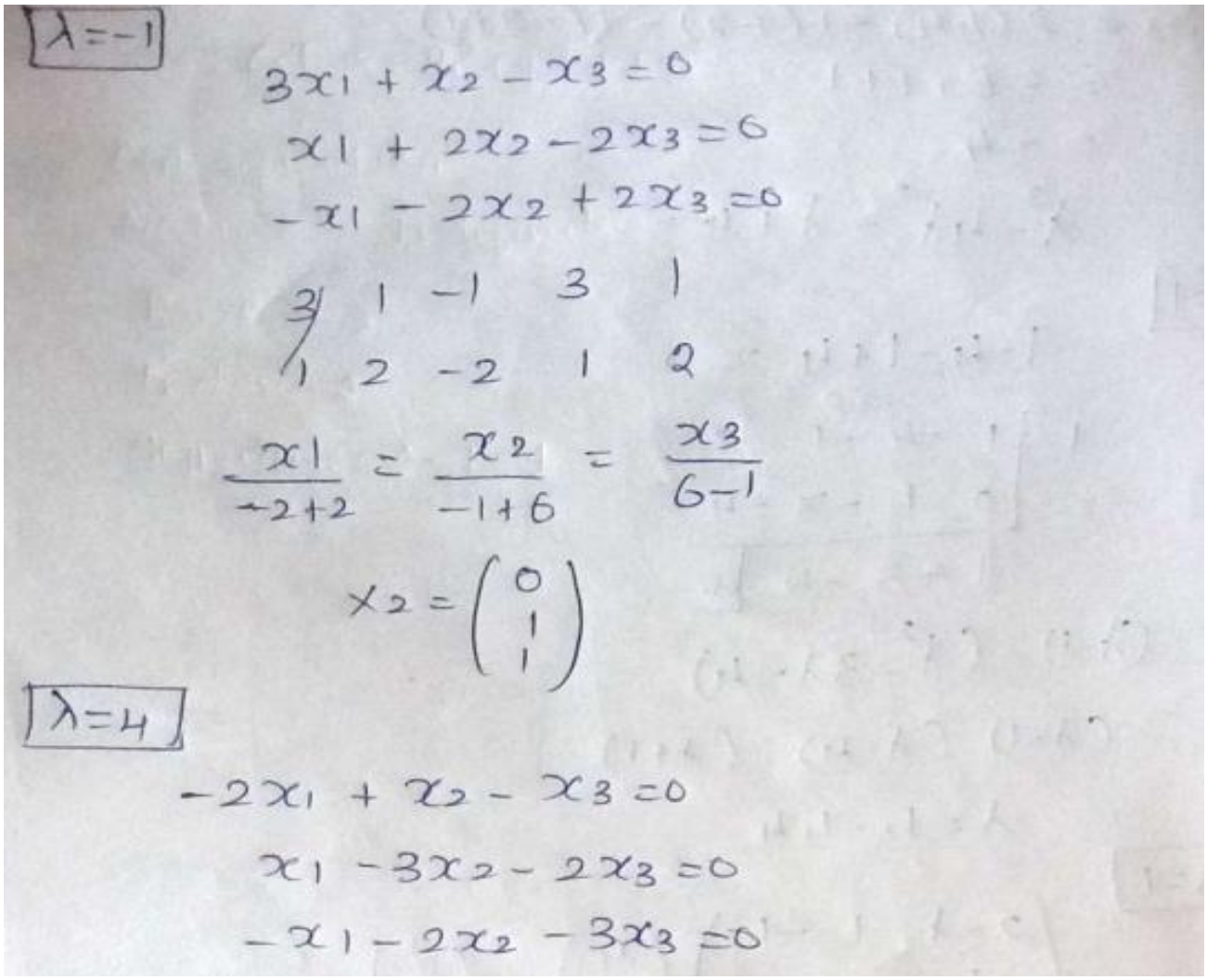
$-x_1 - 2x_2 + x_3(1-\lambda) = 0$

$x_1 + x_2 - x_3 = 0$

$x_1 + 0x_2 - 2x_3 = 0$

$-x_1 - 2x_2 + 0x_3 = 0$


1 -1 1 1
1 0 -2 0
x1/-2 = x2/-1+2 = x3/-1
X1 = (-2, 1, -1)


λ=-1
3x1 + x2 - x3 = 0
x1 + 2x2 - 2x3 = 0
-x1 - 2x2 + 2x3 = 0
3 1 -1 3 1
1 2 -2 1 2
x1/-2+2 = x2/-1+6 = x3/6-1
X2 = (0, 1, 1)
λ=4
-2x1 + x2 - x3 = 0
x1 - 3x2 - 2x3 = 0
-x1 - 2x2 - 3x3 = 0

$$\begin{array}{cccc} -2 & 1 & -1 & -2 \\ 1 & -3 & -2 & 1 \end{array} \quad \begin{array}{c} 1 \\ -3 \end{array}$$

$$\frac{x_1}{-2-3} = \frac{x_2}{-1-2} = \frac{x_3}{6-1}$$

$$\frac{x_1}{-5} = \frac{x_2}{-3} = \frac{x_3}{5}$$

$$\begin{array}{cccc} -3 & -2 & 1 & -3 \\ -1 & -2 & -3 & -1 \end{array} \quad \begin{array}{c} -2 \\ \end{array}$$

$$\frac{x_1}{9-4} = \frac{x_2}{2+3} = \frac{x_3}{-2-3}$$

$$X_3 = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$

$$N = \begin{bmatrix} 0 & 2 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$

$$N = \begin{bmatrix} 0/\sqrt{2} & 2/\sqrt{6} & 1/\sqrt{3} \\ 1/\sqrt{2} & -1/\sqrt{6} & 1/\sqrt{3} \\ 1/\sqrt{2} & 1/\sqrt{6} & -1/\sqrt{3} \end{bmatrix}$$

$$N^T = \begin{bmatrix} 0/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \\ 2/\sqrt{6} & -1/\sqrt{6} & 1/\sqrt{6} \\ 1/\sqrt{3} & 1/\sqrt{3} & -1/\sqrt{3} \end{bmatrix}$$

$$AN = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 1 & -2 \\ -1 & -2 & 1 \end{bmatrix} \begin{bmatrix} 0/\sqrt{2} & 2/\sqrt{6} & 1/\sqrt{6} \\ 1/\sqrt{2} & -1/\sqrt{6} & 1/\sqrt{3} \\ 1/\sqrt{2} & 1/\sqrt{6} & -1/\sqrt{3} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{0+1-1}{\sqrt{2}} & \frac{4-1-1}{\sqrt{6}} & \frac{2+1+1}{\sqrt{3}} \\ \frac{0+1-2}{\sqrt{2}} & \frac{2-1-2}{\sqrt{6}} & \frac{1+1+2}{\sqrt{3}} \\ \frac{0-2+1}{\sqrt{2}} & \frac{-2+2+1}{\sqrt{6}} & \frac{-1-2-1}{\sqrt{3}} \end{bmatrix}$$

$$AN = \begin{bmatrix} 0 & 2/\sqrt{6} & 4/\sqrt{3} \\ -1/\sqrt{3} & -1/\sqrt{6} & 4/\sqrt{3} \\ -1/\sqrt{2} & 1/\sqrt{6} & -4/\sqrt{3} \end{bmatrix}$$

$$N^T AN = \begin{pmatrix} 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 2/\sqrt{6} & -1/\sqrt{6} & 1/\sqrt{6} \\ 1/\sqrt{3} & 1/\sqrt{3} & -1/\sqrt{3} \end{pmatrix} \begin{pmatrix} 0 & 2/\sqrt{6} & 4/\sqrt{3} \\ -1/\sqrt{2} & -1/\sqrt{6} & 4/\sqrt{3} \\ -1/\sqrt{2} & 1/\sqrt{6} & -4/\sqrt{3} \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$