UNIT I
MATRICES
PART A

Remember:

1. Find the sum and product of all the eigen values of $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$.
2. Given A=$\begin{pmatrix} -1 & 0 & 0 \\ 2 & -3 & 0 \\ 1 & 4 & 2 \end{pmatrix}$. Find the eigen values of A^2.
3. Find the eigen values of A^{-1} where A=$\begin{pmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{pmatrix}$.
4. Find the eigen values of the inverse of the matrix A=$\begin{pmatrix} 2 & 1 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & 4 \end{pmatrix}$.
5. If 3 and 6 are the two eigen values of A=$\begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$, write down all the eigen values of A^{-1}.
6. The product of two eigen values of $\begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$ is 16.Find the third eigen value of A.
7. If 1 and 2 are the eigenvalues of a 2 X 2 matrix A, what are the eigenvalues of A^2 and A^{-1}?
8. If 2, 3 are the eigen values of $\begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ b & 0 & 2 \end{pmatrix}$, find the value of b.
9. If 2, -1, -3 are the eigen values of the matrix A, find the eigen values of the matrix A^2-2I.
10. If the sum of two eigen values of a matrix and trace of a 3x3 matrix are equal, find the value of $|A|$.
11. If the eigen values of the matrix A of order 3x3 2, 3 and 1, the find the eigen values of adjoint of A.
12. If λ is the eigen value of the matrix A, then prove that λ^2 is the eigen value of A^2.
13. Write down the Quadratic form corresponding to the matrix A=$\begin{bmatrix} 0 & 5 & -1 \\ 5 & 1 & 6 \\ -1 & 6 & 2 \end{bmatrix}$.
14. Find the nature of the quadratic form $x_1^2 + 2x_2^2 + x_3^2 - 2x_1x_2 + 2x_2x_3$.
15. Write down the matrix of the quadratic form $2x^2+8z^2+4xy+10xz-2yz$.
16. Find the nature of the quadratic form whose matrix is $\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$.
17. Find the symmetric matrix A, whose eigen values are 1 and 3 with corresponding eigen vectors $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

Understand:

18. State Cayley-Hamilton theorem
19. Can A=$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ be diagonalized? Why?

Apply:

20. Using Cayley Hamilton theorem to find (A^4-4A^3-5A^2+A+2I) when A= $\begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$.
21. Check whether the matrix B is orthogonal? Justify. B = $\begin{pmatrix} cos\theta & sin\theta & 0 \\ -sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

PART-B

Remember:

1. Find the characteristic equation of the matrix A given $A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$.

   Hence find A^{-1} and A^4.
2. If $A = \begin{pmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{pmatrix}$, verify Cayley-Hamilton theorem and hence find A^{-1}.
3. Find A^n using Cayley-Hamilton theorem, taking $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$. Hence find A^3.
4. Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{bmatrix}$
5. Find the eigen values and eigen vectors of $\begin{bmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & -1 \end{bmatrix}$.
6. Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$.
7. Find the eigen values and eigen vectors of $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$.
8. Find the eigen values and eigen vectors of $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$.
9. Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix}$.
10. Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$.

11. Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$.

12. Find the eigen values and eigen vectors of $\begin{bmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{bmatrix}$.

13. Find the eigen vectors of a 3x3 real symmetric matrix A corresponding to the eigen values 2,3,6 are $(1, 0, -1)^T$, $(1, 1, 1)^T$ and $(-1, 2, -1)^T$ respectively. Find the matrix A.

14. If the eigen values of $A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$ are 0, 3, 15, find the eigen vectors of A and diagonalize the matrix A.

15. Find a change of variables that reduces the quadratic form $3x_1^2 + 5x_2^2 + 3x_3^2 - 2x_1x_2 + 2x_1x_3 - 2x_2x_3$ to a sum of squares and express the quadratic form in terms of new variables.

Apply:

1. Using Cayley-Hamilton theorem find the inverse of $A = \begin{pmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{pmatrix}$
2. Using Cayley-Hamilton theorem find the inverse of $A = \begin{pmatrix} -1 & 0 & 3 \\ 8 & 1 & 7 \\ -3 & 0 & 8 \end{pmatrix}$.
3. Using Cayley-Hamilton theorem find A^{-1} and A^4, if $A = \begin{pmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{pmatrix}$.
4. Using Cayley-Hamilton theorem find the value of the matrix given by $A^8-5A^7+7A^6-3A^5+A^4-5A^3+8A^2-2A+I$, if the matrix $A = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{pmatrix}$.
5. Reduce the quadratic form $2x^2 + 5y^2 + 3z^2 + 4xy$ to canonical form by orthogonal reduction and state its nature.
6. Reduce the quadratic form $2x_1x_2 + 2x_1x_3 - 2x_2x_3$ to canonical form by orthogonal reduction. Also find its nature.
7. Reduce the given quadratic for Q to its Canonical form using orthogonal transformation. $Q = x^2 + 3y^2 + 3z^2 - 2yz$.
8. Reduce the quadratic form $2x_1x_2 + 2x_2x_3 + 2x_3x_1$ into canonical form.
9. Reduce the quadratic form $x_1^2 + 5x_2^2 + x_3^2 + 2x_1x_2 + 2x_2x_3 + 6x_3x_1$ to the canonical form through orthogonal transformation and find its nature.

10. Reduce the quadratic form $x^2 + 5y^2 + z^2 + 2xy + 2yz + 6zx$ to canonical form and hence find its rank.
11. Reduce the quadratic form $2x_1^2 + x_2^2 + x_3^2 + 2x_1x_2 - 2x_1x_3 - 4x_2x_3$ to canonical form by an orthogonal transformation. Also find the rank, index, signature and nature of the quadratic form.
12. Reduce the quadratic form $3x^2 + 5y^2 + 3z^2 - 2xy - 2yz + 2zx$ into canonical form by orthogonal transformation.
13. Reduce the quadratic form $8x_1^2 + 7x_2^2 + 3x_3^2 - 12x_1x_2 - 8x_2x_3 + 4x_3x_1$ into canonical form by means of an orthogonal transformation.
14. Reduce the quadratic form $6x^2 + 3y^2 + 3z^2 - 4xy - 2yz + 4zx$ into canonical form by an orthogonal reduction. Hence find its rank and nature.
15. Reduce the quadratic form $10x_1^2 + 2x_2^2 + 5x_3^2 + 6x_2x_3 - 4x_3x_1 - 4x_1x_2$ to a canonical form through an orthogonal transformation and hence find rank, index, signature, nature and also give non-zero set of values for x_1, x_2, x_3(if they exist), that will make the quadratic form zero.
16. Reduce the quadratic form $x_1^2 + 2x_2^2 + x_3^2 - 2x_1x_2 + 2x_2x_3$ to a canonical form through an orthogonal transformation and show that is positive semi definite. Also given a non-zero set of values (x_1, x_2, x_3) which makes this quadratic form zero.

Analyze

17. Show that the matrix $\begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & 0 \\ 2 & 0 & 3 \end{pmatrix}$ satisfies the characteristic equation and hence find its inverse
18. Verify Cayley-Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & -2 & 3 \\ 2 & 4 & -2 \\ -1 & 1 & 2 \end{pmatrix}$
19. Verify Cayley-Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}$, find its A^{-1}.
20. Verify Cayley-Hamilton theorem for the matrix $A = \begin{pmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{pmatrix}$ and hence find A^{-1} and A^4