TOPIC:7- Normal Subgroups

Normal Subgroup

Let H be a subgroup of G under $*$. Then H is said to be a normal subgroup of G, for every $x \in G$ and for $h \in H$, if.

$$x * h * x^{-1} \in H$$

$$x * H * x^{-1} \subseteq H$$

Theorem

Prove that the intersection of two normal subgroups of a group is a normal subgroup.

Given H and K are normal subgroups.

$\Rightarrow$ H and K are subgroups of G.

$\Rightarrow$ $H \cap K$ is a subgroup of G.

Now we have to prove that $H \cap K$ is normal

Let $x \in G$ and $h \in H \cap K$.

$\Rightarrow x \in G$ and $h \in H$ and $h \in K$

$x \in G$, $h \in H$ and $x \in G$ and, $h \in K$

$x * h * x^{-1} \in H \rightarrow (1)$

and $x * h * x^{-1} \in K \rightarrow (2)$

[H & K are normal subgp.]

From (1) & (2),

$x * h * x^{-1} \in H \cap K$

$\Rightarrow H \cap K$ is a normal subgroup of G.

Theorem

Let $f : G \rightarrow G'$ be a homomorphism of grou with Kernel K. Then prove that K is a norma subgroup of G.

Given 'e' is an identity in G and 'e'' i an identity in G'.

Let $K = Ker(f) = \{x \in G \mid f(x) = e'\}$

WKT K is a subgroup of G.

Now we have to prove that K is normal.

Let $x \in G$ and $h \in K$.

$$f(x * h * x^{-1}) = f(x) * f(h) * f(x^{-1}) \quad [f \text{ is homo.}]$$

$$= f(x) * e' * f(x^{-1}) \quad [h \in K]$$

$$= f(x) * f(x^{-1})$$

$$= f(x * x^{-1}) \quad [f \text{ is homo.}]$$

$$= f(e) = e'$$

$\Rightarrow x * h * x^{-1} \in K$.

$\therefore$ K is a normal subgroup of G.