SNS COLLEGE OF ENGINEERING

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AN AUTONOMOUS INSTITUTION

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23MAT101 - MATRICES AND CALCULUS

QUESTION BANK

UNIT-III

FUNCTIONS OF SEVERAL VARIABLES

PART A

Remember:

1. If $u = \frac{x}{y} + \frac{y}{z} + \frac{z}{x}$, find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$
2. Find $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$ if $u=y^x$.
3. Given $u(x,y) = x^2 \tan^{-1}\left(\frac{y}{x}\right)$, find the value of $x^2u_{xx} + 2xyu_{xy} + y^2u_{yy}$.
4. If $u = f(y-z, z-x, x-y)$, $find\ \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$.
5. If $u = x^2 + y^2$ and $x = at^2, y = 2at$ find $\frac{du}{dt}$.
6. If $u = x^3y^2 + x^2y^3$ where $x = at^2, y = 2at$ then find $\frac{du}{dt}$.
7. Find $\frac{du}{dt}$ if u=sin(x/y), where $x=e^t$, $y=t^2$.
8. If $x^y+y^x=1$, then find $\frac{dy}{dx}$.
9. If $u = \frac{y^2}{2x}, v = \frac{x^2+y^2}{2x}$, $find\ \frac{\partial(u,v)}{\partial(x,y)}$.
10. If $x=u^2-v^2$ and y=2uv, find the Jacobian of x and y with respect ot u and v.
11. If u=2xy, $v=x^2-y^2$, x=rcosθ, y=rsinθ then compute $\frac{\partial(u,v)}{\partial(r,\theta)}$.
12. If x=rcosθ, y=rsinθ then find $\frac{\partial(x,y)}{\partial(r,\theta)}$.
13. If x=rcosθ, y=rsinθ then find $\frac{\partial(r,\theta)}{\partial(x,y)}$.
14. Write the sufficient condition for f(x,y) to have a maximum value at (a,b).

Understand:

15. State the conditions for maxima minima of f(x,y).

Apply:

16. Using Eulers theorem, given u(x,y) is a homogeneous function of degree n, prove that $x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} = n(n-1)u.$
17. If u= x^y, show that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$.
18. Using the definition of total derivative, find the value of $\frac{du}{dt} given u = y^2 - 4ax; x = at^2, y = 2at.$

PART-B

Remember:

1. If $u = \log(\tan x + \tan y + \tan z)$, find $\sum \sin 2x \frac{\partial u}{\partial x}$.
2. If u=xy+yz+zx where x=$\frac{1}{t}$, y=e^t and z=e^{-t} find $\frac{dy}{dt}$.
3. If $u = (x-y)f\left(\frac{y}{x}\right)$, then find $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$.
4. Find the Taylor expansion of $x^2y^2 + 2x^2y + 3xy^2$ in powers of (x+2) and (y-1) up to 3rd degree terms.
5. Find the Taylor series expansion of $e^x \sin y$ at the point (1, π/4) up to 3rd degree terms.
6. Find the Taylor series expansion of $e^x \cos y$ in the neighborhood of the point (1, π/4) up to 3rd degree terms
7. Find the extreme values of the function $f(x, y) = x^3 + y^3 - 3x - 12y + 20$.
8. Find the maximum minimum values of $x^2 - xy + y^2 - 2x + y$.
9. Find the maximum value of $x^m y^n z^p$ subject to the condition $x + y + z = a$.
10. Find the extreme value of $x^2 + y^2 + z^2$ subject to the condition $x + y + z = 3a$.
11. A rectangular box open at the top , is to have a volume of 32cc. Find the dimensions of the box, that requires the least materials for its construction.
12. A rectangular box open at the top , is to have the capacity of 108 cu.ms. Find the dimensions of the box, requiring the least materials for its construction.

13. Find the dimensions of the rectangular box, open at the top, of maximum capacity whose surface area is 432 square meter.

14. Find the volume of the greatest rectangular parallelepiped inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.

15. Find the length of the shortest line from the point $\left(0,0,\frac{25}{9}\right)$ to the surface z=xy.

16. Find the Jacobian of $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)}$ of the transformation $x=r\sin\theta\cos\phi, y=r\sin\theta\sin\phi$ and $z=r\cos\theta$.

17. Find the Jacobian of $u=x+y+z, v=xy+yz+z, w=x^2+y^2+z^2$.

18. Find the Jacobian of y_1,y_2,y_3 with respect to x_1,x_2,x_3 if $y_1=\frac{x_2x_3}{x_1}, y_2=\frac{x_3x_1}{x_2}, y_3=\frac{x_1x_2}{x_3}$.

19. If $u=\frac{yz}{x}, v=\frac{zx}{y}, w=\frac{xy}{z}$, find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.

Understand:

20. Discuss the maxima minima of the function $f(x,y)=x^4+y^4-2x^2+4xy-2y^2$.

21. Discuss the maxima and minima of $f(x,y)=x^3y^2(1-x-y)$.

Apply:

22. If $u=x^y$, show that $u_{xxy}=u_{xyx}$.

23. If $u=\log(x^2+y^2)+\tan^{-1}\left(\frac{y}{x}\right)$ prove that $u_{xx}+u_{yy}=0$.

24. $w=f(y-z,z-x,x-y)$, then show that $\frac{\partial w}{\partial x}+\frac{\partial w}{\partial y}+\frac{\partial w}{\partial z}=0$.

25. If $z=f(x,y)$, where $x=u^2-v^2, y=2uv$, prove that $\frac{\partial^2 z}{\partial u^2}+\frac{\partial^2 z}{\partial v^2}=4(u^2+v^2)\left(\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}\right)$.

26. If $x=u\cos\alpha-v\sin\alpha, y=u\sin\alpha+v\cos\alpha$ and $v=f(x,y)$, show that $\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}=\frac{\partial^2 v}{\partial u^2}+\frac{\partial^2 v}{\partial v^2}$.

27. If $u = e^{xy}$, show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{u}\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2\right]$.

28. If F is a function of x and y and if $x = e^u \sin v, y = e^u \cos v$, prove that

$$\frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} = e^{-2u}\left[\frac{\partial^2 F}{\partial u^2} + \frac{\partial^2 F}{\partial v^2}\right]$$

29. $u = f(x, y)$, where $x = r\cos\theta, y = r\sin\theta$, prove that

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = \left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2.$$

30. Use Taylor formula to expand the function defined by $f(x, y) = x^3 + y^3 + xy^2$ in powers of (x-1) and (y-2).

31. Expand $x^2 y + 3y - 2$ in powers of (x-1) and (y+2) up to 3rd degree terms.

32. Expand $e^x \sin y$ in powers of x and y as far as the terms of the 3rd degree using Taylor expansion.

33. Expand $e^x \cos y$ at $\left(0, \frac{\pi}{2}\right)$ up to third term using Taylor series.

34. Expand $e^x \log(1+y)$ in powers of x and y up to 3rd degree terms using Taylors theorem.

35. Expand $\sin xy$ at $\left(1, \frac{\pi}{2}\right)$ up to second degree terms using Taylor series.

36. If $x + y + z = u, y + z = uv, z = uvw$ prove that $\frac{\partial(x, y, z)}{\partial(u, v, w)} = u^2 v$.

Analyze:

37. Test for maxima minima for the function $f(x, y) = x^3 + y^3 - 12x - 3y + 20$.

38. Test for an extrema of the function $f(x, y) = x^4 + y^4 - x^2 - y^2 - 1$.

39. Examine the function $f(x, y) = x^3 y^2 (12 - x - y)$ for extreme values.

40. Test for maxima minima of the function $f(x, y) = x^3 y^2 (6 - x - y)$.