SNS COLLEGE OF ENGINEERING
COIMBATORE-641 107
ENGINEERING MATHEMATICS-I

UNIT I
MATRICES
PART A

Remember:

1. Find the sum and product of all the eigen values of $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$.
2. Given A=$\begin{pmatrix} -1 & 0 & 0 \\ 2 & -3 & 0 \\ 1 & 4 & 2 \end{pmatrix}$. Find the eigen values of A^2.
3. Find the eigen values of A^{-1} where A=$\begin{pmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{pmatrix}$.
4. Find the eigen values of the inverse of the matrix A=$\begin{pmatrix} 2 & 1 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & 4 \end{pmatrix}$.
5. If 3 and 6 are the two eigen values of A=$\begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$, write down all the eigen values of A^{-1}.
6. The product of two eigen values of $\begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$ is 16.Find the third eigen value of A.

Understand:

7. State Cayley-Hamilton theorem
8. Can A=$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ be diagonalized? Why?

Apply:

9. Using Cayley Hamilton theorem to find ($A^4-4A^3-5A^2+A+2I$) when A= $\begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$.
10. Check whether the matrix B is orthogonal? Justify. B = $\begin{pmatrix} cos\theta & sin\theta & 0 \\ -sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

PART-B

Remember:

1. Find the characteristic equation of the matrix A given $A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$. Hence find A^{-1} and A^4.
2. If $A = \begin{pmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{pmatrix}$, verify Cayley-Hamilton theorem and hence find A^{-1}.
3. Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix}$.

Apply:

1. Reduce the quadratic form $x^2 + 5y^2 + z^2 + 2xy + 2yz + 6zx$ to canonical form and hence find its rank.
2. Reduce the quadratic form $6x^2 + 3y^2 + 3z^2 - 4xy - 2yz + 4zx$ into canonical form by an orthogonal reduction. Hence find its rank and nature.
3. Reduce the quadratic form $10x_1^2 + 2x_2^2 + 5x_3^2 + 6x_2x_3 - 4x_3x_1 - 4x_1x_2$ to a canonical form through an orthogonal transformation and hence find rank, index, signature, nature and also give non-zero set of values for x_1, x_2, x_3(if they exist), that will make the quadratic form zero.

Analyze

4. Show that the matrix $\begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & 0 \\ 2 & 0 & 3 \end{pmatrix}$ satisfies the characteristic equation and hence find its inverse
5. Verify Cayley-Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & -2 & 3 \\ 2 & 4 & -2 \\ -1 & 1 & 2 \end{pmatrix}$

UNIT II
SEQUENCE AND SERIES
PART A

Remember:

1. Define limit of a sequence.
2. Describe convergence sequence.
3. Find the nature of the series 1+2+3+4+........+n+...
4. Define Bounded sequence
5. Define oscillating sequence with example.
6. Define monotonic sequence

Understand:

7. Distinguish between sequence and series.

8. Give an example for decreasing and increasing sequence.
9. State the necessary conditions for convergence.
10. Using comparison test, prove that the series $\frac{1}{1.2}+\frac{2}{3.5}+\frac{3}{5.7}+\frac{4}{7.9}+\cdots$ is divergent.

Analyse:

11. Test the convergence of the series$\sum_{n=1}^{\infty}\frac{1}{n^2+1}$
12. Test the convergence for the sequence $\left\{\frac{1}{n}\right\}_{n=1}^{n=\infty}$.
13. Test the convergence of the series 1- $\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\cdots\infty$
14. Test the convergence of the series $\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\cdots+\frac{1}{n!}$.
15. Test whether the series $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n-1}$ is convergent or not

PART-B

Remember:

1. Find the nature of the series $\sum_{n=2}^{\infty}\frac{1}{n(\log n)^p}$ by Cauchy's integral test.
2. Determine the convergence of the alternating series $\sum_{n=1}^{\infty}\frac{cosn\pi}{n^2+1}$ for absolute and conditional convergence.

Understand:

3. Discuss the convergence and divergence of the following series

$$\frac{1}{2^3}-\frac{1}{3^3}(1+2)+\frac{1}{4^3}(1+2+3)-\frac{1}{5^3}(1+2+3+4).........$$

Apply:

4. Show that the direct summation of **n** terms that the series $\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\cdots$ is convergent.
5. Using comparison test, examine the convergence of the series

$$\frac{1}{1.2.3}+\frac{3}{2.3.4}+\frac{5}{3.4.5}+....\infty \; .$$

6. Using D' Alembert's ratio test, examine the convergence or divergence of the series

Analyse:

7. Test the convergence of the series $\frac{1}{4\cdot7\cdot10}+\frac{4}{7\cdot10\cdot13}+\frac{9}{10\cdot13\cdot16}+\cdots\cdots$
8. Test the convergence of the series$\sum_{n=0}^{\infty}ne^{-n^2}$.
9. Test the convergence of the series $1+\frac{3}{7}x+\frac{3.6}{7.10}x^2+\frac{3.6.9}{7.10.13}x^3+.........$

   by D' Alembert's ratio test
10. Test the convergence of the series $1+\frac{2^p}{2!}+\frac{3^p}{3!}+\frac{4^p}{4!}+\cdots$ by D' Alembert's ratio test.

UNIT-III
APPLICATIONS OF DIFFERENTIAL CALCULUS
PART A

Remember:

1) For the centenary $y = c\cosh\frac{x}{c}$, find the curvature.
2) Find the radius of curvature for $y=e^x$ at the point where it cuts the y-axis.
3) Define circle of curvature.
4) Find the centre of curvature for $y=x^2$ at the origin.
5) Find the curvature of the curve $2x^2+2y^2+x-2y+1=0$.
6) Find the envelope of the family of straight lines $y = mx + \frac{1}{m}$, where m is a parameter.
7) Find the envelope of the lines $\frac{x}{t} + yt = 2c$, t being a parameter.
8) Find the envelope of the straight lines $x\cos\theta+y\sin\theta=\alpha$ where θ is the parameter.

PART-B

Remember:

1. Find the radius of curvature of the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at $\left(\frac{a}{4},\frac{a}{4}\right)$.
2. Find the equation of the circle of curvature at $\left(\frac{a}{4},\frac{a}{4}\right)$ on $\sqrt{x} + \sqrt{y} = \sqrt{a}$.
3. Find the equation of the circle of curvature of the parabola $y^2 = 12x$ at the point (3,6).
4. Find the equation of the circle of curvature of the rectangular hyperbola xy=12 at the point (3,4).
5. Find the radius of curvature at the point (0,c) on the curve $y = c\cosh\frac{x}{c}$.
6. the point $\left(\frac{3a}{2},\frac{3a}{2}\right)$ on the curve $x^3+y^3=3axy$.
7. Find the radius of the curvature of the curve $x^3+xy^2-6y^2=0$ at (3,3).
8. Find the evolute of the cycloid $x = a(\theta - \sin\theta), y = a(1-\cos\theta)$.
9. Find the evolute of $\sqrt{x} + \sqrt{y} = \sqrt{a}$.

Apply:

10. Prove that the radius of the curve $xy^2 = a^3 - x^3$ at the point (a,0) is $\frac{3a}{2}$.
11. If $y = \frac{ax}{a+x}$, prove that $\left(\frac{2\rho}{a}\right)^{2/3} = \left(\frac{x}{y}\right)^2 + \left(\frac{y}{x}\right)^2$, where ρ is the radius of curvature.
12. Show that the evolutes of the parabola $y^2 = 4ax$ is the curve $27ay^2 = 4(x-2a)^3$

UNIT-IV

DIFFERENTIAL CALCULUS OF SEVERAL VARIABLES
PART A

Remember:

11. If $u = \frac{x}{y} + \frac{y}{z} + \frac{z}{x}$, find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$
12. Find $\frac{\partial U}{\partial x}$ and $\frac{\partial u}{\partial y}$ if $u=y^x$.
13. Given $u(x,y) = x^2 \tan^{-1}\left(\frac{y}{x}\right)$, find the value of $x^2u_{xx} + 2xyu_{xy} + y^2u_{yy}$.
14. If $u = f(y-z, z-x, x-y)$, $find \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$.
15. If $u = x^2 + y^2$ and $x = at^2, y = 2at$ find $\frac{du}{dt}$.
16. If $u = x^3y^2 + x^2y^3$ where $x = at^2, y = 2at$ then find $\frac{du}{dt}$.
17. Find $\frac{du}{dt}$ if u=sin(x/y), where $x=e^t$, $y=t^2$.
18. If $x^y+y^x=1$, then find $\frac{dy}{dx}$.

Understand:

19. State the conditions for maxima minima of f(x,y).

Apply:

20. Using Eulers theorem, given u(x,y) is a homogeneous function of degree n, prove that $x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} = n(n-1)u.$
21. If $u= x^y$, show that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$.
22. Using the definition of total derivative, find the value of $\frac{du}{dt} given u = y^2 - 4ax; x = at^2, y = 2at.$

PART-B

Remember:

1. If $u = \log(\tan x + \tan y + \tan z)$, find $\sum \sin 2x \frac{\partial u}{\partial x}$.
2. If u=xy+yz+zx where $x=\frac{1}{t}$, $y=e^t$ and $z=e^{-t}$ find $\frac{dy}{dt}$.
3. If $u = (x-y)f\left(\frac{y}{x}\right)$, then find $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$.

4. Find the Taylor expansion of $x^2y^2 + 2x^2y + 3xy^2$ in powers of (x+2) and (y-1) up to 3rd degree terms.
5. Find the Taylor series expansion of $e^x \sin y$ at the point (1, π/4) up to 3rd degree terms.
6. Find the maximum value of $x^m y^n z^p$ subject to the condition $x + y + z = a$.
7. Find the extreme value of $x^2 + y^2 + z^2$ subject to the condition $x + y + z = 3a$.
8. A rectangular box open at the top , is to have a volume of 32cc. Find the dimensions of the box, that requires the least materials for its construction.
9. A rectangular box open at the top , is to have the capacity of 108 cu.ms. Find the dimensions of the box, requiring the least materials for its construction.
10. Find the dimensions of the rectangular box, open at the top, of maximum capacity whose surface area is 432 square meter.

Understand:

11. Discuss the maxima minima of the function $f(x, y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2$.
12. Discuss the maxima and minima of $f(x, y) = x^3y^2(1 - x - y)$.

Apply:

13. If $u=x^y$, show that $u_{xxy}=u_{xyx}$.
14. If $u = \log(x^2 + y^2) + \tan^{-1}\left(\frac{y}{x}\right)$ prove that $u_{xx} + u_{yy} = 0$.
15. $w = f(y - z, z - x, x - y)$, then show that $\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z} = 0$.
16. If $z = f(x, y)$, where $x = u^2 - v^2, y = 2uv$, prove that

$$\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2} = 4(u^2 + v^2)\left(\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}\right).$$

17. If $x = u\cos\alpha - v\sin\alpha, y = u\sin\alpha + v\cos\alpha$ and $v = f(x, y)$, show that

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = \frac{\partial^2 v}{\partial u^2} + \frac{\partial^2 v}{\partial v^2}.$$

18. Use Taylor formula to expand the function defined by $f(x, y) = x^3 + y^3 + xy^2$ in powers of (x-1) and (y-2).
19. Expand $x^2y + 3y - 2$ in powers of (x-1) and (y+2) up to 3rd degree terms.
20. Expand $e^x \sin y$ in powers of x and y as far as the terms of the 3rd degree using Taylor expansion.

Analyze:

21. Test for maxima minima for the function $f(x,y)=x^3+y^3-12x-3y+20$.
22. Test for an extrema of the function $f(x,y)=x^4+y^4-x^2-y^2-1$.
23. Examine the function $f(x,y)=x^3y^2(12-x-y)$ for extreme values.
24. Test for maxima minima of the function $f(x,y)=x^3y^2(6-x-y)$.

UNIT V
PART A(Question Bank)

1. Solve the equation $(D^2-6D+13)y=0$.
2. Solve $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=0$
3. Solve $(D^3+D^2+4D+4)y=0$
4. Find the PI of $(D^2-9)y= e^{-3x}$.
5. Find the PI of $(D^2+4)y = \sin 2x$
6. Find the PI of $(D^2+2D+1)y = e^{-x}\cos x$.
7. Solve the equation $x^2\frac{d^2y}{dx^2}+4x\frac{dy}{dx}+2y = 0$
8. Solve $x^2y^{11}-xy^1+y=0$

PART-B(Question Bank)

1. Solve the equation $(D^2+a^2)y = \sec ax$ by the method of variation of parameters
2. Solve $\frac{d^2y}{dx^2}+4y= \tan 2x$ by the method of variation of parameters
3. Solve $\frac{d^2y}{dx^2}+y= \operatorname{cosec} x$ by the method of variation of parameters
4. Solve $(D^2+1)y = x\sin x$ by the method of variation of parameters
5. Solve the equation $(D^2-3D+2)y = 2\cos(2x+3)+2e^{x}$.
6. Solve $(D^2-4D+3)y = \cos 2x+2x^2$.
7. Solve $(D^2+3D+2)y = \sin 2x+x^2$.
8. Solve the equation $(D^2+5D+4)y = e^{-x}\sin 2x$
9. Solve $(D^2+2D+5)y = e^{-x}x^2$.
10. Solve the equation $(D^2+4)y=x^2\cos 2x$.
11. Solve $(x^2D^2-2xD-4)y=x^2+2\log x$ by Euler's(Cauchy's) method
12. Solve $(x^2D^2-3xD+4)y= x^2\cos(\log x)$ by Euler's(Cauchy's) method