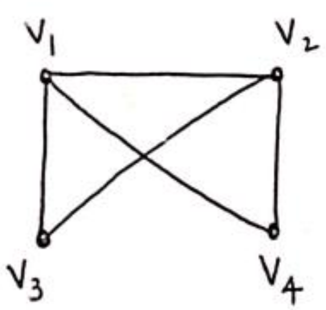
Adjacency Matrix

Let $G(V,E)$ be a simple graph with n vertices ordered from v_1 to v_n, then the adjacency matrix $A = [a_{ij}]_{n\times n}$ of G is an $n \times n$ symmetric matrix defined by the elements

$$a_{ij} = \begin{cases} 1 & \text{when } v_i \text{ is adjacent to } v_j \\ 0 & \text{otherwise} \end{cases}$$

It is denoted by A_G or $A(G)$.



$$A_G = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

Properties of Adjacency Matrix

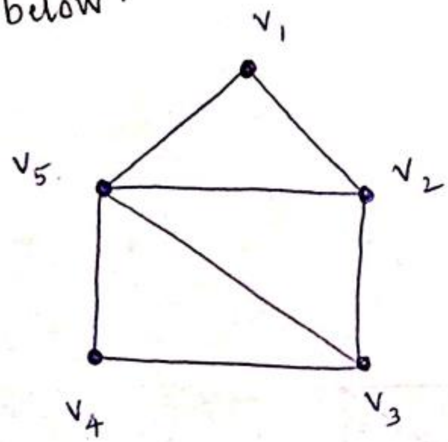
(a) An Adjacency matrix completely defines a simple graph.

(b) The Adjacency matrix is symmetric.

(c) Any element of the Adjacency matrix is either 0 or 1. Therefore it is also called as, bit matrix or boolean matrix.

(d) G is null $\Leftrightarrow$ $A(G)$ is the zero matrix of order n.

1) Obtain the adjacency matrix of the graph given below.



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The Adjacency matrix

$$
A(G) = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array}
\begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\
\begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}
$$

2.

Draw the graph G whose incidence matrix is given below :

$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 \end{array} \\ \left[\begin{array}{ccccccc} 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{array} \right] \end{array}$$