TOPIC:10- Lagrange's theorem

Lagrange's Theorem

Let G be a finite group of order 'n' and H be any subgroup of G. Then the order of H divides the order of G. i.e) $O(H) / O(G)$.

$O(G) = n$.

Let $(H, *)$ be a subgroup of G whose order is 'm'. $O(H) = m$.

Let $h_1, h_2, \ldots, h_m$ be the 'm' different elements of H.

The right coset $H * a$ of H in G is defined by

$$H * a = \{h_1 * a, h_2 * a, \ldots, h_m * a\}, \; a \in G.$$

Since there is a one-one correspondence between the elements of H and $H * a$, the elements of $H * a$ are distinct.

Hence each right coset of H in G has 'm' distinct elements.

WKT any right cosets of H in G are either disjoint or identical.

The number of distinct right cosets of H in G is finite (say k).

The union of these K distinct cosets of H in G is equal to G.

Let these K distinct right cosets be

$H * a_1, H * a_2, \ldots, H * a_K$.

Then $G = (H * a_1) \cup (H * a_2) \cup \cdots \cup (H * a_K)$

$\therefore O(G) = O(H * a_1) + O(H * a_2) + \cdots + O(H * a_K)$

$n = m + m + \cdots + m$ (K times)

$n = Km$

$\Rightarrow K = \frac{n}{m}$ (i.e.) $\frac{O(G)}{O(H)} = K$

$\Rightarrow O(H)$ is a divisor of $O(G)$

$\Rightarrow O(H)$ divides $O(G)$.

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