Euler Graph

Euler Path

A path of a graph G is called an Eulerian path, if it contains each edge of the graph exactly once.

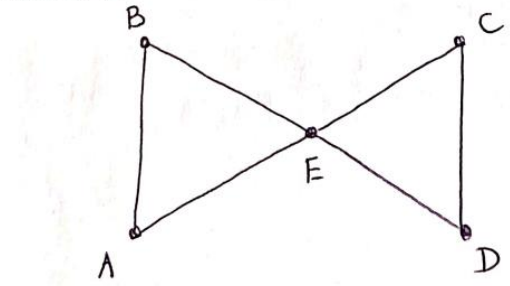
Eulerian Circuit or Eulerian Cycle

A circuit or cycle of a graph G is called an Eulerian circuit or cycle, if it includes each edge of G exactly once.

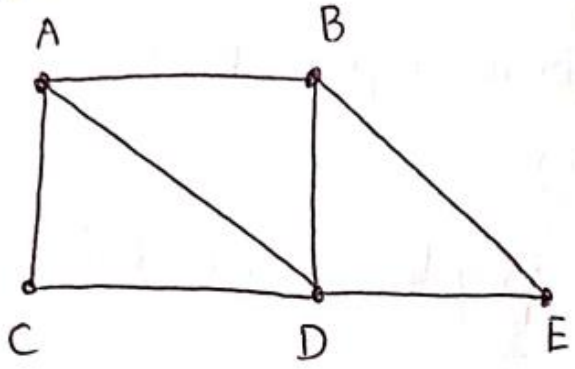
Eulerian Graph

Any graph containing an Eulerian circuit or cycle is called an Eulerian graph.

Example



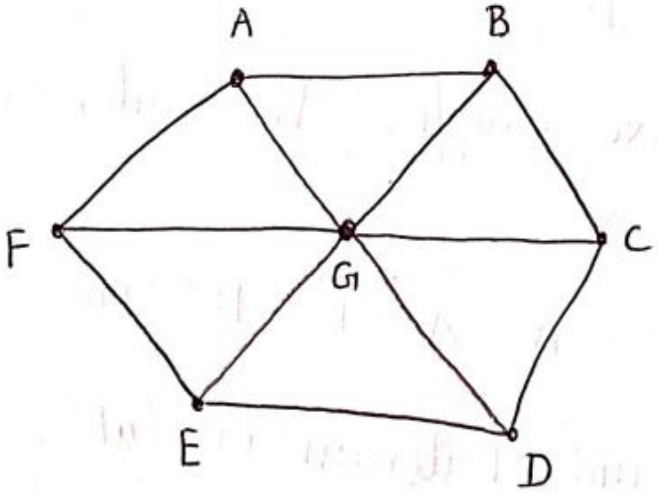
(1) Find an Euler path or Euler circuit, if it exists in each of the three graphs below.



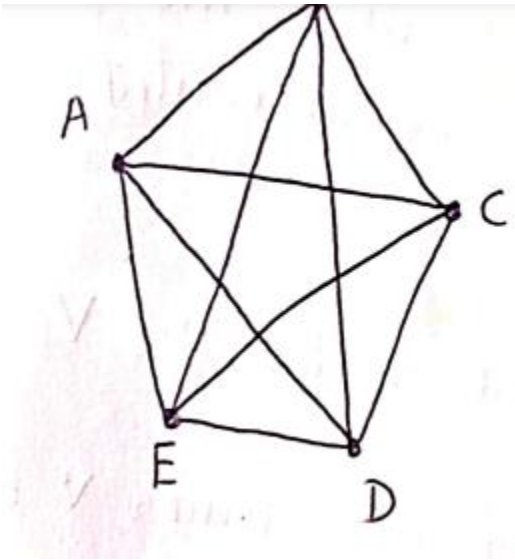
Euler path : A - C - D - A - B - E - D - B

we can not find Euler circuit.

∴ given graph is non-Eulerian. (deg(A) = 3 not even)



Given graph is non-Eulerian (deg(A) = odd)



Euler circuit : A - B - C - D - E - A - C - E - B - D - A

∴ Given graph is an Eulerian graph.
(All vertices are even)

Hamiltonian Path

A path of a graph G is called a Hamiltonian path, if it includes each vertex of G exactly once.

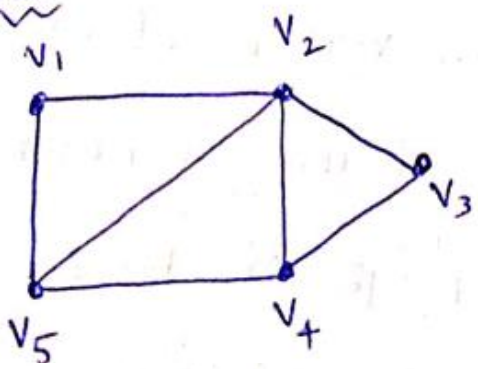
Hamiltonian Circuit

A circuit of a graph G is called a Hamiltonian circuit, if it includes each vertex of G exactly once, except the starting and ending vertices.

Hamiltonian Graph

Any graph containing a Hamiltonian circuit is called a Hamiltonian graph.

Example



Hamiltonian Path : $v_1 - v_2 - v_3 - v_4 - v_5$

Given an example of a graph which is

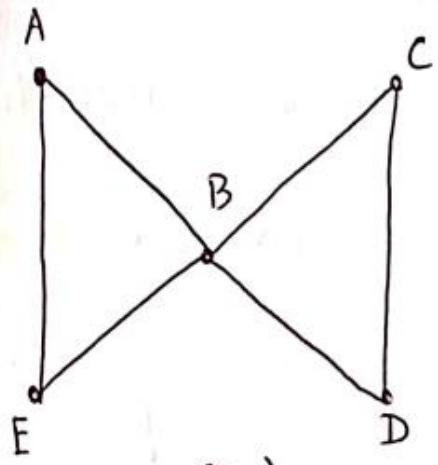
i) Eulerian but not Hamiltonian

ii) Hamiltonian but not Eulerian

iii) Both Eulerian and Hamiltonian

iv) neither Eulerian nor Hamiltonian.

(i)



(G_1)

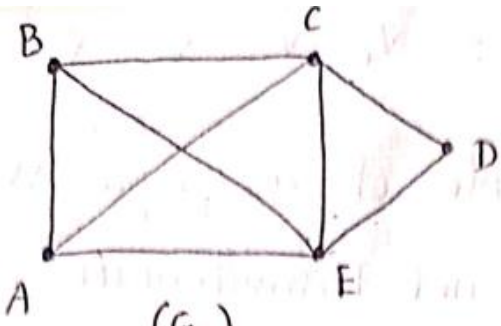
The degrees of each vertices of G_1 are even.

$\therefore$ G_1 is Euler graph. (A - B - C - D - B - E - A Euler circuit).

We can not find Hamiltonian cycle as the vertex B is repealed twice.

$\therefore$ G_1 is not a Hamiltonian graph.

(ii)



(G_2)

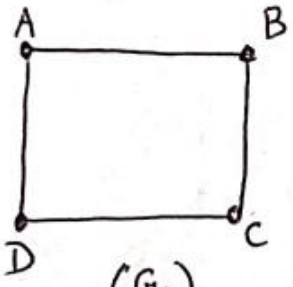
Since the degree of the vertex A is 3 & (not even)

$\therefore$ G_2 is not an Euler graph.

Since G_2 contains the Hamiltonian cycle, name

A - B - C - D - E - A. $\therefore$ G_2 is Hamiltonian Gra

(iii)



(G_3)

Since G_3 contains the Hamiltonian cycle, namly

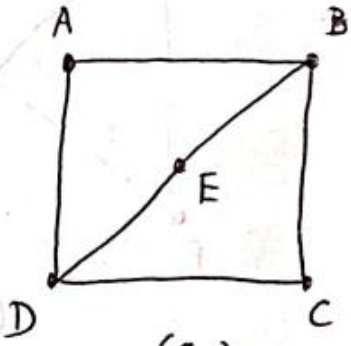
A - B - C - D - A

$\therefore$ G_3 is Hamiltonian graph

since all the vertices are of even degree.

$\therefore$ G_3 is Eulerian

(iv)



(G_4)

Since deg. of B and D ar not even numbers.

$\therefore$ G_4 is not an Euler grap

As no cycle passes throu each of the vertices exactly