TOPIC: 5.5-PROBLEMS ON METHOD OF VARIATION OF PARAMETERS

Solve $(D^2+a^2)y = \sec ax$ using Methods of Variation of parameters.

Given $(D^2+a^2)y = \sec ax$

A.E: $m^2+a^2=0$

$m^2 = -a^2$

$m = \pm ai$

$C.F = C_1 \cos ax + C_2 \sin ax.$

$f_1 = \cos ax \qquad f_2 = \sin ax$

$f_1' = -a\sin ax \qquad f_2' = a\cos ax$

$f_1 f_2' - f_2 f_1' = a\cos ax \cos ax + \sin ax \, a \sin ax = a$

$y = C.F + P.I.$

$$P.I = Pf_1 + Qf_2$$

$$P = -\int \frac{f_2 x}{f_1 f_2' - f_2 f_1'}\,dx = -\int \frac{\sin ax \sec ax}{a}\,dx$$

$$= -\frac{1}{a}\int \tan ax\,dx = -\frac{1}{a}\left[-\frac{\log(\cos ax)}{a}\right]$$

$$P = \frac{1}{a^2}\log[\cos ax]$$

$$Q = \int \frac{f_1 x}{f_1 f_2' - f_2 f_1'}\,dx = \int \frac{\cos ax \sec ax}{a}\,dx$$

$$= \frac{1}{a}\int dx = \frac{1}{a}x.$$

$$y = C.F + P.I = C_1 \cos ax + C_2 \sin ax + \frac{1}{a^2}\log(\cos ax)[\cos ax] + \frac{1}{a}x \sin ax.$$

2) Solve $(D^2+a^2)y = \tan ax$

$(D^2+a^2)y = \tan ax$

$(m^2+a^2) = 0$

$m = \pm ai$

$C.F = A\cos ax + B\sin ax.$

$f_1 = \cos ax \qquad f_2 = \sin ax$

$f_1' = -a\sin ax \qquad f_2' = a\cos ax$

$f_1f_2' - f_2f_1' = a\cos^2 ax + a\sin^2 ax = a.$

$P.I = Pf_1 + Qf_2$

$$P = -\int \frac{f_2 X}{f_1f_2' - f_2f_1'}\,dx = -\int \frac{\sin ax(\tan ax)}{a}\,dx$$

$$= -\frac{1}{a}\int \frac{\sin^2 ax}{\cos ax}\,dx = -\frac{1}{a}\int \frac{1-\cos^2 ax}{\cos ax}\,dx.$$

$\because [\sin^2 ax = 1-\cos^2 ax.]$

$$= -\frac{1}{a}\int \sec ax + \frac{1}{a}\int \cos ax\,dx$$

$$= -\frac{1}{a}\cdot\frac{1}{a}\log(\sec ax + \tan ax) + \frac{1}{a}\cdot\frac{1}{a}\sin ax$$

$$P = -\frac{1}{a^2}\log[\sec ax + \tan ax] + \frac{1}{a^2}\sin ax.$$

$$Q = \int \frac{f_1 X}{f_1 f_2' - f_2 f_1'}\,dx = \int \frac{\cos ax \tan ax}{a}\,dx$$

$$= \frac{1}{a}\int \sin ax\,dx = -\frac{1}{a^2}\cos ax.$$

$$P.I = \cos ax\left[\frac{1}{a^2}\sin ax - \frac{1}{a^2}\log(\sec ax + \tan ax)\right] - \frac{\cos ax}{a^2}.$$

$$y = C.F + P.I.$$

$$= A\cos ax + B\sin ax + \cos ax\left[\frac{1}{a^2}\sin ax - \frac{1}{a^2}\log(\sec ax + \tan ax) - \frac{\cos ax}{a^2}\right]$$