



TOPIC: 5.5-PROBLEMS ON METHOD OF VARIATION OF PARAMETERS

Solve $(D^2 + a^2)y = \sec ax$ using methods of variation of parameters.

Given $(D^2 + a^2)y = \sec ax$

A.E: $m^2 + a^2 = 0$
 $m^2 = -a^2$
 $m = \pm ai$

C.F = $C_1 \cos ax + C_2 \sin ax$.

$f_1 = \cos ax$

$f_2 = \sin ax$

$f_1' = -a \sin ax$

$f_2' = a \cos ax$

$f_1 f_2' - f_2 f_1' = a \cos ax \cos ax + \sin ax a \sin ax = a$

$y = C.F + P.I.$



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$$P.I = P f_1 + Q f_2$$

$$P = \int \frac{f_2 x}{f_1 f_2' - f_2 f_1'} dx = - \int \frac{\sin ax \sec ax}{a} dx$$

$$= -\frac{1}{a} \int \tan ax dx = -\frac{1}{a} \left[-\frac{\log(\cos ax)}{a} \right]$$

$$P = \frac{1}{a^2} \log[\cos ax]$$

$$Q = \int \frac{f_1 x}{f_1 f_2' - f_2 f_1'} dx = \int \frac{\cos ax \sec ax}{a} dx$$

$$= \frac{1}{a} \int dx = \frac{1}{a} x$$

$$y = C.F + P.I = C_1 \cos ax + C_2 \sin ax + \frac{1}{a^2} \log(\cos ax) + \frac{1}{a} x \sin ax$$



2) solve $(D^2 + a^2)y = \tan ax$

$$(D^2 + a^2)y = \tan ax$$

$$(m^2 + a^2) = 0$$

$$m = \pm ai$$

$$C.F = A \cos ax + B \sin ax.$$

$$f_1 = \cos ax \quad f_2 = \sin ax$$

$$f_1' = -a \sin ax \quad f_2' = a \cos ax$$

$$f_1 f_2' - f_2 f_1' = a \cos^2 ax + a \sin^2 ax = a.$$

$$P.I = P f_1 + Q f_2$$

$$P = - \int \frac{f_2 x}{f_1 f_2' - f_2 f_1'} dx = - \int \frac{\sin ax (\tan ax) dx}{a}$$

$$= -\frac{1}{a} \int \frac{\sin^2 ax}{\cos ax} dx = -\frac{1}{a} \int \frac{1 - \cos^2 ax}{\cos ax} dx.$$

$$\begin{aligned} \therefore (\sin^2 ax = 1 - \cos^2 ax) &= -\frac{1}{a} \int \sec ax + \frac{1}{a} \int \cos ax dx \\ &= -\frac{1}{a} \cdot \frac{1}{a} \log(\sec ax + \tan ax) + \frac{1}{a} \cdot \frac{1}{a} \sin ax \end{aligned}$$



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$$P = -\frac{1}{a^2} \log(\sec ax + \tan ax) + \frac{1}{a^2} \sin ax.$$

$$Q = \int \frac{f_1 X}{f_1 f_2' - f_2 f_1'} dx = \int \frac{\cos ax \tan ax}{a} dx$$

$$= \frac{1}{a} \int \sin ax dx = -\frac{1}{a^2} \cos ax.$$

$$P \cdot I = \cos ax \left[\frac{1}{a^2} \sin ax - \frac{1}{a^2} \log(\sec ax + \tan ax) \right] - \frac{\cos ax}{a^2}.$$

$$y = C.F + P.I.$$

$$= A \cos ax + B \sin ax + \cos ax$$

$$\left[\frac{1}{a^2} \sin ax - \frac{1}{a^2} \log(\sec ax + \tan ax) - \frac{\cos ax}{a^2} \right]$$